Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

6-Hyperbolic-functions/6.5-Hyperbolic-secant/180-6.5.7-d-hyper-^m-a+b-c-sech-^n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [220]. This is test number [180].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (220)	0.00 (0)
Mathematica	100.00 (220)	0.00 (0)
Fricas	100.00 (220)	0.00 (0)
Maple	81.82 (180)	18.18 (40)
Maxima	66.82 (147)	33.18 (73)
Mupad	55.00 (121)	45.00 (99)
Giac	46.36 (102)	53.64 (118)
Sympy	4.55 (10)	95.45 (210)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

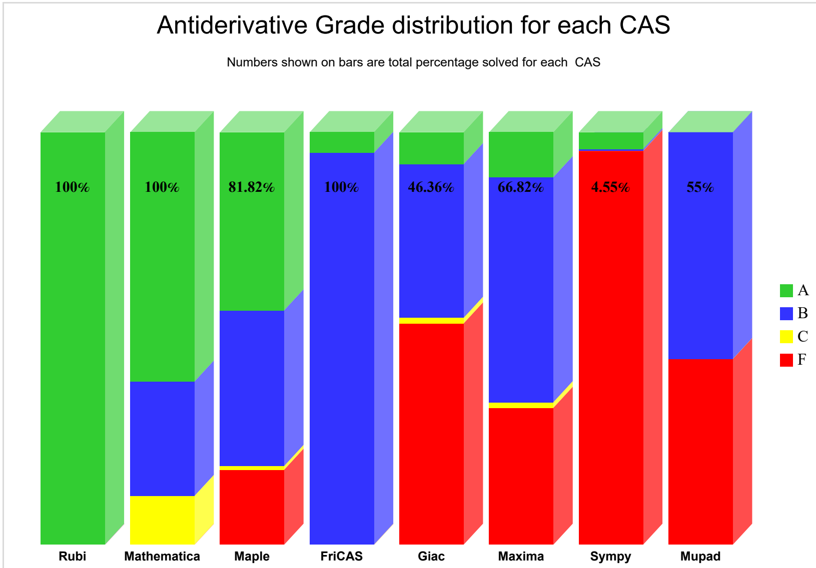
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

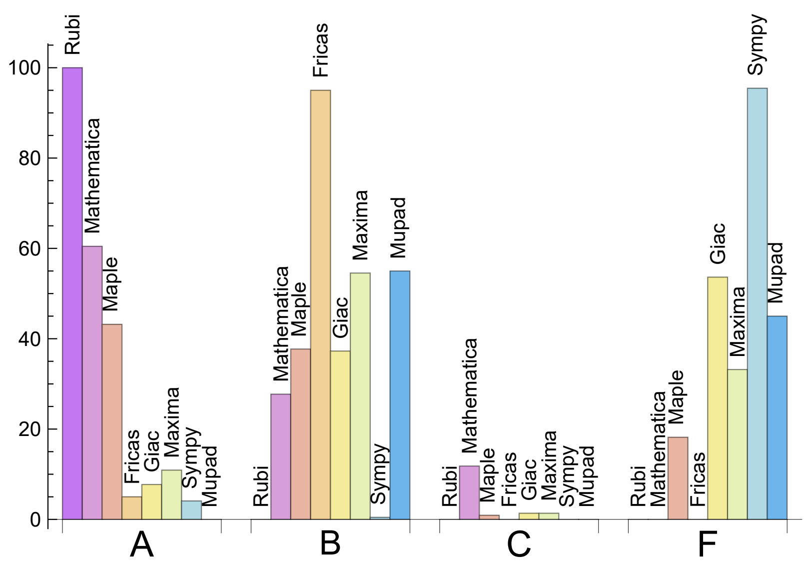
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.182	0.000	1.818	0.000
Mathematica	60.455	27.727	11.818	0.000
Maple	43.182	37.727	0.909	18.182
Maxima	10.909	54.545	1.364	33.182
Giac	7.727	37.273	1.364	53.636
Fricas	5.000	95.000	0.000	0.000
Sympy	4.091	0.455	0.000	95.455
Mupad	0.000	55.000	0.000	45.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	40	100.00	0.00	0.00
Maxima	73	98.63	0.00	1.37
Mupad	99	0.00	100.00	0.00
Giac	118	72.03	0.00	27.97
Sympy	210	82.86	17.14	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.27
Rubi	0.32
Giac	0.33
Fricas	0.33
Mupad	1.77
Sympy	1.94
Mathematica	2.53
Maple	26.35

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	89.60	1.87	81.00	1.54
Rubi	90.27	1.06	77.50	1.03
Maple	169.44	1.84	119.00	1.68
Giac	177.94	2.51	150.00	2.32
Mathematica	262.41	2.54	132.50	1.66
Mupad	347.68	4.70	238.00	3.79
Maxima	450.10	4.45	244.00	3.00
Fricas	3076.75	28.80	1651.00	21.68

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

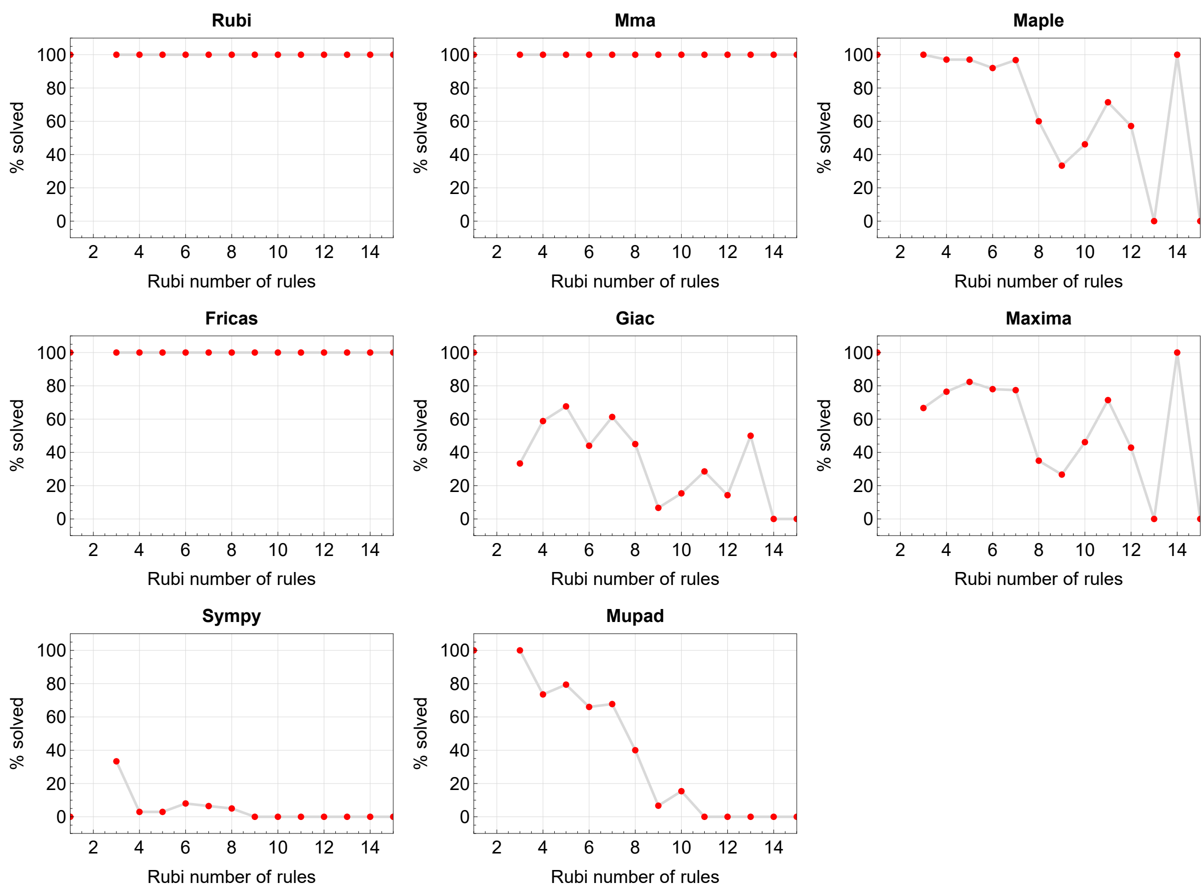


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

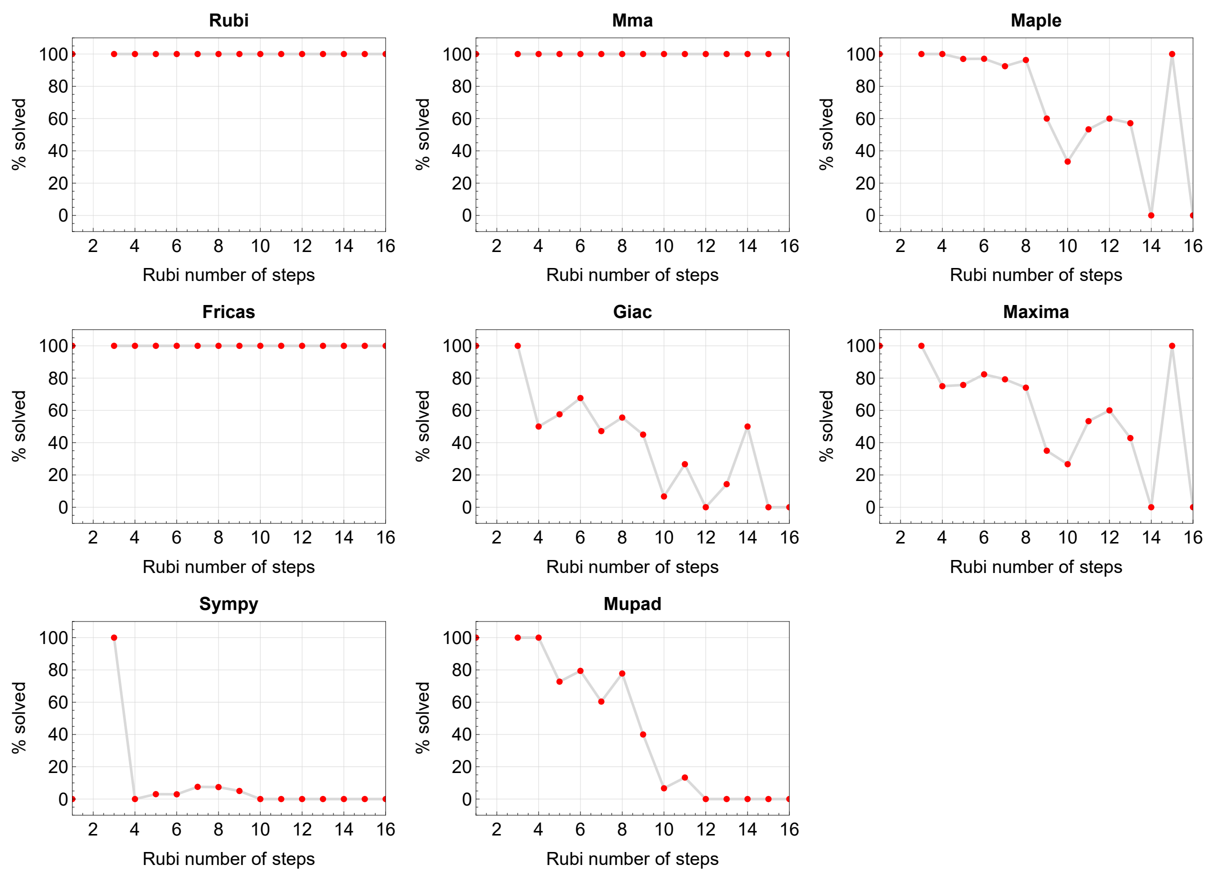


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

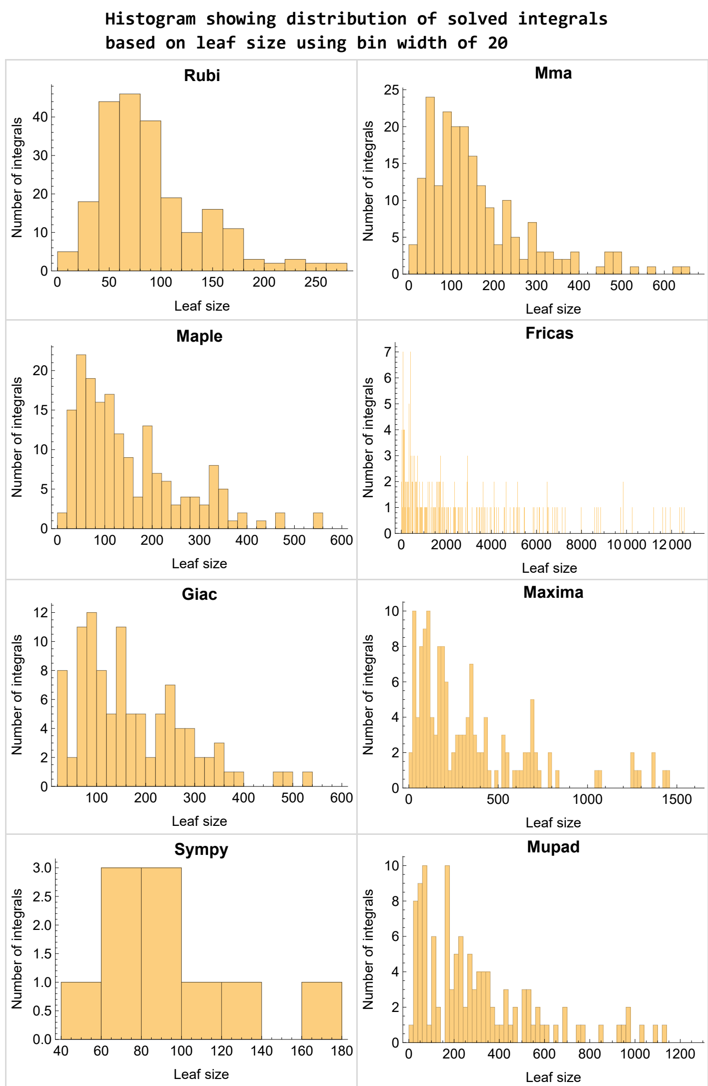


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

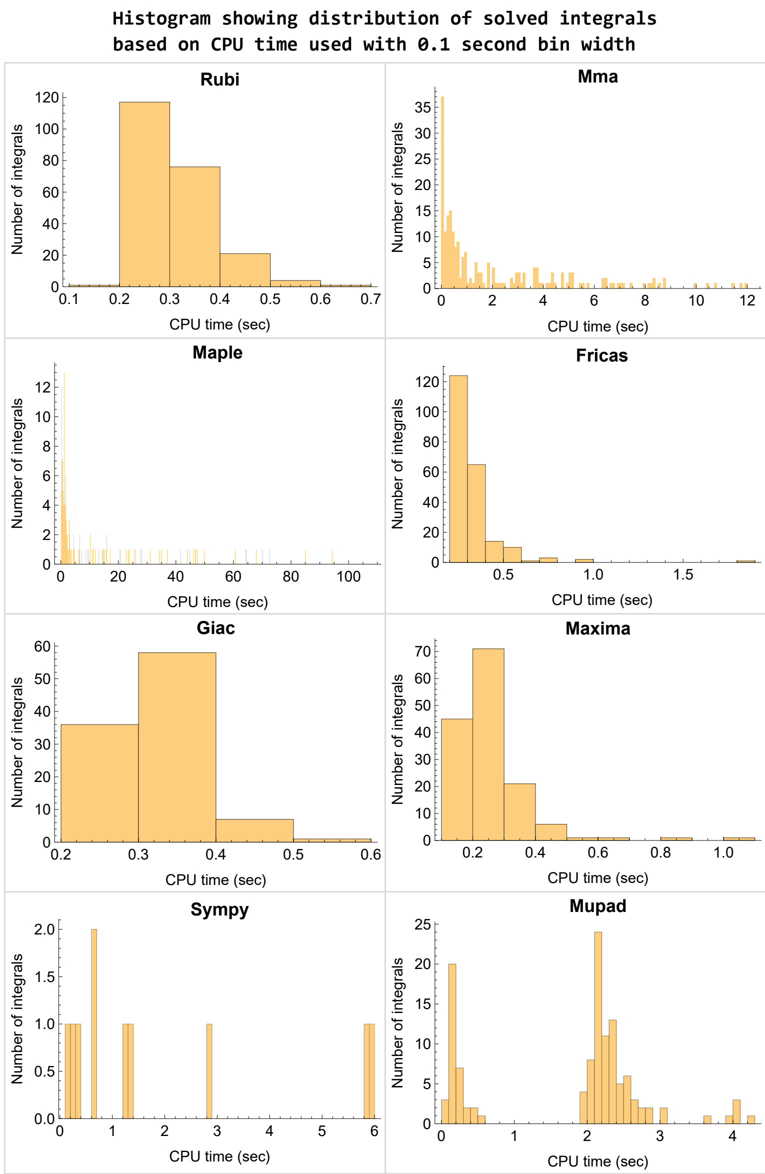


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

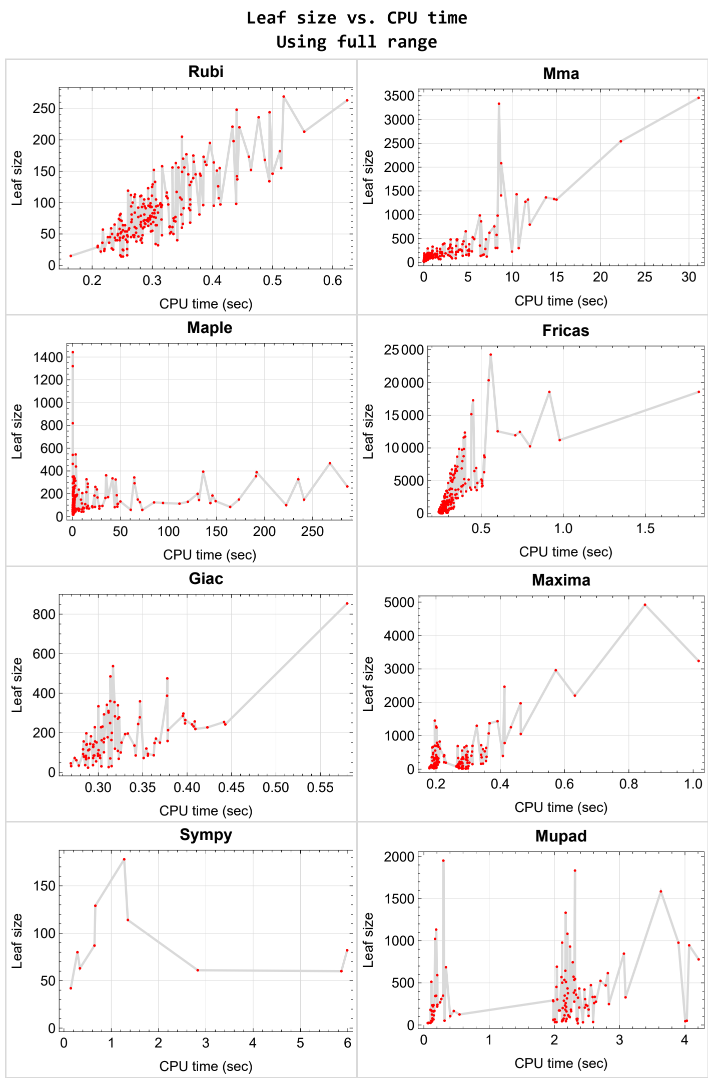


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{103, 113, 115, 117, 125, 127, 129, 131, 138\}$

Mathematica $\{25, 26, 27, 28, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 66, 68, 69, 91, 96, 98, 147, 149, 151, 153, 155, 157, 158, 160, 162, 164, 166, 168, 169\}$

Maple $\{170, 171\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

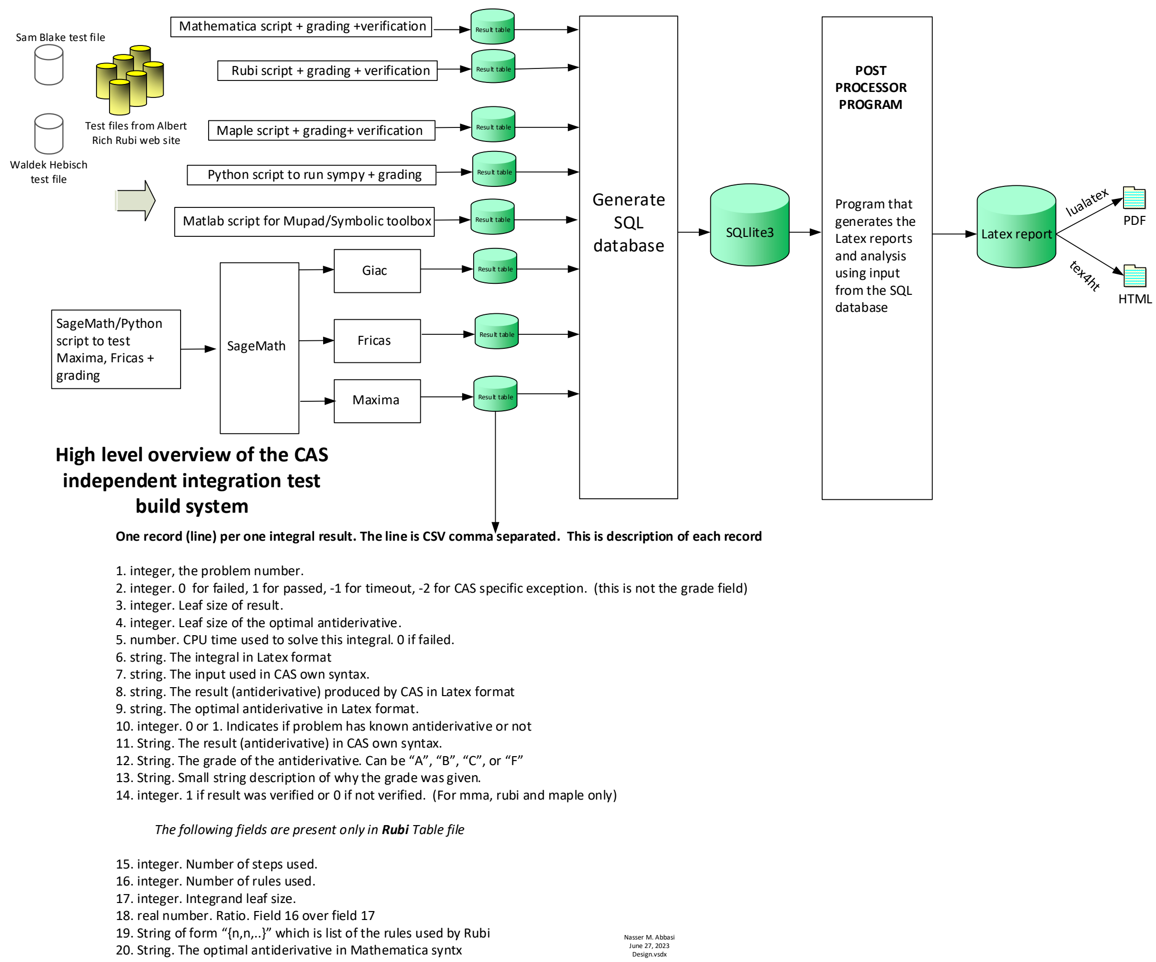
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

B grade { }

C grade { 50, 56, 170, 173 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 15, 18, 20, 21, 23, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 71, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 88, 89, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 111, 113, 115, 116, 117, 119, 121, 123, 125, 127, 128, 129, 131, 133, 135, 136, 137, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 159, 161, 163, 165, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 188, 190, 192, 193, 194, 195, 196, 197, 198, 201, 202, 203, 204, 205, 208, 209, 210, 211, 212, 214, 216, 217, 219, 220 }

B grade { 5, 7, 13, 14, 16, 17, 19, 22, 24, 25, 27, 30, 32, 35, 38, 43, 48, 70, 72, 79, 80, 81, 82, 87, 90, 91, 92, 96, 112, 114, 118, 120, 122, 124, 126, 130, 132, 134, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 160, 162, 164, 180, 181, 191, 199, 200, 206, 207, 213, 215, 218 }

C grade { 26, 28, 29, 31, 33, 34, 36, 37, 39, 40, 41, 42, 44, 45, 46, 47, 66, 68, 69, 108, 110, 158, 166, 168, 169, 189 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 28, 29, 31, 36, 37, 39, 44, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 92, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 136, 137, 142, 152, 163, 174, 180, 189, 198, 207, 216 }

B grade { 22, 24, 25, 26, 27, 30, 32, 33, 34, 35, 38, 40, 41, 42, 43, 45, 46, 48, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 120, 122, 132, 134, 135, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 172, 173, 175 }

C grade { 170, 171 }

F normal fail { 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 3, 4, 49, 50, 51, 57, 59, 65, 171, 172 }

B grade { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 3, 4, 6, 12, 20, 49, 51, 52, 57, 59, 60, 65, 103, 104, 105, 106, 115, 118, 127, 138, 140, 170, 171, 172 }

B grade { 1, 2, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 50, 53, 54, 55, 56, 58, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 75, 78, 80, 82, 84, 87, 89, 91, 93, 96, 98, 100, 102, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169 }

C grade { 173, 174, 175 }

F normal fail { 26, 28, 29, 31, 34, 36, 37, 39, 44, 45, 47, 74, 76, 77, 79, 81, 83, 85, 86, 88, 90, 92, 94, 95, 97, 99, 101, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

F(-1) timedout fail { }

F(-2) exception fail { 42 }

2.1.6 Giac

A grade { 4, 6, 8, 12, 16, 17, 20, 52, 56, 57, 58, 106, 107, 108, 118, 206, 208 }

B grade { 1, 2, 3, 5, 7, 9, 10, 11, 13, 14, 15, 18, 19, 21, 22, 23, 24, 49, 50, 51, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 102, 103, 104, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 170, 171, 172, 205, 207, 210, 212, 213, 214, 215, 216, 217, 219 }

C grade { 173, 174, 175 }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169 }

F(-1) timedout fail { }

F(-2) exception fail { 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 209, 211, 218, 220 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152, 163, 180, 189, 198, 207, 216 }

C grade { }

F normal fail { }

F(-1) timedout fail { 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 51, 103, 105, 113, 115, 125, 127, 207, 216 }

B grade { 142 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 104, 106, 107, 108, 109, 110, 112, 114, 116, 117, 118, 119, 124, 126, 128, 129, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 151, 153, 154, 155, 156, 157, 158, 160, 162, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 220 }

F(-1) timedout fail { 9, 17, 18, 19, 41, 42, 43, 44, 57, 65, 66, 67, 93, 94, 111, 120, 121, 122, 123, 130, 131, 132, 133, 134, 135, 150, 152, 159, 161, 163, 169, 186, 191, 192, 218, 219 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	94	54	78	129	114	0	130	73
N.S.	1	1.34	0.77	1.11	1.84	1.63	0.00	1.86	1.04
time (sec)	N/A	0.277	0.308	10.949	0.201	0.246	0.000	0.294	2.353

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	40	53	56	111	85	0	85	44
N.S.	1	0.91	1.20	1.27	2.52	1.93	0.00	1.93	1.00
time (sec)	N/A	0.250	0.019	2.915	0.187	0.249	0.000	0.296	2.116

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	58	57	45	62	67	0	92	55
N.S.	1	1.35	1.33	1.05	1.44	1.56	0.00	2.14	1.28
time (sec)	N/A	0.245	0.199	0.842	0.187	0.252	0.000	0.283	2.133

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	22	35	25	36	38	0	45	26
N.S.	1	0.92	1.46	1.04	1.50	1.58	0.00	1.88	1.08
time (sec)	N/A	0.220	0.015	1.020	0.197	0.257	0.000	0.269	0.079

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	26	84	36	80	180	0	72	79
N.S.	1	0.96	3.11	1.33	2.96	6.67	0.00	2.67	2.93
time (sec)	N/A	0.223	0.027	1.300	0.201	0.259	0.000	0.273	0.131

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	25	37	44	39	91	0	34	34
N.S.	1	0.93	1.37	1.63	1.44	3.37	0.00	1.26	1.26
time (sec)	N/A	0.244	0.031	3.159	0.201	0.246	0.000	0.278	2.033

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	59	169	70	198	924	0	142	160
N.S.	1	1.09	3.13	1.30	3.67	17.11	0.00	2.63	2.96
time (sec)	N/A	0.256	0.445	6.112	0.207	0.265	0.000	0.287	0.147

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	40	84	73	187	246	0	80	172
N.S.	1	0.89	1.87	1.62	4.16	5.47	0.00	1.78	3.82
time (sec)	N/A	0.248	0.295	11.999	0.198	0.241	0.000	0.284	2.070

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	133	153	109	211	342	0	231	269
N.S.	1	1.17	1.34	0.96	1.85	3.00	0.00	2.03	2.36
time (sec)	N/A	0.375	2.729	13.239	0.207	0.249	0.000	0.305	0.250

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	65	83	69	266	212	0	140	201
N.S.	1	0.90	1.15	0.96	3.69	2.94	0.00	1.94	2.79
time (sec)	N/A	0.289	3.607	27.888	0.204	0.254	0.000	0.294	2.127

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	78	126	90	160	252	0	144	236
N.S.	1	1.07	1.73	1.23	2.19	3.45	0.00	1.97	3.23
time (sec)	N/A	0.294	2.282	20.469	0.203	0.257	0.000	0.291	0.148

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	40	59	43	65	133	0	75	45
N.S.	1	0.89	1.31	0.96	1.44	2.96	0.00	1.67	1.00
time (sec)	N/A	0.241	0.215	11.362	0.186	0.255	0.000	0.286	2.177

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	48	108	72	197	1148	0	139	232
N.S.	1	0.92	2.08	1.38	3.79	22.08	0.00	2.67	4.46
time (sec)	N/A	0.277	3.088	15.116	0.199	0.266	0.000	0.287	2.162

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	45	136	91	140	284	0	111	215
N.S.	1	0.90	2.72	1.82	2.80	5.68	0.00	2.22	4.30
time (sec)	N/A	0.269	4.394	28.010	0.196	0.250	0.000	0.289	2.181

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	116	144	126	354	2930	0	228	316
N.S.	1	1.12	1.38	1.21	3.40	28.17	0.00	2.19	3.04
time (sec)	N/A	0.330	5.079	70.239	0.201	0.280	0.000	0.303	2.192

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	67	151	129	285	408	0	115	115
N.S.	1	0.89	2.01	1.72	3.80	5.44	0.00	1.53	1.53
time (sec)	N/A	0.286	6.648	120.039	0.206	0.253	0.000	0.290	2.155

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	198	651	182	422	727	0	339	686
N.S.	1	1.09	3.58	1.00	2.32	3.99	0.00	1.86	3.77
time (sec)	N/A	0.436	4.737	0.673	0.201	0.262	0.000	0.322	0.339

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	88	119	100	489	403	0	193	348
N.S.	1	0.89	1.20	1.01	4.94	4.07	0.00	1.95	3.52
time (sec)	N/A	0.308	7.117	222.473	0.192	0.255	0.000	0.330	0.294

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	160	480	145	443	595	0	278	592
N.S.	1	1.43	4.29	1.29	3.96	5.31	0.00	2.48	5.29
time (sec)	N/A	0.399	3.724	131.693	0.191	0.257	0.000	0.323	0.205

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	56	93	58	94	276	0	101	288
N.S.	1	0.88	1.45	0.91	1.47	4.31	0.00	1.58	4.50
time (sec)	N/A	0.262	0.318	72.640	0.183	0.259	0.000	0.306	2.145

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	76	134	118	358	3443	0	228	434
N.S.	1	0.92	1.61	1.42	4.31	41.48	0.00	2.75	5.23
time (sec)	N/A	0.305	6.426	143.549	0.190	0.288	0.000	0.296	2.201

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	62	380	148	358	622	0	249	644
N.S.	1	0.89	5.43	2.11	5.11	8.89	0.00	3.56	9.20
time (sec)	N/A	0.288	5.135	241.080	0.187	0.274	0.000	0.313	2.168

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	139	224	192	556	6717	0	341	536
N.S.	1	0.97	1.56	1.33	3.86	46.65	0.00	2.37	3.72
time (sec)	N/A	0.366	9.974	0.404	0.198	0.321	0.000	0.310	2.304

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	93	235	213	664	955	0	355	745
N.S.	1	0.89	2.26	2.05	6.38	9.18	0.00	3.41	7.16
time (sec)	N/A	0.312	4.314	0.849	0.190	0.251	0.000	0.318	2.281

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	117	145	294	327	526	1681	0	0	328
N.S.	1	1.24	2.51	2.79	4.50	14.37	0.00	0.00	2.80
time (sec)	N/A	0.378	3.207	235.072	0.299	0.290	0.000	0.000	3.090

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	71	69	372	170	0	1246	0	0	473
N.S.	1	0.97	5.24	2.39	0.00	17.55	0.00	0.00	6.66
time (sec)	N/A	0.276	3.011	37.000	0.000	0.281	0.000	0.000	2.559

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	75	93	236	148	352	805	0	0	276
N.S.	1	1.24	3.15	1.97	4.69	10.73	0.00	0.00	3.68
time (sec)	N/A	0.301	0.973	8.788	0.283	0.283	0.000	0.000	2.637

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	47	45	328	44	0	595	0	0	42
N.S.	1	0.96	6.98	0.94	0.00	12.66	0.00	0.00	0.89
time (sec)	N/A	0.235	0.989	1.796	0.000	0.268	0.000	0.000	0.127

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	232	65	0	533	0	0	616
N.S.	1	0.98	4.22	1.18	0.00	9.69	0.00	0.00	11.20
time (sec)	N/A	0.258	1.514	1.241	0.000	0.283	0.000	0.000	2.819

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	51	179	120	100	588	0	0	847
N.S.	1	0.96	3.38	2.26	1.89	11.09	0.00	0.00	15.98
time (sec)	N/A	0.256	1.262	0.981	0.284	0.271	0.000	0.000	3.065

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	98	338	111	0	1881	0	0	1586
N.S.	1	1.13	3.89	1.28	0.00	21.62	0.00	0.00	18.23
time (sec)	N/A	0.302	2.979	1.575	0.000	0.320	0.000	0.000	3.631

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	75	77	216	151	195	1753	0	0	248
N.S.	1	1.03	2.88	2.01	2.60	23.37	0.00	0.00	3.31
time (sec)	N/A	0.282	2.931	2.287	0.304	0.280	0.000	0.000	2.835

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	194	220	1330	462	1299	5169	0	0	0
N.S.	1	1.13	6.86	2.38	6.70	26.64	0.00	0.00	0.00
time (sec)	N/A	0.458	14.736	0.231	0.327	0.328	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	114	107	861	264	0	3804	0	0	0
N.S.	1	0.94	7.55	2.32	0.00	33.37	0.00	0.00	0.00
time (sec)	N/A	0.335	6.479	286.110	0.000	0.314	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	131	149	791	325	696	2925	0	0	0
N.S.	1	1.14	6.04	2.48	5.31	22.33	0.00	0.00	0.00
time (sec)	N/A	0.361	11.985	44.871	0.295	0.322	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	84	82	479	70	0	1780	0	0	71
N.S.	1	0.98	5.70	0.83	0.00	21.19	0.00	0.00	0.85
time (sec)	N/A	0.255	3.031	17.108	0.000	0.286	0.000	0.000	2.341

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	106	377	167	0	2376	0	0	0
N.S.	1	1.07	3.81	1.69	0.00	24.00	0.00	0.00	0.00
time (sec)	N/A	0.300	2.152	26.038	0.000	0.320	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	220	238	262	2407	0	0	0
N.S.	1	1.00	2.39	2.59	2.85	26.16	0.00	0.00	0.00
time (sec)	N/A	0.280	3.870	25.407	0.298	0.324	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	147	156	462	214	0	6878	0	0	0
N.S.	1	1.06	3.14	1.46	0.00	46.79	0.00	0.00	0.00
time (sec)	N/A	0.375	3.738	14.421	0.000	0.381	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	123	126	620	327	430	6143	0	0	0
N.S.	1	1.02	5.04	2.66	3.50	49.94	0.00	0.00	0.00
time (sec)	N/A	0.414	7.419	14.547	0.340	0.339	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	242	269	3457	541	2468	12353	0	0	0
N.S.	1	1.11	14.29	2.24	10.20	51.05	0.00	0.00	0.00
time (sec)	N/A	0.539	31.092	0.229	0.413	0.400	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	154	152	1364	348	0	8667	0	0	0
N.S.	1	0.99	8.86	2.26	0.00	56.28	0.00	0.00	0.00
time (sec)	N/A	0.481	13.812	0.207	0.000	0.354	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	187	221	2544	1320	1373	9730	0	0	0
N.S.	1	1.18	13.60	7.06	7.34	52.03	0.00	0.00	0.00
time (sec)	N/A	0.447	22.289	0.235	0.366	0.383	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	116	119	1272	84	0	4829	0	0	103
N.S.	1	1.03	10.97	0.72	0.00	41.63	0.00	0.00	0.89
time (sec)	N/A	0.271	11.492	164.165	0.000	0.324	0.000	0.000	2.221

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	154	173	440	291	0	8742	0	0	0
N.S.	1	1.12	2.86	1.89	0.00	56.77	0.00	0.00	0.00
time (sec)	N/A	0.397	4.490	0.265	0.000	0.393	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	126	133	981	819	533	7275	0	0	0
N.S.	1	1.06	7.79	6.50	4.23	57.74	0.00	0.00	0.00
time (sec)	N/A	0.311	8.346	0.258	0.352	0.341	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	213	236	524	350	0	20341	0	0	0
N.S.	1	1.11	2.46	1.64	0.00	95.50	0.00	0.00	0.00
time (sec)	N/A	0.503	5.530	0.247	0.000	0.545	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	165	168	985	1443	782	15161	0	0	0
N.S.	1	1.02	5.97	8.75	4.74	91.88	0.00	0.00	0.00
time (sec)	N/A	0.496	6.336	0.296	0.414	0.440	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	59	45	44	97	61	0	116	50
N.S.	1	0.97	0.74	0.72	1.59	1.00	0.00	1.90	0.82
time (sec)	N/A	0.295	0.139	0.739	0.190	0.267	0.000	0.283	0.116

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	36	50	31	85	41	0	72	34
N.S.	1	1.20	1.67	1.03	2.83	1.37	0.00	2.40	1.13
time (sec)	N/A	0.289	0.009	0.704	0.191	0.263	0.000	0.302	2.008

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	27	38	28	60	66	23
N.S.	1	1.00	1.06	0.87	1.23	0.90	1.94	2.13	0.74
time (sec)	N/A	0.215	0.028	0.325	0.191	0.257	5.865	0.286	0.071

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	28	93	0	36	62
N.S.	1	1.00	1.46	1.00	1.17	3.88	0.00	1.50	2.58
time (sec)	N/A	0.257	0.015	0.357	0.271	0.264	0.000	0.285	1.986

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	40	81	321	0	84	124
N.S.	1	1.00	1.20	1.00	2.02	8.02	0.00	2.10	3.10
time (sec)	N/A	0.265	0.010	0.822	0.272	0.257	0.000	0.295	0.124

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	43	39	34	112	158	0	61	61
N.S.	1	1.43	1.30	1.13	3.73	5.27	0.00	2.03	2.03
time (sec)	N/A	0.298	0.014	0.983	0.192	0.250	0.000	0.275	1.984

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	68	93	68	184	1112	0	156	283
N.S.	1	0.97	1.33	0.97	2.63	15.89	0.00	2.23	4.04
time (sec)	N/A	0.378	0.019	1.247	0.274	0.284	0.000	0.284	1.984

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	64	71	56	300	343	0	85	292
N.S.	1	1.28	1.42	1.12	6.00	6.86	0.00	1.70	5.84
time (sec)	N/A	0.310	0.016	1.124	0.194	0.258	0.000	0.302	1.980

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	105	58	54	105	78	0	151	66
N.S.	1	1.28	0.71	0.66	1.28	0.95	0.00	1.84	0.80
time (sec)	N/A	0.302	0.315	0.756	0.193	0.256	0.000	0.298	2.006

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	44	72	50	105	414	0	94	114
N.S.	1	0.90	1.47	1.02	2.14	8.45	0.00	1.92	2.33
time (sec)	N/A	0.264	0.065	1.026	0.270	0.265	0.000	0.300	0.151

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	59	52	51	63	80	0	128	65
N.S.	1	1.26	1.11	1.09	1.34	1.70	0.00	2.72	1.38
time (sec)	N/A	0.283	0.239	0.774	0.180	0.257	0.000	0.294	0.147

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	57	80	53	101	653	0	112	172
N.S.	1	1.02	1.43	0.95	1.80	11.66	0.00	2.00	3.07
time (sec)	N/A	0.267	0.075	0.997	0.265	0.278	0.000	0.300	2.081

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	100	109	84	201	1372	0	170	303
N.S.	1	1.11	1.21	0.93	2.23	15.24	0.00	1.89	3.37
time (sec)	N/A	0.271	0.050	1.260	0.274	0.266	0.000	0.307	2.044

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	48	93	70	324	404	0	156	452
N.S.	1	0.91	1.75	1.32	6.11	7.62	0.00	2.94	8.53
time (sec)	N/A	0.282	0.051	1.305	0.207	0.243	0.000	0.296	2.031

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	129	187	122	348	2946	0	293	569
N.S.	1	1.01	1.46	0.95	2.72	23.02	0.00	2.29	4.45
time (sec)	N/A	0.314	0.046	1.951	0.298	0.280	0.000	0.307	2.108

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	72	144	102	671	677	0	197	692
N.S.	1	0.90	1.80	1.28	8.39	8.46	0.00	2.46	8.65
time (sec)	N/A	0.295	0.041	1.541	0.208	0.245	0.000	0.287	2.037

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	99	70	93	130	153	0	177	117
N.S.	1	1.18	0.83	1.11	1.55	1.82	0.00	2.11	1.39
time (sec)	N/A	0.316	1.688	1.307	0.205	0.253	0.000	0.311	2.212

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	81	79	483	79	179	1409	0	163	218
N.S.	1	0.98	5.96	0.98	2.21	17.40	0.00	2.01	2.69
time (sec)	N/A	0.295	7.023	2.067	0.282	0.271	0.000	0.315	0.205

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	81	64	77	160	270	0	152	221
N.S.	1	1.12	0.89	1.07	2.22	3.75	0.00	2.11	3.07
time (sec)	N/A	0.295	1.874	1.274	0.195	0.286	0.000	0.306	0.140

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	95	575	97	221	1992	0	199	344
N.S.	1	1.02	6.18	1.04	2.38	21.42	0.00	2.14	3.70
time (sec)	N/A	0.310	8.229	1.796	0.274	0.292	0.000	0.315	0.175

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	147	163	1430	138	365	3465	0	310	535
N.S.	1	1.11	9.73	0.94	2.48	23.57	0.00	2.11	3.64
time (sec)	N/A	0.347	10.494	2.122	0.295	0.290	0.000	0.311	2.153

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	66	162	116	695	816	0	302	978
N.S.	1	0.89	2.19	1.57	9.39	11.03	0.00	4.08	13.22
time (sec)	N/A	0.303	3.623	1.782	0.206	0.275	0.000	0.312	2.119

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	195	297	186	556	6114	0	485	931
N.S.	1	0.99	1.52	0.95	2.84	31.19	0.00	2.47	4.75
time (sec)	N/A	0.407	10.734	3.166	0.281	0.311	0.000	0.314	2.240

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	97	238	158	1245	1190	0	360	1333
N.S.	1	0.90	2.20	1.46	11.53	11.02	0.00	3.33	12.34
time (sec)	N/A	0.325	5.156	2.060	0.201	0.270	0.000	0.313	2.172

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	145	95	234	526	1713	0	0	260
N.S.	1	1.24	0.81	2.00	4.50	14.64	0.00	0.00	2.22
time (sec)	N/A	0.375	0.841	1.170	0.313	0.294	0.000	0.000	2.609

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	71	79	209	0	1616	0	0	332
N.S.	1	0.93	1.04	2.75	0.00	21.26	0.00	0.00	4.37
time (sec)	N/A	0.301	0.335	0.850	0.000	0.290	0.000	0.000	2.585

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	93	67	152	352	829	0	0	206
N.S.	1	1.24	0.89	2.03	4.69	11.05	0.00	0.00	2.75
time (sec)	N/A	0.308	0.465	0.651	0.283	0.277	0.000	0.000	2.466

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	50	52	125	0	718	0	0	277
N.S.	1	0.96	1.00	2.40	0.00	13.81	0.00	0.00	5.33
time (sec)	N/A	0.248	0.165	0.548	0.000	0.284	0.000	0.000	2.304

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	80	0	487	0	0	108
N.S.	1	1.00	1.00	2.22	0.00	13.53	0.00	0.00	3.00
time (sec)	N/A	0.225	0.077	0.287	0.000	0.281	0.000	0.000	2.523

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	102	66	411	0	0	125
N.S.	1	1.00	1.00	2.83	1.83	11.42	0.00	0.00	3.47
time (sec)	N/A	0.243	0.040	0.284	0.294	0.263	0.000	0.000	0.546

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	53	194	104	0	526	0	0	307
N.S.	1	0.96	3.53	1.89	0.00	9.56	0.00	0.00	5.58
time (sec)	N/A	0.260	0.815	0.544	0.000	0.295	0.000	0.000	2.470

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	50	182	138	91	645	0	0	166
N.S.	1	0.96	3.50	2.65	1.75	12.40	0.00	0.00	3.19
time (sec)	N/A	0.251	2.068	0.751	0.301	0.282	0.000	0.000	0.458

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	93	213	159	0	1518	0	0	946
N.S.	1	1.08	2.48	1.85	0.00	17.65	0.00	0.00	11.00
time (sec)	N/A	0.290	2.028	1.086	0.000	0.280	0.000	0.000	4.065

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	72	214	183	160	1905	0	0	334
N.S.	1	0.94	2.78	2.38	2.08	24.74	0.00	0.00	4.34
time (sec)	N/A	0.296	3.645	1.450	0.340	0.274	0.000	0.000	2.619

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	117	113	321	0	5842	0	0	0
N.S.	1	0.94	0.90	2.57	0.00	46.74	0.00	0.00	0.00
time (sec)	N/A	0.352	0.956	1.796	0.000	0.330	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	165	103	338	696	3739	0	0	0
N.S.	1	1.15	0.72	2.35	4.83	25.97	0.00	0.00	0.00
time (sec)	N/A	0.400	1.584	1.264	0.314	0.323	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	95	88	237	0	3154	0	0	0
N.S.	1	0.95	0.88	2.37	0.00	31.54	0.00	0.00	0.00
time (sec)	N/A	0.315	0.585	0.932	0.000	0.295	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	80	124	201	0	1856	0	0	0
N.S.	1	0.98	1.51	2.45	0.00	22.63	0.00	0.00	0.00
time (sec)	N/A	0.255	0.335	0.553	0.000	0.278	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	72	187	206	150	1489	0	0	0
N.S.	1	0.97	2.53	2.78	2.03	20.12	0.00	0.00	0.00
time (sec)	N/A	0.261	2.454	0.511	0.313	0.268	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	71	113	181	0	1570	0	0	0
N.S.	1	0.97	1.55	2.48	0.00	21.51	0.00	0.00	0.00
time (sec)	N/A	0.253	0.352	0.544	0.000	0.273	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	81	88	222	165	1569	0	0	0
N.S.	1	0.98	1.06	2.67	1.99	18.90	0.00	0.00	0.00
time (sec)	N/A	0.272	0.198	0.565	0.346	0.285	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	111	282	216	0	2069	0	0	0
N.S.	1	1.10	2.79	2.14	0.00	20.49	0.00	0.00	0.00
time (sec)	N/A	0.305	2.770	1.296	0.000	0.307	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	101	96	229	250	244	2958	0	0	0
N.S.	1	0.95	2.27	2.48	2.42	29.29	0.00	0.00	0.00
time (sec)	N/A	0.336	5.444	1.923	0.356	0.315	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	489	271	0	6499	0	0	0
N.S.	1	1.08	3.20	1.77	0.00	42.48	0.00	0.00	0.00
time (sec)	N/A	0.377	5.707	2.889	0.000	0.352	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	244	156	439	1373	11740	0	0	0
N.S.	1	1.20	0.76	2.15	6.73	57.55	0.00	0.00	0.00
time (sec)	N/A	0.509	4.796	3.587	0.367	0.403	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	146	139	335	0	9856	0	0	0
N.S.	1	0.95	0.90	2.18	0.00	64.00	0.00	0.00	0.00
time (sec)	N/A	0.387	2.058	2.288	0.000	0.366	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	152	214	308	0	6806	0	0	0
N.S.	1	1.07	1.51	2.17	0.00	47.93	0.00	0.00	0.00
time (sec)	N/A	0.321	3.211	1.272	0.000	0.345	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	108	113	258	260	353	5109	0	0	0
N.S.	1	1.05	2.39	2.41	3.27	47.31	0.00	0.00	0.00
time (sec)	N/A	0.283	3.957	1.225	0.342	0.321	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	159	294	0	6037	0	0	0
N.S.	1	1.00	1.29	2.39	0.00	49.08	0.00	0.00	0.00
time (sec)	N/A	0.291	0.847	1.277	0.000	0.318	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	250	310	369	5447	0	0	0
N.S.	1	1.00	2.00	2.48	2.95	43.58	0.00	0.00	0.00
time (sec)	N/A	0.292	5.045	1.295	0.357	0.314	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	111	131	235	0	5006	0	0	0
N.S.	1	1.05	1.24	2.22	0.00	47.23	0.00	0.00	0.00
time (sec)	N/A	0.280	1.364	1.272	0.000	0.315	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	156	125	328	395	5887	0	0	0
N.S.	1	1.08	0.87	2.28	2.74	40.88	0.00	0.00	0.00
time (sec)	N/A	0.330	0.985	1.296	0.408	0.332	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	175	247	314	0	7993	0	0	0
N.S.	1	1.14	1.61	2.05	0.00	52.24	0.00	0.00	0.00
time (sec)	N/A	0.377	4.965	2.956	0.000	0.386	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	57	62	92	327	0	108	433
N.S.	1	1.00	1.19	1.29	1.92	6.81	0.00	2.25	9.02
time (sec)	N/A	0.295	0.029	4.524	0.190	0.256	0.000	0.322	2.381

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	46	45	49	78	1072	80	119	173
N.S.	1	0.94	0.92	1.00	1.59	21.88	1.63	2.43	3.53
time (sec)	N/A	0.267	0.017	2.523	0.271	0.273	0.285	0.319	0.114

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	38	41	52	42	155	0	72	163
N.S.	1	1.19	1.28	1.62	1.31	4.84	0.00	2.25	5.09
time (sec)	N/A	0.281	0.010	1.976	0.198	0.259	0.000	0.294	2.258

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	27	29	28	27	359	42	83	72
N.S.	1	0.93	1.00	0.97	0.93	12.38	1.45	2.86	2.48
time (sec)	N/A	0.226	0.023	0.980	0.180	0.256	0.144	0.300	2.187

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	23	36	0	23	23
N.S.	1	1.00	1.00	1.07	1.53	2.40	0.00	1.53	1.53
time (sec)	N/A	0.159	0.001	0.779	0.194	0.252	0.000	0.288	0.059

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	37	44	24	65	69	0	55	167
N.S.	1	1.32	1.57	0.86	2.32	2.46	0.00	1.96	5.96
time (sec)	N/A	0.259	0.039	0.795	0.268	0.261	0.000	0.298	0.162

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	26	41	30	47	39	0	30	25
N.S.	1	1.44	2.28	1.67	2.61	2.17	0.00	1.67	1.39
time (sec)	N/A	0.273	0.018	1.595	0.188	0.261	0.000	0.304	0.102

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	41	52	37	108	378	0	84	76
N.S.	1	1.32	1.68	1.19	3.48	12.19	0.00	2.71	2.45
time (sec)	N/A	0.260	0.170	1.816	0.200	0.273	0.000	0.356	2.142

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	38	49	64	170	140	0	70	161
N.S.	1	1.12	1.44	1.88	5.00	4.12	0.00	2.06	4.74
time (sec)	N/A	0.270	0.023	3.337	0.194	0.259	0.000	0.314	2.185

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	66	62	67	251	1099	0	120	179
N.S.	1	1.29	1.22	1.31	4.92	21.55	0.00	2.35	3.51
time (sec)	N/A	0.281	0.272	4.538	0.189	0.271	0.000	0.354	0.102

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	75	395	93	649	721	0	278	1022
N.S.	1	0.97	5.13	1.21	8.43	9.36	0.00	3.61	13.27
time (sec)	N/A	0.343	2.322	31.116	0.188	0.282	0.000	0.347	0.172

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	71	107	81	333	2591	129	244	349
N.S.	1	0.92	1.39	1.05	4.32	33.65	1.68	3.17	4.53
time (sec)	N/A	0.296	0.319	20.833	0.269	0.278	0.664	0.345	2.158

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	61	281	83	325	435	0	196	513
N.S.	1	1.03	4.76	1.41	5.51	7.37	0.00	3.32	8.69
time (sec)	N/A	0.322	1.841	14.876	0.199	0.268	0.000	0.333	0.115

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	46	82	42	55	1180	63	162	182
N.S.	1	0.96	1.71	0.88	1.15	24.58	1.31	3.38	3.79
time (sec)	N/A	0.259	0.144	9.357	0.188	0.268	0.338	0.304	2.253

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	50	45	120	176	0	79	163
N.S.	1	1.10	1.25	1.12	3.00	4.40	0.00	1.98	4.08
time (sec)	N/A	0.232	0.442	1.097	0.185	0.260	0.000	0.284	0.124

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	55	84	50	161	665	0	150	308
N.S.	1	1.04	1.58	0.94	3.04	12.55	0.00	2.83	5.81
time (sec)	N/A	0.283	0.255	6.523	0.270	0.271	0.000	0.326	0.268

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	40	82	64	71	106	0	68	60
N.S.	1	1.11	2.28	1.78	1.97	2.94	0.00	1.89	1.67
time (sec)	N/A	0.322	3.203	7.219	0.188	0.258	0.000	0.323	2.126

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	64	82	64	206	637	0	148	240
N.S.	1	1.16	1.49	1.16	3.75	11.58	0.00	2.69	4.36
time (sec)	N/A	0.296	0.229	10.165	0.280	0.267	0.000	0.363	0.207

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	160	94	268	201	0	100	183
N.S.	1	1.09	3.48	2.04	5.83	4.37	0.00	2.17	3.98
time (sec)	N/A	0.327	1.443	15.994	0.194	0.257	0.000	0.325	2.166

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	69	77	84	282	1252	0	150	207
N.S.	1	1.33	1.48	1.62	5.42	24.08	0.00	2.88	3.98
time (sec)	N/A	0.293	0.278	23.417	0.202	0.263	0.000	0.369	2.380

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	66	256	163	613	425	0	170	511
N.S.	1	1.03	4.00	2.55	9.58	6.64	0.00	2.66	7.98
time (sec)	N/A	0.335	4.266	34.228	0.213	0.243	0.000	0.365	2.177

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	105	107	132	696	2548	0	219	377
N.S.	1	1.22	1.24	1.53	8.09	29.63	0.00	2.55	4.38
time (sec)	N/A	0.311	0.544	49.839	0.201	0.277	0.000	0.409	2.207

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	105	301	135	1453	1323	0	475	1834
N.S.	1	0.95	2.74	1.23	13.21	12.03	0.00	4.32	16.67
time (sec)	N/A	0.378	8.129	149.029	0.196	0.257	0.000	0.378	2.315

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	94	128	113	652	4658	178	387	573
N.S.	1	0.91	1.24	1.10	6.33	45.22	1.73	3.76	5.56
time (sec)	N/A	0.297	0.783	111.364	0.288	0.299	1.274	0.377	2.293

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	91	479	124	788	881	0	359	1133
N.S.	1	0.99	5.21	1.35	8.57	9.58	0.00	3.90	12.32
time (sec)	N/A	0.356	3.783	84.906	0.203	0.259	0.000	0.347	0.189

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	64	100	59	85	2519	87	271	347
N.S.	1	0.90	1.41	0.83	1.20	35.48	1.23	3.82	4.89
time (sec)	N/A	0.260	1.313	60.575	0.188	0.293	0.645	0.322	0.189

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	74	102	83	332	470	0	182	502
N.S.	1	1.01	1.40	1.14	4.55	6.44	0.00	2.49	6.88
time (sec)	N/A	0.261	1.111	1.508	0.194	0.246	0.000	0.292	2.120

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	114	86	300	2376	0	283	360
N.S.	1	0.99	1.36	1.02	3.57	28.29	0.00	3.37	4.29
time (sec)	N/A	0.306	0.690	46.661	0.280	0.277	0.000	0.319	2.333

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	62	162	111	172	359	0	135	234
N.S.	1	1.02	2.66	1.82	2.82	5.89	0.00	2.21	3.84
time (sec)	N/A	0.353	6.786	47.320	0.200	0.262	0.000	0.341	0.128

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	83	110	110	314	1701	0	247	324
N.S.	1	1.02	1.36	1.36	3.88	21.00	0.00	3.05	4.00
time (sec)	N/A	0.317	1.347	46.877	0.285	0.276	0.000	0.398	2.361

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	61	343	149	366	354	0	158	260
N.S.	1	1.02	5.72	2.48	6.10	5.90	0.00	2.63	4.33
time (sec)	N/A	0.344	3.654	67.918	0.203	0.258	0.000	0.378	2.172

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	94	101	119	422	1830	0	227	384
N.S.	1	1.16	1.25	1.47	5.21	22.59	0.00	2.80	4.74
time (sec)	N/A	0.323	0.731	94.218	0.293	0.264	0.000	0.423	2.298

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	71	303	199	826	521	0	213	547
N.S.	1	1.03	4.39	2.88	11.97	7.55	0.00	3.09	7.93
time (sec)	N/A	0.355	2.900	130.205	0.212	0.259	0.000	0.379	2.316

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	94	98	149	727	2632	0	242	411
N.S.	1	1.22	1.27	1.94	9.44	34.18	0.00	3.14	5.34
time (sec)	N/A	0.317	0.855	173.231	0.206	0.275	0.000	0.443	2.316

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	109	175	129	703	941	0	334	1083
N.S.	1	0.98	1.58	1.16	6.33	8.48	0.00	3.01	9.76
time (sec)	N/A	0.286	5.076	1.804	0.196	0.268	0.000	0.300	2.199

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	158	269	185	1277	1652	0	537	1952
N.S.	1	0.97	1.65	1.13	7.83	10.13	0.00	3.29	11.98
time (sec)	N/A	0.329	5.115	2.408	0.201	0.270	0.000	0.317	0.299

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	63	98	183	131	736	0	0	421
N.S.	1	0.90	1.40	2.61	1.87	10.51	0.00	0.00	6.01
time (sec)	N/A	0.298	0.331	4.321	0.269	0.330	0.000	0.000	2.460

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	70	196	184	637	683	0	0	183
N.S.	1	1.19	3.32	3.12	10.80	11.58	0.00	0.00	3.10
time (sec)	N/A	0.358	1.871	2.880	0.356	0.315	0.000	0.000	2.524

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	115	77	112	0	0	238
N.S.	1	1.00	0.91	2.56	1.71	2.49	0.00	0.00	5.29
time (sec)	N/A	0.282	0.136	1.741	0.264	0.296	0.000	0.000	2.482

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	54	174	104	291	419	0	0	105
N.S.	1	1.17	3.78	2.26	6.33	9.11	0.00	0.00	2.28
time (sec)	N/A	0.321	0.838	1.219	0.293	0.269	0.000	0.000	0.404

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	38	51	76	114	0	51
N.S.	1	1.00	1.13	1.65	2.22	3.30	4.96	0.00	2.22
time (sec)	N/A	0.223	0.216	0.606	0.195	0.257	1.351	0.000	0.317

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	172	108	83	436	0	0	470
N.S.	1	1.00	3.74	2.35	1.80	9.48	0.00	0.00	10.22
time (sec)	N/A	0.270	1.501	0.367	0.281	0.301	0.000	0.000	2.785

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	52	42	119	100	115	0	0	228
N.S.	1	1.13	0.91	2.59	2.17	2.50	0.00	0.00	4.96
time (sec)	N/A	0.292	0.145	1.137	0.198	0.289	0.000	0.000	2.506

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	77	193	133	429	749	0	0	977
N.S.	1	1.24	3.11	2.15	6.92	12.08	0.00	0.00	15.76
time (sec)	N/A	0.360	2.063	1.792	0.295	0.306	0.000	0.000	3.900

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	100	171	187	862	0	0	523
N.S.	1	1.05	1.37	2.34	2.56	11.81	0.00	0.00	7.16
time (sec)	N/A	0.315	0.282	2.879	0.200	0.319	0.000	0.000	2.706

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	87	107	380	192	1435	2705	0	0	779
N.S.	1	1.23	4.37	2.21	16.49	31.09	0.00	0.00	8.95
time (sec)	N/A	0.435	4.373	4.693	0.391	0.309	0.000	0.000	4.209

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	70	109	184	154	853	0	0	0
N.S.	1	0.92	1.43	2.42	2.03	11.22	0.00	0.00	0.00
time (sec)	N/A	0.319	0.495	22.782	0.276	0.323	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	105	228	259	1053	1479	0	0	0
N.S.	1	1.15	2.51	2.85	11.57	16.25	0.00	0.00	0.00
time (sec)	N/A	0.375	3.164	15.772	0.464	0.295	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	48	81	115	108	485	0	0	0
N.S.	1	0.94	1.59	2.25	2.12	9.51	0.00	0.00	0.00
time (sec)	N/A	0.286	0.995	9.768	0.199	0.262	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	97	326	236	597	1846	0	0	0
N.S.	1	1.14	3.84	2.78	7.02	21.72	0.00	0.00	0.00
time (sec)	N/A	0.346	4.767	6.527	0.346	0.303	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	44	79	60	106	476	0	0	53
N.S.	1	0.90	1.61	1.22	2.16	9.71	0.00	0.00	1.08
time (sec)	N/A	0.260	0.660	6.326	0.189	0.267	0.000	0.000	2.347

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	112	221	254	187	1690	0	0	0
N.S.	1	1.20	2.38	2.73	2.01	18.17	0.00	0.00	0.00
time (sec)	N/A	0.279	3.741	0.572	0.298	0.303	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	84	115	206	209	1031	0	0	0
N.S.	1	1.01	1.39	2.48	2.52	12.42	0.00	0.00	0.00
time (sec)	N/A	0.322	0.343	10.170	0.202	0.335	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	121	142	268	288	1070	3624	0	0	0
N.S.	1	1.17	2.21	2.38	8.84	29.95	0.00	0.00	0.00
time (sec)	N/A	0.454	4.116	15.900	0.364	0.307	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	108	130	260	384	3624	0	0	0
N.S.	1	0.98	1.18	2.36	3.49	32.95	0.00	0.00	0.00
time (sec)	N/A	0.354	1.382	23.767	0.224	0.439	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	161	182	350	362	2961	9849	0	0	0
N.S.	1	1.13	2.17	2.25	18.39	61.17	0.00	0.00	0.00
time (sec)	N/A	0.544	6.345	34.979	0.573	0.395	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	148	173	754	389	3239	5463	0	0	0
N.S.	1	1.17	5.09	2.63	21.89	36.91	0.00	0.00	0.00
time (sec)	N/A	0.485	7.975	191.714	1.017	0.337	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	73	136	184	206	1741	0	0	0
N.S.	1	0.95	1.77	2.39	2.68	22.61	0.00	0.00	0.00
time (sec)	N/A	0.306	2.919	145.706	0.225	0.285	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	139	155	1317	343	2201	6464	0	0	0
N.S.	1	1.12	9.47	2.47	15.83	46.50	0.00	0.00	0.00
time (sec)	N/A	0.434	15.010	64.282	0.632	0.343	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	75	131	187	209	1753	0	0	0
N.S.	1	0.93	1.62	2.31	2.58	21.64	0.00	0.00	0.00
time (sec)	N/A	0.316	1.450	46.045	0.210	0.289	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	139	164	1317	335	1255	7158	0	0	0
N.S.	1	1.18	9.47	2.41	9.03	51.50	0.00	0.00	0.00
time (sec)	N/A	0.424	11.778	41.615	0.433	0.355	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	69	129	82	193	1666	0	0	94
N.S.	1	0.95	1.77	1.12	2.64	22.82	0.00	0.00	1.29
time (sec)	N/A	0.292	1.801	43.921	0.231	0.275	0.000	0.000	2.235

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	146	177	301	352	402	6538	0	0	0
N.S.	1	1.21	2.06	2.41	2.75	44.78	0.00	0.00	0.00
time (sec)	N/A	0.373	8.313	1.308	0.310	0.342	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	128	155	292	419	4132	0	0	0
N.S.	1	0.98	1.19	2.25	3.22	31.78	0.00	0.00	0.00
time (sec)	N/A	0.374	1.114	64.557	0.226	0.505	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	182	213	2083	394	1971	11606	0	0	0
N.S.	1	1.17	11.45	2.16	10.83	63.77	0.00	0.00	0.00
time (sec)	N/A	0.565	8.752	136.056	0.463	0.394	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	151	172	354	692	10255	0	0	0
N.S.	1	0.99	1.13	2.33	4.55	67.47	0.00	0.00	0.00
time (sec)	N/A	0.413	1.302	191.058	0.268	0.797	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	232	263	3334	468	4920	24263	0	0	0
N.S.	1	1.13	14.37	2.02	21.21	104.58	0.00	0.00	0.00
time (sec)	N/A	0.646	8.501	268.036	0.850	0.558	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	207	248	1405	545	718	17283	0	0	0
N.S.	1	1.20	6.79	2.63	3.47	83.49	0.00	0.00	0.00
time (sec)	N/A	0.458	8.738	3.264	0.341	0.451	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	C	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	29	32	25	26	33	183	0	72	0
N.S.	1	1.10	0.86	0.90	1.14	6.31	0.00	2.48	0.00
time (sec)	N/A	0.333	0.022	0.399	0.281	0.264	0.000	0.289	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	13	18	0	26	0
N.S.	1	1.00	1.00	1.21	0.93	1.29	0.00	1.86	0.00
time (sec)	N/A	0.262	0.007	0.355	0.293	0.264	0.000	0.312	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	79	22	18	0	31	0
N.S.	1	1.00	1.29	5.64	1.57	1.29	0.00	2.21	0.00
time (sec)	N/A	0.268	0.017	0.164	0.300	0.258	0.000	0.315	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	B	C	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	95	33	404	0	83	0
N.S.	1	1.00	0.79	2.79	0.97	11.88	0.00	2.44	0.00
time (sec)	N/A	0.329	0.023	0.215	0.279	0.269	0.000	0.286	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	26	13	70	0	31	0
N.S.	1	1.00	1.00	1.62	0.81	4.38	0.00	1.94	0.00
time (sec)	N/A	0.257	0.007	0.317	0.281	0.258	0.000	0.269	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	20	33	22	69	0	36	0
N.S.	1	1.00	1.25	2.06	1.38	4.31	0.00	2.25	0.00
time (sec)	N/A	0.268	0.015	0.315	0.285	0.260	0.000	0.294	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	88	108	0	0	4594	0	0	0
N.S.	1	1.06	1.30	0.00	0.00	55.35	0.00	0.00	0.00
time (sec)	N/A	0.327	0.647	0.000	0.000	0.505	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	137	192	0	0	8852	0	0	0
N.S.	1	1.10	1.54	0.00	0.00	70.82	0.00	0.00	0.00
time (sec)	N/A	0.472	0.418	0.000	0.000	0.518	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	64	56	0	0	2394	0	0	0
N.S.	1	1.08	0.95	0.00	0.00	40.58	0.00	0.00	0.00
time (sec)	N/A	0.271	0.265	0.000	0.000	0.345	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	91	150	0	0	4316	0	0	0
N.S.	1	1.05	1.72	0.00	0.00	49.61	0.00	0.00	0.00
time (sec)	N/A	0.389	0.280	0.000	0.000	0.390	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	45	90	43	0	1608	0	0	32
N.S.	1	1.12	2.25	1.08	0.00	40.20	0.00	0.00	0.80
time (sec)	N/A	0.241	0.200	0.132	0.000	0.332	0.000	0.000	2.437

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	134	0	0	2949	0	0	0
N.S.	1	1.00	2.27	0.00	0.00	49.98	0.00	0.00	0.00
time (sec)	N/A	0.234	0.611	0.000	0.000	0.350	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	61	56	0	0	3597	0	0	0
N.S.	1	1.09	1.00	0.00	0.00	64.23	0.00	0.00	0.00
time (sec)	N/A	0.281	0.133	0.000	0.000	0.344	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	75	0	0	1303	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	27.15	0.00	0.00	0.00
time (sec)	N/A	0.326	0.403	0.000	0.000	0.299	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	95	156	0	0	5247	0	0	0
N.S.	1	1.14	1.88	0.00	0.00	63.22	0.00	0.00	0.00
time (sec)	N/A	0.311	0.580	0.000	0.000	0.397	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	88	149	0	0	2341	0	0	0
N.S.	1	1.05	1.77	0.00	0.00	27.87	0.00	0.00	0.00
time (sec)	N/A	0.381	0.458	0.000	0.000	0.358	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	156	191	0	0	12548	0	0	0
N.S.	1	1.25	1.53	0.00	0.00	100.38	0.00	0.00	0.00
time (sec)	N/A	0.378	0.942	0.000	0.000	0.600	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	84	80	0	0	4226	0	0	0
N.S.	1	1.11	1.05	0.00	0.00	55.61	0.00	0.00	0.00
time (sec)	N/A	0.296	0.521	0.000	0.000	0.508	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	134	197	0	0	8582	0	0	0
N.S.	1	1.07	1.58	0.00	0.00	68.66	0.00	0.00	0.00
time (sec)	N/A	0.520	0.585	0.000	0.000	0.521	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	65	65	56	0	2312	0	0	45
N.S.	1	1.14	1.14	0.98	0.00	40.56	0.00	0.00	0.79
time (sec)	N/A	0.255	0.093	0.127	0.000	0.349	0.000	0.000	4.005

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	91	152	0	0	4140	0	0	0
N.S.	1	1.03	1.73	0.00	0.00	47.05	0.00	0.00	0.00
time (sec)	N/A	0.293	0.195	0.000	0.000	0.380	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	76	159	0	0	4123	0	0	0
N.S.	1	1.09	2.27	0.00	0.00	58.90	0.00	0.00	0.00
time (sec)	N/A	0.318	0.520	0.000	0.000	0.381	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	3349	0	0	0
N.S.	1	1.00	1.78	0.00	0.00	41.35	0.00	0.00	0.00
time (sec)	N/A	0.386	0.328	0.000	0.000	0.377	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	280	0	0	12452	0	0	0
N.S.	1	1.00	1.65	0.00	0.00	73.25	0.00	0.00	0.00
time (sec)	N/A	0.366	6.957	0.000	0.000	0.736	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	72	84	0	0	2678	0	0	0
N.S.	1	1.09	1.27	0.00	0.00	40.58	0.00	0.00	0.00
time (sec)	N/A	0.314	0.453	0.000	0.000	0.386	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	97	169	0	0	4569	0	0	0
N.S.	1	1.08	1.88	0.00	0.00	50.77	0.00	0.00	0.00
time (sec)	N/A	0.386	0.445	0.000	0.000	0.403	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	48	42	0	0	1650	0	0	0
N.S.	1	1.14	1.00	0.00	0.00	39.29	0.00	0.00	0.00
time (sec)	N/A	0.263	0.225	0.000	0.000	0.319	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	107	0	0	2856	0	0	0
N.S.	1	1.00	1.78	0.00	0.00	47.60	0.00	0.00	0.00
time (sec)	N/A	0.354	0.221	0.000	0.000	0.342	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	0	1430	0	0	19
N.S.	1	1.00	1.00	1.20	0.00	57.20	0.00	0.00	0.76
time (sec)	N/A	0.237	0.076	0.242	0.000	0.291	0.000	0.000	2.359

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	62	0	0	1059	0	0	0
N.S.	1	1.00	2.14	0.00	0.00	36.52	0.00	0.00	0.00
time (sec)	N/A	0.203	0.037	0.000	0.000	0.285	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	61	124	0	0	3663	0	0	0
N.S.	1	1.09	2.21	0.00	0.00	65.41	0.00	0.00	0.00
time (sec)	N/A	0.295	0.345	0.000	0.000	0.361	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	94	0	0	1365	0	0	0
N.S.	1	1.00	1.77	0.00	0.00	25.75	0.00	0.00	0.00
time (sec)	N/A	0.344	0.083	0.000	0.000	0.314	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	110	159	0	0	6475	0	0	0
N.S.	1	1.22	1.77	0.00	0.00	71.94	0.00	0.00	0.00
time (sec)	N/A	0.328	0.819	0.000	0.000	0.459	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	73	129	0	0	3360	0	0	0
N.S.	1	1.07	1.90	0.00	0.00	49.41	0.00	0.00	0.00
time (sec)	N/A	0.334	0.635	0.000	0.000	0.408	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	96	169	0	0	5170	0	0	0
N.S.	1	1.12	1.97	0.00	0.00	60.12	0.00	0.00	0.00
time (sec)	N/A	0.418	0.336	0.000	0.000	0.424	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	54	49	0	0	2194	0	93	0
N.S.	1	1.10	1.00	0.00	0.00	44.78	0.00	1.90	0.00
time (sec)	N/A	0.280	0.274	0.000	0.000	0.322	0.000	0.356	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	128	0	0	1733	0	86	0
N.S.	1	1.00	2.51	0.00	0.00	33.98	0.00	1.69	0.00
time (sec)	N/A	0.358	0.551	0.000	0.000	0.311	0.000	0.362	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	48	98	46	0	2034	61	85	35
N.S.	1	1.12	2.28	1.07	0.00	47.30	1.42	1.98	0.81
time (sec)	N/A	0.249	0.483	0.155	0.000	0.324	2.828	0.342	2.592

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	107	0	0	2095	0	72	0
N.S.	1	1.00	1.88	0.00	0.00	36.75	0.00	1.26	0.00
time (sec)	N/A	0.243	0.612	0.000	0.000	0.328	0.000	0.351	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	99	155	0	0	6939	0	0	0
N.S.	1	1.25	1.96	0.00	0.00	87.84	0.00	0.00	0.00
time (sec)	N/A	0.331	0.674	0.000	0.000	0.470	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	98	120	0	0	3941	0	254	0
N.S.	1	1.11	1.36	0.00	0.00	44.78	0.00	2.89	0.00
time (sec)	N/A	0.435	0.384	0.000	0.000	0.432	0.000	0.442	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	146	178	0	0	11939	0	0	0
N.S.	1	1.24	1.51	0.00	0.00	101.18	0.00	0.00	0.00
time (sec)	N/A	0.490	0.576	0.000	0.000	0.708	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	81	126	0	0	5184	0	297	0
N.S.	1	1.07	1.66	0.00	0.00	68.21	0.00	3.91	0.00
time (sec)	N/A	0.355	0.628	0.000	0.000	0.501	0.000	0.396	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	97	290	0	0	3559	0	242	0
N.S.	1	1.08	3.22	0.00	0.00	39.54	0.00	2.69	0.00
time (sec)	N/A	0.402	1.462	0.000	0.000	0.473	0.000	0.406	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	79	124	0	0	4644	0	285	0
N.S.	1	1.16	1.82	0.00	0.00	68.29	0.00	4.19	0.00
time (sec)	N/A	0.284	1.042	0.000	0.000	0.477	0.000	0.395	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	95	290	0	0	4989	0	264	0
N.S.	1	1.08	3.30	0.00	0.00	56.69	0.00	3.00	0.00
time (sec)	N/A	0.393	0.437	0.000	0.000	0.506	0.000	0.398	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	73	112	61	0	3994	82	257	50
N.S.	1	1.18	1.81	0.98	0.00	64.42	1.32	4.15	0.81
time (sec)	N/A	0.270	0.338	0.118	0.000	0.463	5.987	0.409	4.028

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	110	130	0	0	6299	0	232	0
N.S.	1	1.16	1.37	0.00	0.00	66.31	0.00	2.44	0.00
time (sec)	N/A	0.283	0.340	0.000	0.000	0.518	0.000	0.407	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	143	242	0	0	18563	0	0	0
N.S.	1	1.31	2.22	0.00	0.00	170.30	0.00	0.00	0.00
time (sec)	N/A	0.372	0.960	0.000	0.000	0.915	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	155	155	0	0	11205	0	854	0
N.S.	1	1.17	1.17	0.00	0.00	84.25	0.00	6.42	0.00
time (sec)	N/A	0.542	0.648	0.000	0.000	0.978	0.000	0.580	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	205	221	0	0	18565	0	0	0
N.S.	1	1.12	1.21	0.00	0.00	101.45	0.00	0.00	0.00
time (sec)	N/A	0.375	1.860	0.000	0.000	1.826	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [173] had the largest ratio of [1.10000000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.34	21	0.333
2	A	6	5	0.91	21	0.238
3	A	7	6	1.35	21	0.286
4	A	6	5	0.92	19	0.263
5	A	6	5	0.96	19	0.263
6	A	6	5	0.93	21	0.238
7	A	8	7	1.09	21	0.333
8	A	5	4	0.89	21	0.190
9	A	8	7	1.17	23	0.304
10	A	6	5	0.90	23	0.217
11	A	8	7	1.07	23	0.304
12	A	6	5	0.89	21	0.238
13	A	6	5	0.92	21	0.238
14	A	6	5	0.90	23	0.217
15	A	9	8	1.12	23	0.348
16	A	5	4	0.89	23	0.174
17	A	9	8	1.09	23	0.348
18	A	6	5	0.89	23	0.217
19	A	11	10	1.43	23	0.435
20	A	6	5	0.88	21	0.238
21	A	6	5	0.92	21	0.238
22	A	6	5	0.89	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	6	0.97	23	0.261
24	A	5	4	0.89	23	0.174
25	A	9	8	1.24	23	0.348
26	A	7	6	0.97	23	0.261
27	A	8	7	1.24	23	0.304
28	A	6	5	0.96	21	0.238
29	A	7	6	0.98	21	0.286
30	A	6	5	0.96	23	0.217
31	A	8	7	1.13	23	0.304
32	A	6	5	1.03	23	0.217
33	A	11	10	1.13	23	0.435
34	A	7	6	0.94	23	0.261
35	A	10	9	1.14	23	0.391
36	A	7	6	0.98	21	0.286
37	A	8	7	1.07	21	0.333
38	A	7	6	1.00	23	0.261
39	A	10	9	1.06	23	0.391
40	A	6	5	1.02	23	0.217
41	A	13	12	1.11	23	0.522
42	A	9	8	0.99	23	0.348
43	A	12	11	1.18	23	0.478
44	A	8	7	1.03	21	0.333
45	A	9	8	1.12	21	0.381
46	A	8	7	1.06	23	0.304
47	A	13	12	1.11	23	0.522
48	A	7	6	1.02	23	0.261
49	A	5	5	0.97	21	0.238
50	C	6	5	1.20	21	0.238
51	A	3	3	1.00	21	0.143
52	A	4	4	1.00	19	0.211
53	A	4	4	1.00	19	0.211
54	A	6	5	1.43	21	0.238
55	A	6	6	0.97	21	0.286
56	C	6	5	1.28	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	7	6	1.28	23	0.261
58	A	5	4	0.90	23	0.174
59	A	5	4	1.26	23	0.174
60	A	5	4	1.02	21	0.190
61	A	6	5	1.11	21	0.238
62	A	5	4	0.91	23	0.174
63	A	7	6	1.01	23	0.261
64	A	5	4	0.90	23	0.174
65	A	5	4	1.18	23	0.174
66	A	5	4	0.98	23	0.174
67	A	5	4	1.12	23	0.174
68	A	5	4	1.02	21	0.190
69	A	8	7	1.11	21	0.333
70	A	5	4	0.89	23	0.174
71	A	9	8	0.99	23	0.348
72	A	5	4	0.90	23	0.174
73	A	8	7	1.24	23	0.304
74	A	5	4	0.93	23	0.174
75	A	7	6	1.24	23	0.261
76	A	5	4	0.96	21	0.190
77	A	4	3	1.00	21	0.143
78	A	4	3	1.00	23	0.130
79	A	6	5	0.96	23	0.217
80	A	5	4	0.96	23	0.174
81	A	7	6	1.08	23	0.261
82	A	5	4	0.94	23	0.174
83	A	5	4	0.94	23	0.174
84	A	9	8	1.15	23	0.348
85	A	5	4	0.95	21	0.190
86	A	5	4	0.98	21	0.190
87	A	5	4	0.97	23	0.174
88	A	5	4	0.97	23	0.174
89	A	5	4	0.98	23	0.174
90	A	7	6	1.10	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
91	A	5	4	0.95	23	0.174
92	A	9	8	1.08	23	0.348
93	A	11	10	1.20	23	0.435
94	A	5	4	0.95	21	0.190
95	A	6	5	1.07	21	0.238
96	A	6	5	1.05	23	0.217
97	A	6	5	1.00	23	0.217
98	A	6	5	1.00	23	0.217
99	A	6	5	1.05	23	0.217
100	A	7	6	1.08	23	0.261
101	A	8	7	1.14	23	0.304
102	A	7	6	1.00	21	0.286
103	A	7	6	0.94	21	0.286
104	A	9	8	1.19	21	0.381
105	A	6	5	0.93	19	0.263
106	A	1	1	1.00	12	0.083
107	A	7	6	1.32	19	0.316
108	A	8	7	1.44	21	0.333
109	A	7	6	1.32	21	0.286
110	A	7	6	1.12	21	0.286
111	A	7	6	1.29	21	0.286
112	A	6	5	0.97	23	0.217
113	A	7	6	0.92	23	0.261
114	A	8	7	1.03	23	0.304
115	A	7	6	0.96	21	0.286
116	A	5	4	1.10	14	0.286
117	A	7	6	1.04	21	0.286
118	A	8	7	1.11	23	0.304
119	A	7	6	1.16	23	0.261
120	A	6	5	1.09	23	0.217
121	A	7	6	1.33	23	0.261
122	A	8	7	1.03	23	0.304
123	A	8	7	1.22	23	0.304
124	A	6	5	0.95	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	8	7	0.91	23	0.304
126	A	8	7	0.99	23	0.304
127	A	7	6	0.90	21	0.286
128	A	5	4	1.01	14	0.286
129	A	7	6	0.99	21	0.286
130	A	8	7	1.02	23	0.304
131	A	7	6	1.02	23	0.261
132	A	6	5	1.02	23	0.217
133	A	7	6	1.16	23	0.261
134	A	8	7	1.03	23	0.304
135	A	7	6	1.22	23	0.261
136	A	5	4	0.98	14	0.286
137	A	5	4	0.97	14	0.286
138	A	7	6	0.90	23	0.261
139	A	8	7	1.19	23	0.304
140	A	7	6	1.00	23	0.261
141	A	9	8	1.17	23	0.348
142	A	5	4	1.00	21	0.190
143	A	6	5	1.00	14	0.357
144	A	7	6	1.13	21	0.286
145	A	10	9	1.24	23	0.391
146	A	7	6	1.05	23	0.261
147	A	11	10	1.23	23	0.435
148	A	7	6	0.92	23	0.261
149	A	8	7	1.15	23	0.304
150	A	7	6	0.94	23	0.261
151	A	10	9	1.14	23	0.391
152	A	7	6	0.90	21	0.286
153	A	8	7	1.20	14	0.500
154	A	7	6	1.01	21	0.286
155	A	13	12	1.17	23	0.522
156	A	7	6	0.98	23	0.261
157	A	13	12	1.13	23	0.522
158	A	11	10	1.17	23	0.435

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	7	6	0.95	23	0.261
160	A	11	10	1.12	23	0.435
161	A	7	6	0.93	23	0.261
162	A	12	11	1.18	23	0.478
163	A	7	6	0.95	21	0.286
164	A	10	9	1.21	14	0.643
165	A	7	6	0.98	21	0.286
166	A	15	14	1.17	23	0.609
167	A	7	6	0.99	23	0.261
168	A	15	14	1.13	23	0.609
169	A	12	11	1.20	14	0.786
170	C	11	11	1.10	12	0.917
171	A	7	7	1.00	12	0.583
172	A	7	7	1.00	12	0.583
173	C	11	11	1.00	10	1.100
174	A	7	7	1.00	10	0.700
175	A	7	7	1.00	10	0.700
176	A	7	6	1.06	17	0.353
177	A	11	10	1.10	17	0.588
178	A	10	9	1.08	17	0.529
179	A	12	11	1.05	17	0.647
180	A	8	7	1.12	15	0.467
181	A	8	7	1.00	12	0.583
182	A	9	8	1.09	15	0.533
183	A	10	9	1.00	17	0.529
184	A	10	9	1.14	17	0.529
185	A	9	8	1.05	17	0.471
186	A	13	12	1.25	17	0.706
187	A	11	10	1.11	17	0.588
188	A	16	15	1.07	17	0.882
189	A	9	8	1.14	15	0.533
190	A	10	9	1.03	12	0.750
191	A	11	10	1.09	15	0.667
192	A	12	11	1.00	17	0.647

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	10	9	1.00	16	0.562
194	A	7	6	1.09	17	0.353
195	A	10	9	1.08	17	0.529
196	A	9	8	1.14	17	0.471
197	A	11	10	1.00	17	0.588
198	A	7	6	1.00	15	0.400
199	A	5	4	1.00	12	0.333
200	A	9	8	1.09	15	0.533
201	A	10	9	1.00	17	0.529
202	A	10	9	1.22	17	0.529
203	A	7	6	1.07	17	0.353
204	A	10	9	1.12	17	0.529
205	A	9	8	1.10	17	0.471
206	A	9	8	1.00	17	0.471
207	A	8	7	1.12	15	0.467
208	A	6	5	1.00	12	0.417
209	A	11	10	1.25	15	0.667
210	A	13	12	1.11	17	0.706
211	A	14	13	1.24	17	0.765
212	A	7	6	1.07	17	0.353
213	A	9	8	1.08	17	0.471
214	A	10	9	1.16	17	0.529
215	A	11	10	1.08	17	0.588
216	A	9	8	1.18	15	0.533
217	A	9	8	1.16	12	0.667
218	A	13	12	1.31	15	0.800
219	A	14	13	1.17	17	0.765
220	A	11	10	1.12	16	0.625

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \left(a+b\,\text{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx$

3.1.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \left(a+b\,\text{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx=\frac{3}{8}(a-4b)x-\frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{b\tanh(c+dx)}{d}$$

output
```
3/8*(a-4*b)*x-1/8*(5*a-4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)^3*
sinh(d*x+c)/d+b*tanh(d*x+c)/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \left(a+b\,\text{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx=\frac{12(a-4b)(c+dx)-8(a-b)\sinh(2(c+dx))+a\sinh(4(c+dx))+32b\tanh(c+dx)}{32d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]
```

output
```
(12*(a - 4*b)*(c + d*x) - 8*(a - b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x)
] + 32*b*Tanh[c + d*x])/(32*d)
```

3.1.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4620, 360, 25, 1471, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 360}$$

$$\frac{\frac{1}{4}\int -\frac{-4b\tanh^4(c+dx)+4a\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{a\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{a\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{-4b\tanh^4(c+dx)+4a\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{-8b\tanh^2(c+dx)+3a-4b}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{(5a-4b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{a\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3(a-4b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+8b\tanh(c+dx)\right)-\frac{(5a-4b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{a\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(3(a-4b)\text{arctanh}(\tanh(c+dx))+8b\tanh(c+dx))-\frac{(5a-4b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{a\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]
```

output
```
((a*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + ((3*(a - 4*b)*ArcTanh[Tan
h[c + d*x]] + 8*b*Tanh[c + d*x])/2 - ((5*a - 4*b)*Tanh[c + d*x])/(2*(1 - T
anh[c + d*x]^2)))/4)/d
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.1.4 Maple [A] (verified)

Time = 10.95 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
parts	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
risch	$\frac{3ax}{8}-\frac{3bx}{2}+\frac{a\,\mathrm{e}^{4dx+4c}}{64d}-\frac{\mathrm{e}^{2dx+2c}a}{8d}+\frac{\mathrm{e}^{2dx+2c}b}{8d}+\frac{\mathrm{e}^{-2dx-2c}a}{8d}-\frac{\mathrm{e}^{-2dx-2c}b}{8d}-\frac{a\,\mathrm{e}^{-4dx-4c}}{64d}-\frac{2b}{d(\mathrm{e}^{2dx+2c}+1)}$

input
```
int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(
1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx$$
$$=\frac{a\sinh\left(dx+c\right)^5+\left(10\,a\cosh\left(dx+c\right)^2-7\,a+8\,b\right)\sinh\left(dx+c\right)^3+8\left(3\left(a-4\,b\right)dx-8\,b\right)\cosh\left(dx+c\right)+}{64\,d\cosh\left(dx+c\right)}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/64*(a*sinh(d*x + c)^5 + (10*a*cosh(d*x + c)^2 - 7*a + 8*b)*sinh(d*x + c)
^3 + 8*(3*(a - 4*b)*d*x - 8*b)*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 - 3*(7
*a - 8*b)*cosh(d*x + c)^2 - 8*a + 72*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

3.1.6 Sympy [F]

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx=\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\sinh^4\left(c+dx\right)dx$$

input
```
integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**4,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**4, x)
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(64)=128$.

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.84

$$\begin{aligned}\int &\left(a+b\mathrm{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx\\ &=\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)\\ &\quad-\frac{1}{8}\,b\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)})}\right)\end{aligned}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) - 1/8*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (
17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)))
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(64) = 128.

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.86

$$\int \left(a+b\text{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx$$
$$=\frac{24\,(dx+c)(a-4\,b)+ae^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx+2\,c)}-\left(18\,ae^{(4\,dx+4\,c)}-72\,be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+}\right.}{64\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="giac")
```

output
```
1/64*(24*(d*x + c)*(a - 4*b) + a*e^(4*d*x + 4*c) - 8*a*e^(2*d*x + 2*c) + 8
*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) - 72*b*e^(4*d*x + 4*c) - 8*a*e^
(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + a)*e^(-4*d*x - 4*c) - 128*b/(e^(2*d*
x + 2*c) + 1))/d
```

3.1.9 Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \left(a+b\text{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx=\frac{3\,a\,x}{8}-\frac{3\,b\,x}{2}-\frac{a\sinh(2\,c+2\,d\,x)}{4\,d}+\frac{a\sinh(4\,c+4\,d\,x)}{32\,d}+\frac{b\sinh(2\,c+2\,d\,x)}{4\,d}+\frac{b\sinh(c+d\,x)}{d\cosh\left(c+d\,x\right)}$$

input
```
int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

output
```
(3*a*x)/8 - (3*b*x)/2 - (a*sinh(2*c + 2*d*x))/(4*d) + (a*sinh(4*c + 4*d*x)
)/(32*d) + (b*sinh(2*c + 2*d*x))/(4*d) + (b*sinh(c + d*x))/(d*cosh(c + d*x
))
```

3.2 $\int (a+b\text{sech}^2(c+dx)) \sinh^3(c+dx)\,dx$

3.2.1 Optimal result

Integrand size = 21, antiderivative size = 44

$$\int (a+b\text{sech}^2(c+dx)) \sinh^3(c+dx)\,dx = -\frac{(a-b)\cosh(c+dx)}{d} + \frac{a\cosh^3(c+dx)}{3d} + \frac{b\text{sech}(c+dx)}{d}$$

output
```
-(a-b)*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d+b*sech(d*x+c)/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int (a+b\text{sech}^2(c+dx)) \sinh^3(c+dx)\,dx = -\frac{3a\cosh(c+dx)}{4d} + \frac{b\cosh(c+dx)}{d} + \frac{a\cosh(3(c+dx))}{12d} + \frac{b\text{sech}(c+dx)}{d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]
```

output
```
(-3*a*Cosh[c + d*x])/(4*d) + (b*Cosh[c + d*x])/d + (a*Cosh[3*(c + d*x)])/(
12*d) + (b*Sech[c + d*x])/d
```

3.2.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \left(b\sec(ic+idx)^2+a\right)\sin(ic+idx)^3dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)\text{sech}^2(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(-a\cosh^2(c+dx)+b\text{sech}^2(c+dx)+a\left(1-\frac{b}{a}\right)\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a-b)\cosh(c+dx)-\frac{1}{3}a\cosh^3(c+dx)-b\text{sech}(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]
```

output
```
-(((a - b)*Cosh[c + d*x] - (a*Cosh[c + d*x]^3)/3 - b*Sech[c + d*x])/d)
```

3.2.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.2.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	56
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	56
parts	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	58
risch	$\frac{a\,e^{3dx+3c}}{24d}-\frac{3\,e^{dx+c}a}{8d}+\frac{e^{dx+c}b}{2d}-\frac{3\,e^{-dx-c}a}{8d}+\frac{e^{-dx-c}b}{2d}+\frac{a\,e^{-3dx-3c}}{24d}+\frac{2b\,e^{dx+c}}{d\left(e^{2dx+2c}+1\right)}$	111

input
```
int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(sinh(d*x+c)^2/cosh(d*x+c)+2
/cosh(d*x+c)))
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(42) = 84$.

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93

$$\int (a + b\text{sech}^2(c + dx)) \sinh^3(c + dx) \, dx = \frac{a \cosh(dx + c)^4 + a \sinh(dx + c)^4 - 4(2a - 3b) \cosh(dx + c)^2 + 2(3a \cosh(dx + c)^2 - 4a + 6b) \sinh(}{24 d \cosh(dx + c)}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/24*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 4*(2*a - 3*b)*cosh(d*x + c)^
2 + 2*(3*a*cosh(d*x + c)^2 - 4*a + 6*b)*sinh(d*x + c)^2 - 9*a + 36*b)/(d*c
osh(d*x + c))
```

3.2.6 Sympy [F]

$$\int (a + b\text{sech}^2(c + dx)) \sinh^3(c + dx) \, dx = \int (a + b \operatorname{sech}^2(c + dx)) \sinh^3(c + dx) \, dx$$

input
```
integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**3, x)
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(42) = 84$.

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.52

$$\int (a + b\operatorname{sech}^2(c + dx)) \sinh^3(c + dx) \, dx = \frac{1}{24} a \left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d} \right) + \frac{1}{2} b \left(\frac{e^{(-dx-c)}}{d} + \frac{5\,e^{(-2\,dx-2\,c)} + 1}{d(e^{(-dx-c)} + e^{(-3\,dx-3\,c)})} \right)$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/24*a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
 - 3*c)/d) + 1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x
- c) + e^(-3*d*x - 3*c))))
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(42) = 84$.

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93

$$\int (a + b\operatorname{sech}^2(c + dx)) \sinh^3(c + dx) \, dx = \frac{a(e^{(dx+c)} + e^{(-dx-c)})^3 - 12\,a(e^{(dx+c)} + e^{(-dx-c)}) + 12\,b(e^{(dx+c)} + e^{(-dx-c)}) + \frac{48\,b}{e^{(dx+c)} + e^{(-dx-c)}}}{24\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/24*(a*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a*(e^(d*x + c) + e^(-d*x - c))
 + 12*b*(e^(d*x + c) + e^(-d*x - c)) + 48*b/(e^(d*x + c) + e^(-d*x - c)))/
d
```

3.2.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \left(a+b\text{sech}^2(c+dx)\right)\sinh^3(c+dx)\,dx=\frac{a\cosh(c+d\,x)^3}{3\,d}-\frac{\cosh(c+d\,x)\,\left(a-b\right)}{d}+\frac{b}{d\cosh\left(c+d\,x\right)}$$

input
```
int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

output
```
(a*cosh(c + d*x)^3)/(3*d) - (cosh(c + d*x)*(a - b))/d + b/(d*cosh(c + d*x)
)
```

3.3 $\int (a + b\text{sech}^2(c + dx)) \sinh^2(c + dx)\, dx$

3.3.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int (a + b\text{sech}^2(c + dx)) \sinh^2(c + dx)\, dx = -\frac{1}{2}(a - 2b)x + \frac{a\cosh(c + dx)\sinh(c + dx)}{2d} - \frac{b\tanh(c + dx)}{d}$$

output
```
-1/2*(a-2*b)*x+1/2*a*cosh(d*x+c)*sinh(d*x+c)/d-b*tanh(d*x+c)/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int (a + b\text{sech}^2(c + dx)) \sinh^2(c + dx)\, dx = \frac{a(-c - dx)}{2d} + \frac{b\text{arctanh}(\tanh(c + dx))}{d} + \frac{a\sinh(2(c + dx))}{4d} - \frac{b\tanh(c + dx)}{d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]
```

output
```
(a*(-c - d*x))/(2*d) + (b*ArcTanh[Tanh[c + d*x]])/d + (a*Sinh[2*(c + d*x)]
)/(4*d) - (b*Tanh[c + d*x])/d
```

3.3.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 4620, 360, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\left(\sin(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \left(b\sec(ic+idx)^2+a\right)\sin(ic+idx)^2dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 360}$$

$$\frac{\frac{a\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{a-2b\tanh^2(c+dx)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{1}{2}\left(-(a-2b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-2b\tanh(c+dx)\right)+\frac{a\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}(-((a-2b)\text{arctanh}(\tanh(c+dx)))-2b\tanh(c+dx))+\frac{a\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]
```

output
```
((-((a - 2*b)*ArcTanh[Tanh[c + d*x]]) - 2*b*Tanh[c + d*x])/2 + (a*Tanh[c +
 d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.3.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b(dx+c-\tanh(dx+c))}{d}$	45
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b(dx+c-\tanh(dx+c))}{d}$	45
parts	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b(dx+c-\tanh(dx+c))}{d}$	47
risch	$-\frac{ax}{2}+bx+\frac{e^{2dx+2c}a}{8d}-\frac{e^{-2dx-2c}a}{8d}+\frac{2b}{d(e^{2dx+2c}+1)}$	58

input
```
int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(d*x+c-tanh(d*x+c)))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int (a+b\operatorname{sech}^2(c+dx))\sinh^2(c+dx)\,dx$$
$$=\frac{a\sinh(dx+c)^3-4\,((a-2\,b)dx-2\,b)\cosh(dx+c)+\left(3\,a\cosh(dx+c)^2+a-8\,b\right)\sinh(dx+c)}{8\,d\cosh(dx+c)}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/8*(a*sinh(d*x + c)^3 - 4*((a - 2*b)*d*x - 2*b)*cosh(d*x + c) + (3*a*cosh
(d*x + c)^2 + a - 8*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

3.3.6 Sympy [F]

$$\int (a + b\text{sech}^2(c + dx)) \sinh^2(c + dx)\, dx = \int (a + b\,\text{sech}^2(c + dx)) \sinh^2(c + dx)\, dx$$

input
```
integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**2, x)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int (a + b\text{sech}^2(c + dx)) \sinh^2(c + dx)\, dx = -\frac{1}{8}\,a\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)} + 1)}\right)$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + b*(x + c/d - 2/(d*
(e^(-2*d*x - 2*c) + 1)))
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int (a + b\text{sech}^2(c + dx)) \sinh^2(c + dx)\, dx = -\frac{4\,(dx + c)(a - 2\,b) - ae^{(2\,dx+2\,c)} - \frac{ae^{(4\,dx+4\,c)} - 2\,be^{(4\,dx+4\,c)} + 14\,be^{(2\,dx+2\,c)} - a}{e^{(4\,dx+4\,c)} + e^{(2\,dx+2\,c)}}}{8\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/8*(4*(d*x + c)*(a - 2*b) - a*e^(2*d*x + 2*c) - (a*e^(4*d*x + 4*c) - 2*b
*e^(4*d*x + 4*c) + 14*b*e^(2*d*x + 2*c) - a)/(e^(4*d*x + 4*c) + e^(2*d*x +
 2*c)))/d
```

3.3.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int (a+b\text{sech}^2(c+dx)) \sinh^2(c+dx)\, dx = \frac{a\cosh(c+d\,x)\,\sinh(c+d\,x)}{2\,d} - \frac{\frac{a\,d\,x}{2}-b\,d\,x}{d} - \frac{b\sinh(c+d\,x)}{d\cosh\left(c+d\,x\right)}$$

input
```
int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

output
```
(a*cosh(c + d*x)*sinh(c + d*x))/(2*d) - ((a*d*x)/2 - b*d*x)/d - (b*sinh(c
+ d*x))/(d*cosh(c + d*x))
```

3.4 $\int (a+b\operatorname{sech}^2(c+dx))\sinh(c+dx)\,dx$

3.4.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (a+b\operatorname{sech}^2(c+dx))\sinh(c+dx)\,dx=\frac{a\cosh(c+dx)}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

output
```
a*cosh(d*x+c)/d-b*sech(d*x+c)/d
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int (a+b\operatorname{sech}^2(c+dx))\sinh(c+dx)\,dx=\frac{a\cosh(c)\cosh(dx)}{d}-\frac{b\operatorname{sech}(c+dx)}{d}+\frac{a\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x],x]
```

output
```
(a*Cosh[c]*Cosh[d*x])/d - (b*Sech[c + d*x])/d + (a*Sinh[c]*Sinh[d*x])/d
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 26, 4621, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a+b\sec(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \left(b\sec(ic+idx)^2+a\right)\sin(ic+idx)dx$$

$$\Big\downarrow \text{4621}$$

$$\frac{\int \left(a\cosh^2(c+dx)+b\right)\text{sech}^2(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(b\text{sech}^2(c+dx)+a\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\cosh(c+dx)-b\text{sech}(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x],x]
```

output
```
(a*Cosh[c + d*x] - b*Sech[c + d*x])/d
```

3.4.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.4.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-b\,\mathrm{sech}(dx+c)+\frac{a}{\mathrm{sech}(dx+c)}}{d}$	25
default	$\frac{-b\,\mathrm{sech}(dx+c)+\frac{a}{\mathrm{sech}(dx+c)}}{d}$	25
parts	$\frac{a\cosh(dx+c)}{d}-\frac{b\,\mathrm{sech}(dx+c)}{d}$	25
risch	$\frac{\mathrm{e}^{dx+c}a}{2d}+\frac{\mathrm{e}^{-dx-c}a}{2d}-\frac{2b\,\mathrm{e}^{dx+c}}{d(\mathrm{e}^{2dx+2c}+1)}$	54

input
```
int((a+b*sech(d*x+c)^2)*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b*sech(d*x+c)+a/sech(d*x+c))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)\sinh(c+dx)\,dx = \frac{a\cosh\left(dx+c\right)^2+a\sinh\left(dx+c\right)^2+a-2\,b}{2\,d\cosh\left(dx+c\right)}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a - 2*b)/(d*cosh(d*x + c))
```

3.4.6 Sympy [F]

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)\sinh(c+dx)\,dx = \int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\sinh\left(c+dx\right)dx$$

input
```
integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x), x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)\sinh(c+dx)\,dx = \frac{a\cosh\left(dx+c\right)}{d} - \frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="maxima")
```

output
```
a*cosh(d*x + c)/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \left(a+b\text{sech}^2(c+dx)\right)\sinh(c+dx)\,dx = \frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(a*(e^(d*x + c) + e^(-d*x - c)) - 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(a+b\text{sech}^2(c+dx)\right)\sinh(c+dx)\,dx = -\frac{b-a\cosh(c+d\,x)^2}{d\cosh\left(c+d\,x\right)}$$

input
```
int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

output
```
-(b - a*cosh(c + d*x)^2)/(d*cosh(c + d*x))
```

3.5 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

3.5.1 Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=-\frac{(a+b)\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

output
```
-(a+b)*arctanh(cosh(d*x+c))/d+b*sech(d*x+c)/d
```

3.5.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(27) = 54.

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.11

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

output
```
-((a*Log[Cosh[c/2 + (d*x)/2]])/d) - (b*Log[Cosh[(c + d*x)/2]])/d + (a*Log[
Sinh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[(c + d*x)/2]])/d + (b*Sech[c + d*x])
/d
```

3.5.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 26, 4621, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\left(a+b\sec(ic+idx)^2\right)}{\sin(ic+idx)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{b\sec(ic+idx)^2+a}{\sin(ic+idx)}dx$$

$$\Big\downarrow \; 4621$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)\operatorname{sech}^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 359$$

$$-\frac{(a+b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)-b\operatorname{sech}(c+dx)}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{(a+b)\operatorname{arctanh}(\cosh(c+dx))-b\operatorname{sech}(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

output
```
-(((a + b)*ArcTanh[Cosh[c + d*x]] - b*Sech[c + d*x])/d)
```

3.5.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.5.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(\frac{1}{\cosh(dx+c)}-2\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	36
default	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(\frac{1}{\cosh(dx+c)}-2\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	36
risch	$\frac{2b\,\mathrm{e}^{dx+c}}{d\left(\mathrm{e}^{2dx+2c}+1\right)}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)b}{d}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)b}{d}$	85

input
```
int(csch(d*x+c)*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a*arctanh(exp(d*x+c))+b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(27) = 54.

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 6.67

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$= \frac{2\,b\cosh\left(dx+c\right)-\left(\left(a+b\right)\cosh\left(dx+c\right)^2+2\left(a+b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a+b\right)\sinh\left(dx+c\right)^2}{}$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
(2*b*cosh(d*x + c) - ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*si
nh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*
x + c) + 1) + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x
+ c) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*x + c)
- 1) + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x
+ c) + d*sinh(d*x + c)^2 + d)
```

3.5.6 Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)\operatorname{csch}\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*csch(c + d*x), x)
```

3.5.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(27) = 54$.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = -b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right) + \frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*
(e^(-2*d*x - 2*c) + 1))) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.67

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = -\frac{(a+b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a+b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*((a + b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a + b)*log(e^(d*x + c
) + e^(-d*x - c) - 2) - 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.93

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{b}{d\cosh\left(c+d\,x\right)} - \frac{2\,\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a\,\sqrt{-d^2}+b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^2+2\,a\,b+b^2}}\right)\,\sqrt{a^2+2\,a\,b+b^2}}{\sqrt{-d^2}}$$

input
```
int((a + b/cosh(c + d*x)^2)/sinh(c + d*x),x)
```

output
```
b/(d*cosh(c + d*x)) - (2*atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) + b*(-d^2)^
(1/2)))/(d*(2*a*b + a^2 + b^2)^(1/2)))*(2*a*b + a^2 + b^2)^(1/2))/(-d^2)^(
1/2)
```

3.6 $\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

3.6.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=-\frac{(a+b)\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

output
```
-(a+b)*coth(d*x+c)/d-b*tanh(d*x+c)/d
```

3.6.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=-\frac{a\coth(c+dx)}{d}-\frac{b\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

output
```
-((a*Coth[c + d*x])/d) - (b*Coth[c + d*x])/d - (b*Tanh[c + d*x])/d
```

3.6.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4620, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(c+dx) \left(a+b\text{sech}^2(c+dx)\right) \, dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{a+b\sec(ic+idx)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{b\sec(ic+idx)^2+a}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \coth^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 244$$

$$\frac{\int \left((a+b)\coth^2(c+dx)-b\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{-((a+b)\coth(c+dx))-b\tanh(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

output
```
(-((a + b)*Coth[c + d*x]) - b*Tanh[c + d*x])/d
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.6.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

method	result	size
derivativedivides	$\frac{-\coth(dx+c)a+b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	44
default	$\frac{-\coth(dx+c)a+b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	44
risch	$-\frac{2\left(\mathrm{e}^{2dx+2c}a+a+2b\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)\left(\mathrm{e}^{2dx+2c}-1\right)}$	48

input
```
int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a+b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c)))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.37

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = -\frac{4\left((a+b)\cosh(dx+c) - b\sinh(dx+c)\right)}{d\cosh(dx+c)^3 + 3\,d\cosh(dx+c)\sinh(dx+c)^2 + d\sinh(dx+c)^3 - d\cosh(dx+c) + \left(3\,d\cosh(dx+\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-4*((a + b)*cosh(d*x + c) - b*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh
(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cos
h(d*x + c)^2 + d)*sinh(d*x + c))
```

3.6.6 Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \int \left(a+b\operatorname{sech}^2(c+dx)\right)\operatorname{csch}^2(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**2, x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)} + \frac{4\,b}{d(e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
2*a/(d*(e^(-2*d*x - 2*c) - 1)) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=-\frac{2\left(ae^{(2\,dx+2\,c)}+a+2\,b\right)}{d(e^{(4\,dx+4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
-2*(a*e^(2*d*x + 2*c) + a + 2*b)/(d*(e^(4*d*x + 4*c) - 1))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=-\frac{2\left(a+2\,b+a\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^2,x)
```

output
```
-(2*(a + 2*b + a*exp(2*c + 2*d*x)))/(d*(exp(4*c + 4*d*x) - 1))
```

3.7 $\int \operatorname{csch}^3(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right) \, dx$

3.7.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \operatorname{csch}^3(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right) \, dx = \frac{(a+3b)\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{b\operatorname{sech}(c+dx)}{d}$$

output
```
1/2*(a+3*b)*arctanh(cosh(d*x+c))/d-1/2*(a+b)*coth(d*x+c)*csch(d*x+c)/d-b*s
ech(d*x+c)/d
```

3.7.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. $2(54) = 108$.

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.13

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = -\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{3b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/8*(a*Csch[(c + d*x)/2]^2)/d - (b*Csch[(c + d*x)/2]^2)/(8*d) + (a*Log[Co
sh[(c + d*x)/2]])/(2*d) + (3*b*Log[Cosh[(c + d*x)/2]])/(2*d) - (a*Log[Sinh
[(c + d*x)/2]])/(2*d) - (3*b*Log[Sinh[(c + d*x)/2]])/(2*d) - (a*Sech[(c +
d*x)/2]^2)/(8*d) - (b*Sech[(c + d*x)/2]^2)/(8*d) - (b*Sech[c + d*x])/d
```

3.7.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4621, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\left(a+b\sec(ic+idx)^2\right)}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{b\sec(ic+idx)^2+a}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \; 4621$$

$$\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)\text{sech}^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 361$$

$$\frac{\frac{(a+b)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{\left((a+b)\cosh^2(c+dx)+2b\right)\text{sech}^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\int \frac{\left((a+b)\cosh^2(c+dx)+2b\right)\text{sech}^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{(a+b)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{1}{2}\left((a+3b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)-2b\text{sech}(c+dx)\right)+\frac{(a+b)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}((a+3b)\text{arctanh}(\cosh(c+dx))-2b\text{sech}(c+dx))+\frac{(a+b)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

output
```
(((a + b)*Cosh[c + d*x])/(2*(1 - Cosh[c + d*x]^2)) + ((a + 3*b)*ArcTanh[Co
sh[c + d*x]] - 2*b*Sech[c + d*x])/2)/d
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.7. $\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

3.7.4 Maple [A] (verified)

Time = 6.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b\left(-\frac{1}{2\sinh(dx+c)^{2}\cosh(dx+c)}-\frac{3}{2\cosh(dx+c)}+3\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b\left(-\frac{1}{2\sinh(dx+c)^{2}\cosh(dx+c)}-\frac{3}{2\cosh(dx+c)}+3\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$
risch	$-\frac{\mathrm{e}^{dx+c}\left(a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a-2b\,\mathrm{e}^{2dx+2c}+a+3b\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}-\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{2d}-\frac{3\ln\left(\mathrm{e}^{dx+c}-1\right)b}{2d}+\frac{a\ln\left(\mathrm{e}^{dx+c}\right.}{2d}$

input
```
int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(-1/2/sinh(d*x
+c)^2/cosh(d*x+c)-3/2/cosh(d*x+c)+3*arctanh(exp(d*x+c))))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(50) = 100$.

Time = 0.26 (sec) , antiderivative size = 924, normalized size of antiderivative = 17.11

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*(a + 3*b)*cosh(d*x + c)^5 + 10*(a + 3*b)*cosh(d*x + c)*sinh(d*x +
c)^4 + 2*(a + 3*b)*sinh(d*x + c)^5 + 4*(a - b)*cosh(d*x + c)^3 + 4*(5*(a +
 3*b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 4*(5*(a + 3*b)*cosh(d*x +
 c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x +
c) - ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^
5 + (a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4 + (15*(a + 3*b)*
cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*b)*cosh(d*x + c)^
3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c)^2 +
 (15*(a + 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*si
nh(d*x + c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3*b)*cosh(d*x + c)
^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b)*log(cosh(d*x + c) +
 sinh(d*x + c) + 1) + ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x +
c)*sinh(d*x + c)^5 + (a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4
 + (15*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*
b)*cosh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*
cosh(d*x + c)^2 + (15*(a + 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c
)^2 - a - 3*b)*sinh(d*x + c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3
*b)*cosh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b)*lo
g(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(5*(a + 3*b)*cosh(d*x + c)^4 + 6*
(a - b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + ...
```

3.7.6 Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\int\left(a+b\operatorname{sech}^2(c+dx)\right)\operatorname{csch}^3(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**3, x)
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(50) = 100$.

Time = 0.21 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.67

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$=\frac{1}{2}\,b\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-2\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}-e^{(-6\,dx-6\,c)}-1\right)}\right)$$
$$+\frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*
x - c) - 2*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2*c) + e
^(-4*d*x - 4*c) - e^(-6*d*x - 6*c) - 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(
-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(50) = 100$.

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.63

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$=\frac{(a+3\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a+3\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3\,b\left(e^{(dx+\cdots}\right.\right.}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-4\,e^{(dx+\cdots}}}{4\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
1/4*((a + 3*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a + 3*b)*log(e^(d*x
+ c) + e^(-d*x - c) - 2) - 4*(a*(e^(d*x + c) + e^(-d*x - c))^2 + 3*b*(e^(d
*x + c) + e^(-d*x - c))^2 - 8*b)/((e^(d*x + c) + e^(-d*x - c))^3 - 4*e^(d*
x + c) - 4*e^(-d*x - c)))/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.96

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a\,\sqrt{-d^2}+3\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^2+6\,a\,b+9\,b^2}}\right)\,\sqrt{a^2+6\,a\,b+9\,b^2}}{\sqrt{-d^2}} - \frac{e^{c+d\,x}\,(a+b)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} - \frac{2\,e^{c+d\,x}\,(a+b)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} - \frac{2\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) + 3*b*(-d^2)^(1/2)))/(d*(6*a*b + a^
2 + 9*b^2)^(1/2)))*(6*a*b + a^2 + 9*b^2)^(1/2))/(-d^2)^(1/2) - (exp(c + d*
x)*(a + b))/(d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x)*(a + b))/(d*(exp(
4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(exp(2*c +
 2*d*x) + 1))
```

3.8 $\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

3.8.1 Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\frac{(a+2b)\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}$$

output
```
(a+2*b)*coth(d*x+c)/d-1/3*(a+b)*coth(d*x+c)^3/d+b*tanh(d*x+c)/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\frac{2a\coth(c+dx)}{3d}+\frac{5b\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{b\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

output
```
(2*a*Coth[c + d*x])/(3*d) + (5*b*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*C
sch[c + d*x]^2)/(3*d) - (b*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) + (b*Tanh[
c + d*x])/d
```

3.8.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4620, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(ic+idx)^2}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left((a+b)\coth^4(c+dx)+(-a-2b)\coth^2(c+dx)+b\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}(a+b)\coth^3(c+dx)+(a+2b)\coth(c+dx)+b\tanh(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b)*Coth[c + d*x] - ((a + b)*Coth[c + d*x]^3)/3 + b*Tanh[c + d*x])/
d
```

3.8.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.8.4 Maple [A] (verified)

Time = 12.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{3\sinh(dx+c)^3\cosh(dx+c)}+\frac{4}{3\sinh(dx+c)\cosh(dx+c)}+\frac{8\tanh(dx+c)}{3}\right)}{d}$	73
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{3\sinh(dx+c)^3\cosh(dx+c)}+\frac{4}{3\sinh(dx+c)\cosh(dx+c)}+\frac{8\tanh(dx+c)}{3}\right)}{d}$	73
risch	$-\frac{4\left(3a\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+8b\,\mathrm{e}^{2dx+2c}-a-4b\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3\left(\mathrm{e}^{2dx+2c}+1\right)}$	75

input
```
int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(-1/3/sinh(d*x+c)^3/cosh(d*x+
c)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c)))
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(43) = 86.

Time = 0.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.47

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx =$$

$$-\frac{}{3\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6-2\,d\cosh\left(dx+c\right)^4+\left(15\,d\cos\right.\right.}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-8/3*((a - 2*b)*cosh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c) +
(a - 2*b)*sinh(d*x + c)^2 + a + 4*b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c
)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 2*d*cosh(d*x + c)^4 + (15*d*cosh(d
*x + c)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 2*d*cosh(d*x +
 c))*sinh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 12*d*co
sh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 4*d*cosh(d*x
 + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + 2*d)
```

3.8.6 Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\int \left(a+b\operatorname{sech}^2(c+dx)\right)\operatorname{csch}^4(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**4, x)
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(43)=86$.

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 4.16

$$\begin{aligned}&\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx\\&=\frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)\\&\quad+\frac{16}{3}b\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1)}-\frac{1}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1)}\right)\end{aligned}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 16/3*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) -
 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) -
2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)))
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.78

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$= -\frac{2\left(\frac{3\,b}{e^{(2\,dx+2\,c)}+1}-\frac{3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-12\,be^{(2\,dx+2\,c)}+2\,a+5\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^3}\right)}{3\,d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
-2/3*(3*b/(e^(2*d*x + 2*c) + 1) - (3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*
c) - 12*b*e^(2*d*x + 2*c) + 2*a + 5*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.8.9 Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{\frac{2\,b}{3\,d}+\frac{2\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{3\,d}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a+3\,b)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}$$
$$-\frac{\frac{2\,(2\,a+3\,b)}{3\,d}-\frac{2\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}$$
$$+\frac{2\,b}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

output
```
((2*b)/(3*d) + (2*b*exp(4*c + 4*d*x))/(3*d) - (4*exp(2*c + 2*d*x)*(2*a + 3
*b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) -
1) - ((2*(2*a + 3*b))/(3*d) - (2*b*exp(2*c + 2*d*x))/(3*d))/(exp(4*c + 4*d
*x) - 2*exp(2*c + 2*d*x) + 1) + (2*b)/(3*d*(exp(2*c + 2*d*x) - 1)) - (2*b)
/(d*(exp(2*c + 2*d*x) + 1))
```

3.9 $\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx$

3.9.1 Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx = \frac{1}{8}\left(3a^2-24ab+8b^2\right)x - \frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{(a^2-8ab+4b^2)\tanh(c+dx)}{4d} + \frac{a^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
1/8*(3*a^2-24*a*b+8*b^2)*x-1/8*a*(a-8*b)*cosh(d*x+c)*sinh(d*x+c)/d-1/4*(a^
2-8*a*b+4*b^2)*tanh(d*x+c)/d+1/4*a^2*sinh(d*x+c)^4*tanh(d*x+c)/d-1/3*b^2*t
anh(d*x+c)^3/d
```

3.9.2 Mathematica [A] (verified)

Time = 2.73 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.34

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx$$
$$=\frac{\left(b+a\cosh^2(c+dx)\right)^2 \text{sech}^3(c+dx)\left(32b^2\text{sech}(c)\sinh(dx)+64(3a-2b)b\cosh^2(c+dx)\text{sech}(c)\sinh(dx)\right.}{24d(a+}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]
```

output
```
((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(32*b^2*Sech[c]*Sinh[d*x] + 64*
(3*a - 2*b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*Cosh[c + d*x]^3*(4*(3*
a^2 - 24*a*b + 8*b^2)*d*x - 8*a*(a - 2*b)*Sinh[2*(c + d*x)] + a^2*Sinh[4*(
c + d*x)]) + 32*b^2*Cosh[c + d*x]*Tanh[c]))/(24*d*(a + 2*b + a*Cosh[2*(c +
 d*x)])^2)
```

3.9.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4620, 366, 360, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^2}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{a^2\tanh^5(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{\tanh^4(c+dx)\left(5a^2-4(a+b)^2+4b^2\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 360$$

$$\frac{\frac{1}{4}\left(-\frac{1}{2}\int -\frac{8b^2\tanh^4(c+dx)+2a(a-8b)\tanh^2(c+dx)+a(a-8b)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{a(a-8b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{a^2\tanh^5(c+dx)}{4(1-\tanh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b^2\tanh^4(c+dx)+2a(a-8b)\tanh^2(c+dx)+a(a-8b)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{a(a-8b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{a^2\tanh^5(c+dx)}{4(1-\tanh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 1467$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \left(-8b^2\tanh^2(c+dx)-2(a^2-8ba+4b^2)+\frac{3a^2-24ba+8b^2}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)-\frac{a(a-8b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{a^2\tanh^5(\ldots}{4(1-\tanh^2\ldots}}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left((3a^2-24ab+8b^2)\operatorname{arctanh}(\tanh(c+dx))-2(a^2-8ab+4b^2)\tanh(c+dx)-\frac{8}{3}b^2\tanh^3(c+dx)\right)-\frac{a(a-8b)\ldots}{2(1-\tan\ldots}}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]
```

output
```
((a^2*Tanh[c + d*x]^5)/(4*(1 - Tanh[c + d*x]^2)^2) + (-1/2*(a*(a - 8*b)*Ta
nh[c + d*x])/(1 - Tanh[c + d*x]^2) + ((3*a^2 - 24*a*b + 8*b^2)*ArcTanh[Tan
h[c + d*x]] - 2*(a^2 - 8*a*b + 4*b^2)*Tanh[c + d*x] - (8*b^2*Tanh[c + d*x]
^3)/3)/2)/4)/d
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.9. $\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx$

3.9.4 Maple [A] (verified)

Time = 13.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^2\left(dx+c-\tan\right.}{d}$
default	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^2\left(dx+c-\tan\right.}{d}$
parts	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^2\left(dx+c-\tanh(dx+c)-\frac{\tanh(dx+c)^3}{3}\right)}{d}+\frac{2ab\left(\frac{\sinh(dx+}{2\cosh(dx}\right.}{}$
risch	$\frac{3a^2x}{8}-3abx+b^2x+\frac{a^2e^{4dx+4c}}{64d}-\frac{a^2e^{2dx+2c}}{8d}+\frac{a\,e^{2dx+2c}b}{4d}+\frac{a^2e^{-2dx-2c}}{8d}-\frac{a\,e^{-2dx-2c}b}{4d}-\frac{a^2e^{-4dx-4}}{64d}$

input
```
int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2
*a*b*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^2*(d*
x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3))
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(104) = 208$.

Time = 0.25 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.00

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\sinh^4(c+dx)\,dx$$
$$=\frac{3\,a^2\sinh\left(dx+c\right)^7+3\left(21\,a^2\cosh\left(dx+c\right)^2-5\,a^2+16\,ab\right)\sinh\left(dx+c\right)^5+8\left(3\left(3\,a^2-24\,ab+8\,b^2\right)dx-\right.}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/192*(3*a^2*sinh(d*x + c)^7 + 3*(21*a^2*cosh(d*x + c)^2 - 5*a^2 + 16*a*b)
*sinh(d*x + c)^5 + 8*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*co
sh(d*x + c)^3 + 24*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh
(d*x + c)*sinh(d*x + c)^2 + (105*a^2*cosh(d*x + c)^4 - 30*(5*a^2 - 16*a*b)
*cosh(d*x + c)^2 - 63*a^2 + 528*a*b - 256*b^2)*sinh(d*x + c)^3 + 24*(3*(3*
a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*x + c) + 3*(7*a^2*cosh
(d*x + c)^6 - 5*(5*a^2 - 16*a*b)*cosh(d*x + c)^4 - (63*a^2 - 528*a*b + 256
*b^2)*cosh(d*x + c)^2 - 15*a^2 + 160*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^
3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \left(a + b\text{sech}^2(c + dx)\right)^2 \sinh^4(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**4,x)
```

output
```
Timed out
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(104) = 208$.

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.85

$$\begin{aligned}
&\int \left(a + b\text{sech}^2(c + dx)\right)^2 \sinh^4(c + dx)\, dx \\
&= \frac{1}{64}a^2\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) \\
&\quad + \frac{1}{3}b^2\left(3\,x + \frac{3\,c}{d} - \frac{4\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2\right)}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right) \\
&\quad - \frac{1}{4}ab\left(\frac{12\,(dx+c)}{d} + \frac{e^{(-2\,dx-2\,c)}}{d} - \frac{17\,e^{(-2\,dx-2\,c)} + 1}{d(e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)})}\right)
\end{aligned}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1))) - 1/4*a*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (1
7*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(104) = 208$.

Time = 0.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.03

$$\int \left(a + b\mathrm{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx$$
$$= \frac{3\,a^2e^{(4\,dx+4\,c)} - 24\,a^2e^{(2\,dx+2\,c)} + 48\,abe^{(2\,dx+2\,c)} + 24\,(3\,a^2 - 24\,ab + 8\,b^2)(dx+c) - 3\,\left(18\,a^2e^{(4\,dx+4\,c)} - 144\right.}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="giac")
```

output
```
1/192*(3*a^2*e^(4*d*x + 4*c) - 24*a^2*e^(2*d*x + 2*c) + 48*a*b*e^(2*d*x +
2*c) + 24*(3*a^2 - 24*a*b + 8*b^2)*(d*x + c) - 3*(18*a^2*e^(4*d*x + 4*c) -
 144*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c)
+ 16*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c) - 256*(3*a*b*e^(4*d*x + 4
*c) - 3*b^2*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c
) + 3*a*b - 2*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.36

$$\int \left(a + b\mathrm{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx = x\left(\frac{3\,a^2}{8} - 3\,a\,b + b^2\right) - \frac{\frac{4\,(a\,b-b^2)}{3\,d} + \frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(a\,b-b^2)}{3\,d} + \frac{8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x} + 3\,\mathrm{e}^{4\,c+4\,d\,x} + \mathrm{e}^{6\,c+6\,d\,x} + 1} - \frac{\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a\,b-b^2)}{3\,d} + \frac{4\,a\,b}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x} + \mathrm{e}^{4\,c+4\,d\,x} + 1} - \frac{4\,(a\,b-b^2)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x} + 1\right)} + \frac{\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a\,b - a^2)}{8\,d} - \frac{a^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d} + \frac{a^2\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d} + \frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}\,(a - 2\,b)}{8\,d}$$

input
```
int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
x*((3*a^2)/8 - 3*a*b + b^2) - ((4*(a*b - b^2))/(3*d) + (4*exp(4*c + 4*d*x)
*(a*b - b^2))/(3*d) + (8*a*b*exp(2*c + 2*d*x))/(3*d))/(3*exp(2*c + 2*d*x)
+ 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((4*exp(2*c + 2*d*x)*(a*b -
 b^2))/(3*d) + (4*a*b)/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)
- (4*(a*b - b^2))/(3*d*(exp(2*c + 2*d*x) + 1)) + (exp(2*c + 2*d*x)*(2*a*b
- a^2))/(8*d) - (a^2*exp(- 4*c - 4*d*x))/(64*d) + (a^2*exp(4*c + 4*d*x))/(
64*d) + (a*exp(- 2*c - 2*d*x)*(a - 2*b))/(8*d)
```

3.9. $\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx$

3.10 $\int (a+b\text{sech}^2(c+dx))^2 \sinh^3(c+dx)\,dx$

3.10.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int (a+b\text{sech}^2(c+dx))^2 \sinh^3(c+dx)\,dx = -\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{(2a-b)b\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

output
```
-a*(a-2*b)*cosh(d*x+c)/d+1/3*a^2*cosh(d*x+c)^3/d+(2*a-b)*b*sech(d*x+c)/d+1
/3*b^2*sech(d*x+c)^3/d
```

3.10.2 Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int (a+b\text{sech}^2(c+dx))^2 \sinh^3(c+dx)\,dx = \frac{(-26a^2+168ab-16b^2-3(11a^2-64ab+16b^2)\cosh(2(c+dx))-6a(a-4b)\cosh(4(c+dx))+a^2\cosh(}{96d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^3,x]
```

output
```
((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cosh[2*(c + d*
x)] - 6*a*(a - 4*b)*Cosh[4*(c + d*x)] + a^2*Cosh[6*(c + d*x)])*Sech[c + d*
x]^3)/(96*d)
```

3.10.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 26, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^2\,dx$$

$$\Big\downarrow \text{26}$$

$$i\int \left(b\sec(ic+idx)^2+a\right)^2\sin(ic+idx)^3dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2\text{sech}^4(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(b^2\text{sech}^4(c+dx)+(2a-b)b\text{sech}^2(c+dx)-a^2\cosh^2(c+dx)+a(a-2b)\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}a^2\cosh^3(c+dx)+a(a-2b)\cosh(c+dx)-b(2a-b)\text{sech}(c+dx)-\frac{1}{3}b^2\text{sech}^3(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^3,x]
```

output
```
-((a*(a - 2*b)*Cosh[c + d*x] - (a^2*Cosh[c + d*x]^3)/3 - (2*a - b)*b*Sech[
c + d*x] - (b^2*Sech[c + d*x]^3)/3)/d)
```

3.10.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.10.4 Maple [A] (verified)

Time = 27.89 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{b^2 \operatorname{sech}(dx+c)^3}{3}+2ab \operatorname{sech}(dx+c)-b^2 \operatorname{sech}(dx+c)+\frac{a^2}{3 \operatorname{sech}(dx+c)^3}-\frac{a(a-2b)}{\operatorname{sech}(dx+c)}}{d}$
default	$\frac{\frac{b^2 \operatorname{sech}(dx+c)^3}{3}+2ab \operatorname{sech}(dx+c)-b^2 \operatorname{sech}(dx+c)+\frac{a^2}{3 \operatorname{sech}(dx+c)^3}-\frac{a(a-2b)}{\operatorname{sech}(dx+c)}}{d}$
parts	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b^2\left(\frac{\operatorname{sech}(dx+c)^3}{3}-\operatorname{sech}(dx+c)\right)}{d}+\frac{2ab\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$
risch	$\frac{a^2 e^{3dx+3c}}{24d}-\frac{3a^2 e^{dx+c}}{8d}+\frac{e^{dx+c}ab}{d}-\frac{3a^2 e^{-dx-c}}{8d}+\frac{a\,e^{-dx-c}b}{d}+\frac{a^2 e^{-3dx-3c}}{24d}+\frac{2\,e^{dx+c}b(6a\,e^{4dx+4c}-3b\,e^{4dx\cdots}}{3d(\cdots}$

input
```
int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b^2*sech(d*x+c)^3+2*a*b*sech(d*x+c)-b^2*sech(d*x+c)+1/3*a^2/sech(
d*x+c)^3-a*(a-2*b)/sech(d*x+c))
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(68) = 136.

Time = 0.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.94

$$\int (a+b\mathrm{sech}^2(c+dx))^2\sinh^3(c+dx)\,dx = \frac{a^2\cosh(dx+c)^6 + a^2\sinh(dx+c)^6 - 6(a^2-4ab)\cosh(dx+c)^4 + 3\left(5a^2\cosh(dx+c)^2 - 2a^2 + 8ab\right)}{24}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/24*(a^2*cosh(d*x + c)^6 + a^2*sinh(d*x + c)^6 - 6*(a^2 - 4*a*b)*cosh(d*x
 + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 - 2*a^2 + 8*a*b)*sinh(d*x + c)^4 - 3*(1
1*a^2 - 64*a*b + 16*b^2)*cosh(d*x + c)^2 + 3*(5*a^2*cosh(d*x + c)^4 - 12*(
a^2 - 4*a*b)*cosh(d*x + c)^2 - 11*a^2 + 64*a*b - 16*b^2)*sinh(d*x + c)^2 -
 26*a^2 + 168*a*b - 16*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*
x + c)^2 + 3*d*cosh(d*x + c))
```

3.10.6 Sympy [F]

$$\int (a+b\mathrm{sech}^2(c+dx))^2\sinh^3(c+dx)\,dx = \int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\sinh^3(c+dx)\,dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x)**3, x)
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(68) = 136$.

Time = 0.20 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.69

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\sinh^3(c+dx)\,dx$$
$$=\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+ab\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}\right)-\frac{2}{3}b^2\left(\frac{3\,e^{(-dx-c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{2\,e^{(-3\,dx-3\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\ldots\right)$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d
*x - 3*c)/d) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x
- c) + e^(-3*d*x - 3*c)))) - 2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c
) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3
*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*
d*x - 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
+ 1)))
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(68) = 136$.

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.94

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\sinh^3(c+dx)\,dx$$
$$=\frac{a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+24\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{16\left(6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-3\,b\ldots\right)}{\left(e^{(dx+c)}+e^{(-\ldots}\right.}}{24\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/24*(a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^2*(e^(d*x + c) + e^(-d*x -
 c)) + 24*a*b*(e^(d*x + c) + e^(-d*x - c)) + 16*(6*a*b*(e^(d*x + c) + e^(-
d*x - c))^2 - 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4*b^2)/(e^(d*x + c) +
 e^(-d*x - c))^3)/d
```

3.10.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.79

$$\int (a+b\text{sech}^2(c+dx))^2 \sinh^3(c+dx)\,dx = \frac{e^{c+dx}\,(8\,a\,b-3\,a^2)}{8\,d}+\frac{e^{-c-dx}\,(8\,a\,b-3\,a^2)}{8\,d}+\frac{a^2\,e^{-3\,c-3\,dx}}{24\,d}+\frac{a^2\,e^{3\,c+3\,dx}}{24\,d}-\frac{8\,b^2\,e^{c+dx}}{3\,d\,(3\,e^{2\,c+2\,dx}+3\,e^{4\,c+4\,dx}+e^{6\,c+6\,dx}+1)}+\frac{2\,e^{c+dx}\,(2\,a\,b-b^2)}{d\,(e^{2\,c+2\,dx}+1)}+\frac{8\,b^2\,e^{c+dx}}{3\,d\,(2\,e^{2\,c+2\,dx}+e^{4\,c+4\,dx}+1)}$$

input
```
int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(exp(c + d*x)*(8*a*b - 3*a^2))/(8*d) + (exp(- c - d*x)*(8*a*b - 3*a^2))/(8
*d) + (a^2*exp(- 3*c - 3*d*x))/(24*d) + (a^2*exp(3*c + 3*d*x))/(24*d) - (8
*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) + 1)) + (2*exp(c + d*x)*(2*a*b - b^2))/(d*(exp(2*c + 2*d*x) + 1)
) + (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.11 $\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh^2(c+dx)\,dx$

3.11.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh^2(c+dx)\,dx = -\frac{1}{2}a(a-4b)x+\frac{a(a-4b)\tanh(c+dx)}{2d}+\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
-1/2*a*(a-4*b)*x+1/2*a*(a-4*b)*tanh(d*x+c)/d+1/2*a^2*sinh(d*x+c)^2*tanh(d*
x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

3.11.2 Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.73

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh^2(c+dx)\,dx = \frac{\left(b+a\cosh^2(c+dx)\right)^2\text{sech}^3(c+dx)\left(-4b^2\text{sech}(c)\sinh(dx)-4(6a-b)b\cosh^2(c+dx)\text{sech}(c)\sinh(dx)+\right.}{3d(a+2b+a\cosh(2(c+dx}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]
```

output
```
((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(-4*b^2*Sech[c]*Sinh[d*x] - 4*(
6*a - b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*a*Cosh[c + d*x]^3*(-2*(a
- 4*b)*d*x + a*Sinh[2*(c + d*x)]) - 4*b^2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a
+ 2*b + a*Cosh[2*(c + d*x)])^2)
```

3.11.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4620, 366, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(ic+idx)^2\left(-\left(a+b\sec(ic+idx)^2\right)^2\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \left(b\sec(ic+idx)^2+a\right)^2\sin(ic+idx)^2dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 366$$

$$\frac{\frac{a^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{\tanh^2(c+dx)\left(3a^2-2(a+b)^2+2b^2\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 363$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tanh^3(c+dx)-a(a-4b)\int \frac{\tanh^2(c+dx)}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)+\frac{a^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \ 262$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tanh^3(c+dx)-a(a-4b)\left(\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\tanh(c+dx)\right)\right)+\frac{a^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a^2\tanh^3(c+dx)}{2(1-\tanh^2(c+dx))}+\frac{1}{2}\left(\frac{2}{3}b^2\tanh^3(c+dx)-a(a-4b)(\operatorname{arctanh}(\tanh(c+dx))-\tanh(c+dx))\right)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]
```

output
```
((a^2*Tanh[c + d*x]^3)/(2*(1 - Tanh[c + d*x]^2)) + (-(a*(a - 4*b)*(ArcTanh
[Tanh[c + d*x]] - Tanh[c + d*x])) + (2*b^2*Tanh[c + d*x]^3)/3)/2)/d
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.11.4 Maple [A] (verified)

Time = 20.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab(dx+c-\tanh(dx+c))+b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab(dx+c-\tanh(dx+c))+b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
parts	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}+\frac{2ab(dx+c-\tanh(dx+c}{d}$
risch	$-\frac{a^2x}{2}+2abx+\frac{a^2e^{2dx+2c}}{8d}-\frac{a^2e^{-2dx-2c}}{8d}+\frac{2b\left(6a\,e^{4dx+4c}-3b\,e^{4dx+4c}+12\,e^{2dx+2c}a+6a-b\right)}{3d\left(e^{2dx+2c}+1\right)^3}$

input
```
int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(d*x+c-tanh(d*x
+c))+b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(
d*x+c)))
```

3.11.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(65) = 130$.

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.45

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\sinh^2(c+dx)\,dx = \frac{3\,a^2\sinh\left(dx+c\right)^5-4\left(3\left(a^2-4\,ab\right)dx-12\,ab+2\,b^2\right)\cosh\left(dx+c\right)^3-12\left(3\left(a^2-4\,ab\right)dx-12\,ab+2\,b^2\right)}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/24*(3*a^2*sinh(d*x + c)^5 - 4*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cos
h(d*x + c)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c)*sin
h(d*x + c)^2 + (30*a^2*cosh(d*x + c)^2 + 9*a^2 - 48*a*b + 8*b^2)*sinh(d*x
+ c)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c) + 3*(5*a^
2*cosh(d*x + c)^4 + (9*a^2 - 48*a*b + 8*b^2)*cosh(d*x + c)^2 + 2*a^2 - 16*
a*b - 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*
x + c)^2 + 3*d*cosh(d*x + c))
```

3.11.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\sinh^2(c+dx)\,dx = \int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)^2\sinh^2\left(c+dx\right)dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x)**2, x)
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.19

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\sinh^2(c+dx)\,dx$$
$$= -\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+2ab\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+\frac{2}{3}b^2\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a*b*(x + c/d -
 2/(d*(e^(-2*d*x - 2*c) + 1))) + 2/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d
*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*
x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.97

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\sinh^2(c+dx)\,dx$$
$$= \frac{3a^2e^{(2dx+2c)}-12(a^2-4ab)(dx+c)+3\left(2a^2e^{(2dx+2c)}-8abe^{(2dx+2c)}-a^2\right)e^{(-2dx-2c)}+\frac{16\left(6abe^{(4dx+4c)}-3\right.}{}}{24d}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/24*(3*a^2*e^(2*d*x + 2*c) - 12*(a^2 - 4*a*b)*(d*x + c) + 3*(2*a^2*e^(2*d
*x + 2*c) - 8*a*b*e^(2*d*x + 2*c) - a^2)*e^(-2*d*x - 2*c) + 16*(6*a*b*e^(4
*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 6*a*b - b^2
)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.23

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^2\sinh^2(c+dx)\,dx$$
$$=\frac{\frac{2\,\left(b^2+2\,a\,b\right)}{3\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a\,b-b^2\right)}{3\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+x\,\left(2\,a\,b-\frac{a^2}{2}\right)$$
$$+\frac{\frac{2\,\left(2\,a\,b-b^2\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+2\,a\,b\right)}{3\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a\,b-b^2\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}$$
$$+\frac{2\,\left(2\,a\,b-b^2\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{a^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}+\frac{a^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}$$

input
```
int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
((2*(2*a*b + b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b - b^2))/(3*d))/(2*ex
p(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + x*(2*a*b - a^2/2) + ((2*(2*a*b -
b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d) + (2*exp(4*c + 4*d*
x)*(2*a*b - b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) + 1) + (2*(2*a*b - b^2))/(3*d*(exp(2*c + 2*d*x) + 1)) - (a^2*ex
p(- 2*c - 2*d*x))/(8*d) + (a^2*exp(2*c + 2*d*x))/(8*d)
```

3.12 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\sinh(c+dx)\,dx$

3.12.1 Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\sinh(c+dx)\,dx=\frac{a^2\cosh(c+dx)}{d}-\frac{2ab\,\mathrm{sech}(c+dx)}{d}-\frac{b^2\mathrm{sech}^3(c+dx)}{3d}$$

output
```
a^2*cosh(d*x+c)/d-2*a*b*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

3.12.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\sinh(c+dx)\,dx=\frac{(9a^2-24ab-8b^2+12a(a-2b)\cosh(2(c+dx))+3a^2\cosh(4(c+dx)))\,\mathrm{sech}^3(c+dx)}{24d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x],x]
```

output
```
((9*a^2 - 24*a*b - 8*b^2 + 12*a*(a - 2*b)*Cosh[2*(c + d*x)] + 3*a^2*Cosh[4
*(c + d*x)])*Sech[c + d*x]^3)/(24*d)
```

3.12.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 4621, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a+b\sec(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \left(b\sec(ic+idx)^2+a\right)^2\sin(ic+idx)dx$$

$$\Big\downarrow \text{4621}$$

$$\frac{\int \left(a\cosh^2(c+dx)+b\right)^2\text{sech}^4(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(b^2\text{sech}^4(c+dx)+2ab\text{sech}^2(c+dx)+a^2\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\cosh(c+dx)-2ab\text{sech}(c+dx)-\frac{1}{3}b^2\text{sech}^3(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x],x]
```

output
```
(a^2*Cosh[c + d*x] - 2*a*b*Sech[c + d*x] - (b^2*Sech[c + d*x]^3)/3)/d
```

3.12.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.12.4 Maple [A] (verified)

Time = 11.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{\frac{b^2\operatorname{sech}(dx+c)^3}{3}+2ab\operatorname{sech}(dx+c)-\frac{a^2}{\operatorname{sech}(dx+c)}}{d}$	43
default	$-\frac{\frac{b^2\operatorname{sech}(dx+c)^3}{3}+2ab\operatorname{sech}(dx+c)-\frac{a^2}{\operatorname{sech}(dx+c)}}{d}$	43
parts	$\frac{a^2\cosh(dx+c)}{d}-\frac{2ab\operatorname{sech}(dx+c)}{d}-\frac{b^2\operatorname{sech}(dx+c)^3}{3d}$	44
risch	$\frac{a^2 e^{dx+c}}{2d}+\frac{a^2 e^{-dx-c}}{2d}-\frac{4\,e^{dx+c}b\left(3a\,e^{4dx+4c}+6\,e^{2dx+2c}a+2b\,e^{2dx+2c}+3a\right)}{3d\left(e^{2dx+2c}+1\right)^3}$	98

input
```
int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/3*b^2*sech(d*x+c)^3+2*a*b*sech(d*x+c)-a^2/sech(d*x+c))
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.96

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\sinh(c+dx)\,dx = \frac{3\,a^2\cosh\left(dx+c\right)^4+3\,a^2\sinh\left(dx+c\right)^4+12\left(a^2-2\,ab\right)\cosh\left(dx+c\right)^2+6\left(3\,a^2\cosh\left(dx+c\right)^2+2\,a^2-4\cdots\right.}{6\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\,d\cosh\left(dx+\cdots\right.\right.}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="fricas")
```

output
```
1/6*(3*a^2*cosh(d*x + c)^4 + 3*a^2*sinh(d*x + c)^4 + 12*(a^2 - 2*a*b)*cosh
(d*x + c)^2 + 6*(3*a^2*cosh(d*x + c)^2 + 2*a^2 - 4*a*b)*sinh(d*x + c)^2 +
9*a^2 - 24*a*b - 8*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x +
c)^2 + 3*d*cosh(d*x + c))
```

3.12.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\sinh(c+dx)\,dx = \int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)^2\sinh\left(c+dx\right)\,dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x), x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\sinh(c+dx)\,dx = \frac{a^2\cosh\left(dx+c\right)}{d} - \frac{4\,ab}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)} - \frac{8\,b^2}{3\,d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="maxima")
```

output
```
a^2*cosh(d*x + c)/d - 4*a*b/(d*(e^(d*x + c) + e^(-d*x - c))) - 8/3*b^2/(d*
(e^(d*x + c) + e^(-d*x - c))^3)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.67

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh(c+dx)\,dx = \frac{3\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{8\left(3\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+2\,b^2\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{6\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="giac")
```

output
```
1/6*(3*a^2*(e^(d*x + c) + e^(-d*x - c)) - 8*(3*a*b*(e^(d*x + c) + e^(-d*x
- c))^2 + 2*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

3.12.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh(c+dx)\,dx = \frac{a^2\cosh(c+d\,x)}{d}-\frac{\frac{b^2}{3}+2\,a\,b\cosh(c+d\,x)^2}{d\cosh\left(c+d\,x\right)^3}$$

input
```
int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(a^2*cosh(c + d*x))/d - (b^2/3 + 2*a*b*cosh(c + d*x)^2)/(d*cosh(c + d*x)^3
)
```

3.13 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

3.13.1 Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=-\frac{(a+b)^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

output
```
-(a+b)^2*arctanh(cosh(d*x+c))/d+b*(2*a+b)*sech(d*x+c)/d+1/3*b^2*sech(d*x+c
)^3/d
```

3.13.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(52) = 104.

Time = 3.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.08

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=$$
$$-\frac{4\left(b+a\cosh^2(c+dx)\right)^2\left(-b^2-3b(2a+b)\cosh^2(c+dx)+3(a+b)^2\cosh^3(c+dx)\left(\log\left(\cosh\left(\frac{1}{2}(c+d\right.\right.\right.\right.}{3d(a+2b+a\cosh(2(c+dx)))^2}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(-4*(b + a*Cosh[c + d*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cosh[c + d*x]^2 + 3*(a
 + b)^2*Cosh[c + d*x]^3*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))
*Sech[c + d*x]^3)/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

3.13.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 4621, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sec(ic+idx)^2\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sec(ic+idx)^2+a\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^2\operatorname{sech}^4(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$-\frac{\int \left(b^2\operatorname{sech}^4(c+dx)+b(2a+b)\operatorname{sech}^2(c+dx)-\frac{(a+b)^2}{\cosh^2(c+dx)-1}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b)^2\operatorname{arctanh}(\cosh(c+dx))-b(2a+b)\operatorname{sech}(c+dx)-\frac{1}{3}b^2\operatorname{sech}^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-(((a + b)^2*ArcTanh[Cosh[c + d*x]] - b*(2*a + b)*Sech[c + d*x] - (b^2*Sec
h[c + d*x]^3)/3)/d)
```

3.13.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.13.4 Maple [A] (verified)

Time = 15.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{-2a^{2} \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+2ab\left(\frac{1}{\cosh\left(dx+c\right)}-2 \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b^{2}\left(\frac{1}{3\cosh\left(dx+c\right)^{3}}+\frac{1}{\cosh\left(dx+c\right)}-2 \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$
default	$\frac{-2a^{2} \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+2ab\left(\frac{1}{\cosh\left(dx+c\right)}-2 \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b^{2}\left(\frac{1}{3\cosh\left(dx+c\right)^{3}}+\frac{1}{\cosh\left(dx+c\right)}-2 \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$
risch	$\frac{2b\,\mathrm{e}^{dx+c}\left(6a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}+12\,\mathrm{e}^{2dx+2c}a+10b\,\mathrm{e}^{2dx+2c}+6a+3b\right)}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^{3}}-\frac{a^{2}\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}-\frac{2\ln\left(\mathrm{e}^{dx+c}+1\right)ab}{d}-\ln($

input
```
int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c))
)+b^2*(1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(50) = 100$.

Time = 0.27 (sec) , antiderivative size = 1148, normalized size of antiderivative = 22.08

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + 5*b^2)*cosh(d*x
 + c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + 5*b^2)*sinh(d*x +
c)^3 + 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + 5*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) - 3*((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (
a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4
 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x
+ c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*
cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3
*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 +
 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a
^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d
*x + c) + 1) + 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 +
3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x +...
```

3.13.6 Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\int \left(a+b\operatorname{sech}^2{(c+dx)}\right)^2\operatorname{csch}{(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x), x)
```

3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(50) = 100$.

Time = 0.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.79

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=&\\ -\frac{1}{3}\,b^2&\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(3\,e^{(-dx-c)}+10\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)\\ &-2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)\\ &+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}\end{aligned}$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/3*b^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d - 2*(3*e^(
-d*x - c) + 10*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 2*a*b*(log(e^(-d*x - c
) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2*d*x - 2*c) +
 1))) + a^2*log(tanh(1/2*d*x + 1/2*c))/d
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(50) = 100$.

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.67

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx =$$

$$-\frac{3\,(a^2+2\,ab+b^2)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\,(a^2+2\,ab+b^2)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(6\,ab(e^{(c}\right.}{}}{6\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*(a^2 + 2*a*b + b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*(a^2 +
 2*a*b + b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(6*a*b*(e^(d*x + c)
+ e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4*b^2)/(e^(d*x
+ c) + e^(-d*x - c))^3)/d
```

3.13.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.46

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$

$$=\frac{2\,e^{c+d\,x}\,(b^2+2\,a\,b)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$

$$-\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^2\,\sqrt{-d^2}+b^2\,\sqrt{-d^2}+2\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}}\right)\,\sqrt{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}}{\sqrt{-d^2}}$$

$$+\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x),x)
```

output
```
(2*exp(c + d*x)*(2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8*b^2*exp(c +
 d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
1)) - (2*atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) + b^2*(-d^2)^(1/2) + 2*a*
b*(-d^2)^(1/2)))/(d*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2)))*(4
*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2))/(-d^2)^(1/2) + (8*b^2*exp
(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.14 $\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

3.14.1 Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{2b(a+b)\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
-(a+b)^2*coth(d*x+c)/d-2*b*(a+b)*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

3.14.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 136 vs. 2(50) = 100.

Time = 4.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.72

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{4\left(b+a\cosh^2(c+dx)\right)^2\operatorname{sech}^3(c+dx)\left(-b^2\operatorname{sech}(c)\sinh(dx)+\frac{1}{2}\cosh(c+dx)\left((3a^2+12ab+8b^2)\cosh(dx\right.\right.}{3d(a+2b+a\cosh(2(}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(-(b^2*Sech[c]*Sinh[d*x]) + (
Cosh[c + d*x]*((3*a^2 + 12*a*b + 8*b^2)*Cosh[d*x] + (3*a^2 - 2*b^2)*Cosh[2
*c + d*x])*Coth[c + d*x]*Csch[c]*Sech[c]*Sinh[d*x])/2 - b^2*Cosh[c + d*x]*
Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

3.14.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4620, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\left(a+b\sec(ic+idx)^2\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\left(b\sec(ic+idx)^2+a\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \coth^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^2 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left((a+b)^2\coth^2(c+dx)+b^2\tanh^2(c+dx)-2b(a+b)\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-2b(a+b)\tanh(c+dx)-(a+b)^2\coth(c+dx)+\frac{1}{3}b^2\tanh^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(-((a + b)^2*Coth[c + d*x]) - 2*b*(a + b)*Tanh[c + d*x] + (b^2*Tanh[c + d*
x]^3)/3)/d
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.14.4 Maple [A] (verified)

Time = 28.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{-\coth(dx+c)a^2+2ab\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh\right.}{d}$
default	$\frac{-\coth(dx+c)a^2+2ab\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh\right.}{d}$
risch	$-\frac{2\left(3a^2 e^{6dx+6c}+9a^2 e^{4dx+4c}+12ab\,e^{4dx+4c}+9a^2 e^{2dx+2c}+24ab\,e^{2dx+2c}+16\,e^{2dx+2c}b^2+3a^2+12ab+8b^2\right)}{3d\left(e^{2dx+2c}-1\right)\left(e^{2dx+2c}+1\right)^3}$

input
```
int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^2+2*a*b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c))+b^2
*(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(48) = 96$.

Time = 0.25 (sec) , antiderivative size = 284, normalized size of antiderivative = 5.68

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{4\left(\left(3a^2+6ab+4b^2\right)\cosh\left(dx+c\right)^3+3\left(3a^2+6ab+4b^2\right)\cosh\left(dx+c\right)\operatorname{si}\right.}{3\left(d\cosh\left(dx+c\right)^5+5d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+d\sinh\left(dx+c\right)^5+d\cosh\left(dx+c\right)^3+\left(10d\cosh\right.\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-4/3*((3*a^2 + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 6*a*b + 4*b^2)*
cosh(d*x + c)*sinh(d*x + c)^2 - 2*(3*a*b + 2*b^2)*sinh(d*x + c)^3 + (9*a^2
 + 18*a*b + 8*b^2)*cosh(d*x + c) - 2*(3*(3*a*b + 2*b^2)*cosh(d*x + c)^2 +
3*a*b + 4*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(
d*x + c)^4 + d*sinh(d*x + c)^5 + d*cosh(d*x + c)^3 + (10*d*cosh(d*x + c)^2
 + 3*d)*sinh(d*x + c)^3 + (10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(
d*x + c)^2 - 2*d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^
2 + 2*d)*sinh(d*x + c))
```

3.14.6 Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2\operatorname{csch}^2\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**2, x)
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.80

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{16}{3}b^2\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1)}+\frac{1}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1)}\right) + \frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}-1)}+\frac{8\,ab}{d(e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-16/3*b^2*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c)
- e^(-8*d*x - 8*c) + 1)) + 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) -
 e^(-8*d*x - 8*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) + 8*a*b/(d*(e^
(-4*d*x - 4*c) - 1))
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(48) = 96$.

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.22

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{2\left(\frac{3\,(a^2+2\,ab+b^2)}{e^{(2\,dx+2\,c)}-1}-\frac{6\,abe^{(4\,dx+4\,c)}+3\,b^2e^{(4\,dx+4\,c)}+12\,abe^{(2\,dx+2\,c)}+12\,b^2e^{(2\,dx+2\,c)}+6\,ab+5\,b^2}{\left(e^{(2\,dx+2\,c)}+1\right)^3}\right)}{3\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-2/3*(3*(a^2 + 2*a*b + b^2)/(e^(2*d*x + 2*c) - 1) - (6*a*b*e^(4*d*x + 4*c)
 + 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 12*b^2*e^(2*d*x + 2*c)
 + 6*a*b + 5*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.14.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 215, normalized size of antiderivative = 4.30

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{\frac{2\,(3\,b^2+2\,a\,b)}{3\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{3\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}+\frac{\frac{2\,(b^2+2\,a\,b)}{3\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(b^2+2\,a\,b)}{3\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(3\,b^2+2\,a\,b)}{3\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}$$
$$-\frac{2\,(a^2+2\,a\,b+b^2)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}+\frac{2\,(b^2+2\,a\,b)}{3\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

output
```
((2*(2*a*b + 3*b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d))/(2*
exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(2*a*b + b^2))/(3*d) + (2*e
xp(4*c + 4*d*x)*(2*a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + 3*b^2)
)/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)
- (2*(2*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) + (2*(2*a*b + b^2))/(
3*d*(exp(2*c + 2*d*x) + 1))
```

3.15 $\int \mathrm{csch}^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

3.15.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \mathrm{csch}^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx = \frac{(a+b)(a+5b)\,\mathrm{arctanh}(\cosh(c+dx))}{2d} - \frac{(3a^2+6ab+5b^2)\coth(c+dx)\mathrm{csch}(c+dx)}{6d} - \frac{b(6a+5b)\mathrm{sech}(c+dx)}{3d} + \frac{b^2\mathrm{csch}^2(c+dx)\mathrm{sech}^3(c+dx)}{3d}$$

output
```
1/2*(a+b)*(a+5*b)*arctanh(cosh(d*x+c))/d-1/6*(3*a^2+6*a*b+5*b^2)*coth(d*x+
c)*csch(d*x+c)/d-1/3*b*(6*a+5*b)*sech(d*x+c)/d+1/3*b^2*csch(d*x+c)^2*sech(
d*x+c)^3/d
```

3.15.2 Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.38

$$\int \mathrm{csch}^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx =$$

$$-\frac{\left(b+a\cosh^2(c+dx)\right)^2\left(8b^2+48b(a+b)\cosh^2(c+dx)+3(a+b)\cosh^3(c+dx)\left((a+b)\mathrm{csch}^2\left(\frac{1}{2}(c+dx\right.\right.\right.}{6d(a+2b+a}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-1/6*((b + a*Cosh[c + d*x]^2)^2*(8*b^2 + 48*b*(a + b)*Cosh[c + d*x]^2 + 3*
(a + b)*Cosh[c + d*x]^3*((a + b)*Csch[(c + d*x)/2]^2 - 4*(a + 5*b)*(Log[Co
sh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]) + (a + b)*Sech[(c + d*x)/2]^2))
*Sech[c + d*x]^3)/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

3.15.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 26, 4621, 365, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\left(a+b\sec(ic+idx)^2\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\left(b\sec(ic+idx)^2+a\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \ 4621$$

$$\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^2\operatorname{sech}^4(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 365$$

$$\frac{\frac{1}{3}\int \frac{\left(3a^2\cosh^2(c+dx)+b(6a+5b)\right)\operatorname{sech}^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)-\frac{b^2\operatorname{sech}^3(c+dx)}{3\left(1-\cosh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \ 361$$

$$\frac{\frac{1}{3}\left(\frac{\left(3a^2+6ab+5b^2\right)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{\left(\left(3a^2+6ba+5b^2\right)\cosh^2(c+dx)+2b(6a+5b)\right)\operatorname{sech}^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)\right)-\frac{b^2\operatorname{sech}^3(c+dx)}{3\left(1-\cosh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow 25$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\int \frac{\left(\left(3a^2+6ba+5b^2\right)\cosh^2(c+dx)+2b(6a+5b)\right)\text{sech}^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{\left(3a^2+6ab+5b^2\right)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}\right)-\frac{b^2\text{sech}^3(c+dx)}{3\left(1-\cosh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow 359$$

$$\frac{\frac{1}{3}\left(\frac{1}{2}\left(3(a+b)(a+5b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)-2b(6a+5b)\text{sech}(c+dx)\right)+\frac{\left(3a^2+6ab+5b^2\right)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}\right)-\frac{b^2\text{sech}^3(c+dx)}{3\left(1-\cosh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow 219$$

$$\frac{\frac{1}{3}\left(\frac{\left(3a^2+6ab+5b^2\right)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}+\frac{1}{2}\left(3(a+b)(a+5b)\text{arctanh}(\cosh(c+dx))-2b(6a+5b)\text{sech}(c+dx)\right)\right)-\frac{b^2\text{sech}^3(c+dx)}{3\left(1-\cosh^2(c+dx)\right)}}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(-1/3*(b^2*Sech[c + d*x]^3)/(1 - Cosh[c + d*x]^2) + (((3*a^2 + 6*a*b + 5*b
^2)*Cosh[c + d*x])/(2*(1 - Cosh[c + d*x]^2)) + (3*(a + b)*(a + 5*b)*ArcTan
h[Cosh[c + d*x]] - 2*b*(6*a + 5*b)*Sech[c + d*x])/2)/3)/d
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.15. $\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

3.15.4 Maple [A] (verified)

Time = 70.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(-\frac{1}{2\sinh(dx+c)^{2}\cosh(dx+c)}-\frac{3}{2\cosh(dx+c)}+3\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b^{2}\left(\right.}{d}$
default	$\frac{a^{2}\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(-\frac{1}{2\sinh(dx+c)^{2}\cosh(dx+c)}-\frac{3}{2\cosh(dx+c)}+3\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b^{2}\left(\right.}{d}$
risch	$-\frac{\mathrm{e}^{dx+c}\left(3a^{2}\mathrm{e}^{8dx+8c}+18ab\,\mathrm{e}^{8dx+8c}+15b^{2}\mathrm{e}^{8dx+8c}+12a^{2}\mathrm{e}^{6dx+6c}+24ab\,\mathrm{e}^{6dx+6c}+20b^{2}\mathrm{e}^{6dx+6c}+18a^{2}\mathrm{e}^{4dx+4c}+12ab\,\mathrm{e}^{4dx}\right.}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^{3}\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}$

input
```
int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(-1/2/si
nh(d*x+c)^2/cosh(d*x+c)-3/2/cosh(d*x+c)+3*arctanh(exp(d*x+c)))+b^2*(-1/2/s
inh(d*x+c)^2/cosh(d*x+c)^3-5/6/cosh(d*x+c)^3-5/2/cosh(d*x+c)+5*arctanh(exp
(d*x+c))))
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2930 vs. 2(96) = 192.

Time = 0.28 (sec) , antiderivative size = 2930, normalized size of antiderivative = 28.17

$$\int \operatorname{csch}^{3}(c+dx)\left(a+b\operatorname{sech}^{2}(c+dx)\right)^{2}\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^9 + 54*(a^2 + 6*a*b + 5*b^2)*c
osh(d*x + c)*sinh(d*x + c)^8 + 6*(a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^9 + 8
*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 + 8*(27*(a^2 + 6*a*b + 5*b^2)*cos
h(d*x + c)^2 + 3*a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^7 + 56*(9*(a^2 + 6*a*b
 + 5*b^2)*cosh(d*x + c)^3 + (3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*
x + c)^6 + 4*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^5 + 4*(189*(a^2 + 6*a*
b + 5*b^2)*cosh(d*x + c)^4 + 42*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 +
9*a^2 + 6*a*b - 11*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 + 6*a*b + 5*b^2)*cos
h(d*x + c)^5 + 70*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 6*a
*b - 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(3*a^2 + 6*a*b + 5*b^2)*co
sh(d*x + c)^3 + 8*(63*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(3*a^2 +
6*a*b + 5*b^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^
2 + 3*a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^3 + 8*(27*(a^2 + 6*a*b + 5*b^2)*c
osh(d*x + c)^7 + 21*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 + 5*(9*a^2 + 6
*a*b - 11*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*
sinh(d*x + c)^2 + 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c) - 3*((a^2 + 6*a*b
+ 5*b^2)*cosh(d*x + c)^10 + 10*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*
x + c)^9 + (a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^10 + (a^2 + 6*a*b + 5*b^2)*
cosh(d*x + c)^8 + (45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b
+ 5*b^2)*sinh(d*x + c)^8 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 ...
```

3.15.6 Sympy [F]

$$\int \operatorname{csch}^3(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \, dx = \int \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \operatorname{csch}^3(c+dx) \, dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**3, x)
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(96) = 192.

Time = 0.20 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.40

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{6}b^2\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15\,e^{(-dx-c)}+20\,e^{(-3\,dx-3\,c)}-22\,e^{(-5\,dx-5\,c)}+20\,e\right.}{d(e^{(-2\,dx-2\,c)}-2\,e^{(-4\,dx-4\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,d}}\right.$$
$$+ab\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-2\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}-e^{(-6\,dx-6\,c)}-1)}\right)$$
$$+\frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/6*b^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d - 2*(15*e
^(-d*x - c) + 20*e^(-3*d*x - 3*c) - 22*e^(-5*d*x - 5*c) + 20*e^(-7*d*x - 7
*c) + 15*e^(-9*d*x - 9*c))/(d*(e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 2*e
^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a*b*(3*lo
g(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 2*
e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x -
4*c) - e^(-6*d*x - 6*c) - 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^
(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x -
2*c) - e^(-4*d*x - 4*c) - 1)))
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(96) = 192.

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.19

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{3\left(a^2+6\,ab+5\,b^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\left(a^2+6\,ab+5\,b^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{12\,(a^2(e^{(}}{}}{12\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/12*(3*(a^2 + 6*a*b + 5*b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*(a^2
 + 6*a*b + 5*b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 12*(a^2*(e^(d*x +
c) + e^(-d*x - c)) + 2*a*b*(e^(d*x + c) + e^(-d*x - c)) + b^2*(e^(d*x + c)
 + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) - 16*(3*a*b*(e^(d*x
 + c) + e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 2*b^2)/(e
^(d*x + c) + e^(-d*x - c))^3)/d
```

3.15.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.04

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(a^2\,\sqrt{-d^2}+5\,b^2\,\sqrt{-d^2}+6\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4+12\,a^3\,b+46\,a^2\,b^2+60\,a\,b^3+25\,b^4}}\right)\sqrt{a^4+12\,a^3\,b+46\,a^2\,b^2+60\,a\,b^3+25\,b^4}}{\sqrt{-d^2}} - \frac{e^{c+d\,x}\,(a^2+2\,a\,b+b^2)}{d\,(e^{2\,c+2\,d\,x}-1)} + \frac{8\,b^2\,e^{c+d\,x}}{3\,d\,(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1)} - \frac{2\,e^{c+d\,x}\,(a^2+2\,a\,b+b^2)}{d\,(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1)} - \frac{4\,e^{c+d\,x}\,(b^2+a\,b)}{d\,(e^{2\,c+2\,d\,x}+1)} - \frac{8\,b^2\,e^{c+d\,x}}{3\,d\,(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1)}$$

input
```
int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) + 5*b^2*(-d^2)^(1/2) + 6*a*b*(-d^
2)^(1/2)))/(d*(60*a*b^3 + 12*a^3*b + a^4 + 25*b^4 + 46*a^2*b^2)^(1/2)))*(6
0*a*b^3 + 12*a^3*b + a^4 + 25*b^4 + 46*a^2*b^2)^(1/2))/(-d^2)^(1/2) - (exp
(c + d*x)*(2*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) + (8*b^2*exp(c +
 d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
1)) - (2*exp(c + d*x)*(2*a*b + a^2 + b^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*
c + 2*d*x) + 1)) - (4*exp(c + d*x)*(a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1))
 - (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.16 $\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

3.16.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{(a+b)(a+3b)\coth(c+dx)}{d} - \frac{(a+b)^2\coth^3(c+dx)}{3d} + \frac{b(2a+3b)\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
(a+b)*(a+3*b)*coth(d*x+c)/d-1/3*(a+b)^2*coth(d*x+c)^3/d+b*(2*a+3*b)*tanh(d
*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

3.16.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(75) = 150.

Time = 6.65 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.01

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{\operatorname{csch}(2c)\operatorname{csch}^3(2(c+dx))\left(8a(a+2b)\sinh(2c) - 6(a+2b)^2\sinh(2dx) - 3a^2\sinh(2(c+dx)) - 6ab\sinh(\cdots\right.}{\cdots}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-1/6*(Csch[2*c]*Csch[2*(c + d*x)]^3*(8*a*(a + 2*b)*Sinh[2*c] - 6*(a + 2*b)
^2*Sinh[2*d*x] - 3*a^2*Sinh[2*(c + d*x)] - 6*a*b*Sinh[2*(c + d*x)] + a^2*S
inh[6*(c + d*x)] + 2*a*b*Sinh[6*(c + d*x)] + 3*a^2*Sinh[4*c + 2*d*x] + a^2
*Sinh[4*c + 6*d*x] + 8*a*b*Sinh[4*c + 6*d*x] + 8*b^2*Sinh[4*c + 6*d*x]))/d
```

3.16.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^2}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 355}$$

$$\frac{\int \left((a+b)^2\coth^4(c+dx)+(-a-3b)(a+b)\coth^2(c+dx)-b^2\tanh^2(c+dx)+b(2a+3b)\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b(2a+3b)\tanh(c+dx)-\frac{1}{3}(a+b)^2\coth^3(c+dx)+(a+b)(a+3b)\coth(c+dx)-\frac{1}{3}b^2\tanh^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + b)*(a + 3*b)*Coth[c + d*x] - ((a + b)^2*Coth[c + d*x]^3)/3 + b*(2*a
+ 3*b)*Tanh[c + d*x] - (b^2*Tanh[c + d*x]^3)/3)/d
```

3.16.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.16.4 Maple [A] (verified)

Time = 120.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{4\left(3a^{2}{\mathrm e}^{8dx+8c}+8a^{2}{\mathrm e}^{6dx+6c}+16ab\,{\mathrm e}^{6dx+6c}+6a^{2}{\mathrm e}^{4dx+4c}+24ab\,{\mathrm e}^{4dx+4c}+24\,{\mathrm e}^{4dx+4c}b^{2}-a^{2}-8ab-8b^{2}\right)}{3d\left({\mathrm e}^{2dx+2c}+1\right)^{3}\left({\mathrm e}^{2dx+2c}-1\right)^{3}}$
derivativedivides	$\frac{a^{2}\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)+2ab\left(-\frac{1}{3\sinh(dx+c)^{3}\cosh(dx+c)}+\frac{4}{3\sinh(dx+c)\cosh(dx+c)}+\frac{8\tanh(dx+c)}{3}\right)+b^{2}\left(-\frac{}{3\sinh(}\right.}{d}$
default	$\frac{a^{2}\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)+2ab\left(-\frac{1}{3\sinh(dx+c)^{3}\cosh(dx+c)}+\frac{4}{3\sinh(dx+c)\cosh(dx+c)}+\frac{8\tanh(dx+c)}{3}\right)+b^{2}\left(-\frac{}{3\sinh(}\right.}{d}$

input
```
int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3*(3*a^2*exp(8*d*x+8*c)+8*a^2*exp(6*d*x+6*c)+16*a*b*exp(6*d*x+6*c)+6*a^
2*exp(4*d*x+4*c)+24*a*b*exp(4*d*x+4*c)+24*exp(4*d*x+4*c)*b^2-a^2-8*a*b-8*b
^2)/d/(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(71) = 142$.

Time = 0.25 (sec) , antiderivative size = 408, normalized size of antiderivative = 5.44

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{8\left((a^2-4\,ab-4\,b^2)\cosh\left(dx+c\right)^4+8\left(a^2+2\,ab+2\,b^2\right)\cosh\left(dx+c\right)\right.}{3\left(d\cosh\left(dx+c\right)^8+56\,d\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^5+28\,d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^6+8\,d\cosh\left(\right.\right.}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-8/3*((a^2 - 4*a*b - 4*b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + 2*b^2)*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 - 4*a*b - 4*b^2)*sinh(d*x + c)^4 + 4*(a^2
 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 - 4*a*b - 4*b^2)*cosh(d*x + c)^2 + 2
*a^2 + 4*a*b)*sinh(d*x + c)^2 + 3*a^2 + 12*a*b + 12*b^2 + 8*((a^2 + 2*a*b
+ 2*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*
cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c
)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^
8 - 4*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 2*d)*sinh(d*x + c)^4 +
 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*d*cosh(d
*x + c)^6 - 6*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 -
d*cosh(d*x + c)^3)*sinh(d*x + c) + 3*d)
```

3.16.6 Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2\operatorname{csch}^4\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(71) = 142$.

Time = 0.21 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.80

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{4}{3}a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$
$$+\frac{32}{3}ab\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1)}-\frac{1}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1}\right.$$
$$+\frac{32}{3}b^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-4\,dx-4\,c)}-3\,e^{(-8\,dx-8\,c)}+e^{(-12\,dx-12\,c)}-1)}-\frac{1}{d(3\,e^{(-4\,dx-4\,c)}-3\,e^{(-8\,dx-8\,c)}+e^{(-12\,dx-12\,c)}-}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) - 1))) + 32/3*a*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*
c) - 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c
) - 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1))) + 32/3*b^2*(3*e^(-4*d*x -
 4*c)/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1
)) - 1/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) -
1)))
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.53

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx =$$
$$-\frac{4\left(3\,a^2e^{(8\,dx+8\,c)}+8\,a^2e^{(6\,dx+6\,c)}+16\,abe^{(6\,dx+6\,c)}+6\,a^2e^{(4\,dx+4\,c)}+24\,abe^{(4\,dx+4\,c)}+24\,b^2e^{(4\,dx+4\,c)}-a^2-\right.}{3\,d\left(e^{(4\,dx+4\,c)}-1\right)^3}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-4/3*(3*a^2*e^(8*d*x + 8*c) + 8*a^2*e^(6*d*x + 6*c) + 16*a*b*e^(6*d*x + 6*
c) + 6*a^2*e^(4*d*x + 4*c) + 24*a*b*e^(4*d*x + 4*c) + 24*b^2*e^(4*d*x + 4*
c) - a^2 - 8*a*b - 8*b^2)/(d*(e^(4*d*x + 4*c) - 1)^3)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.53

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{4\left(6\,a^2\,e^{4\,c+4\,d\,x}-a^2-8\,b^2-8\,a\,b+8\,a^2\,e^{6\,c+6\,d\,x}+3\,a^2\,e^{8\,c+8\,d\,x}+24\,b^2\,e^{4\,c+4\,d\,x}+24\,a\,b\,e^{4\,c+4\,d\,x}+16\,a\,b\right.}{3\,d\left(e^{4\,c+4\,d\,x}-1\right)^3}$$

input
```
int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

output
```
-(4*(6*a^2*exp(4*c + 4*d*x) - a^2 - 8*b^2 - 8*a*b + 8*a^2*exp(6*c + 6*d*x)
 + 3*a^2*exp(8*c + 8*d*x) + 24*b^2*exp(4*c + 4*d*x) + 24*a*b*exp(4*c + 4*d
*x) + 16*a*b*exp(6*c + 6*d*x)))/(3*d*(exp(4*c + 4*d*x) - 1)^3)
```

3.17 $\int \left(a+b\,\text{sech}^2(c+dx)\right)^3 \sinh^4(c+dx)\,dx$

3.17.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\begin{aligned}\int \left(a+b\,\text{sech}^2(c+dx)\right)^3 \sinh^4(c+dx)\,dx &= \frac{3}{8}a\left(a^2-12ab+8b^2\right)x-\frac{3a(a^2-12ab+8b^2)\tanh(c+dx)}{8d}\\ &+\frac{b(6a^2-23ab-8b^2)\tanh^3(c+dx)}{8d}-\frac{3(5a-16b)b^2\tanh^5(c+dx)}{40d}\\ &-\frac{3(a-2b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^2}{8d}\\ &+\frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^3}{4d}\end{aligned}$$

output
```
3/8*a*(a^2-12*a*b+8*b^2)*x-3/8*a*(a^2-12*a*b+8*b^2)*tanh(d*x+c)/d+1/8*b*(6
*a^2-23*a*b-8*b^2)*tanh(d*x+c)^3/d-3/40*(5*a-16*b)*b^2*tanh(d*x+c)^5/d-3/8
*(a-2*b)*sinh(d*x+c)^2*tanh(d*x+c)*(a+b-b*tanh(d*x+c)^2)^2/d+1/4*cosh(d*x+
c)*sinh(d*x+c)^3*(a+b-b*tanh(d*x+c)^2)^3/d
```

3.17.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 651 vs. $2(182) = 364$.

Time = 4.74 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.58

$$\int (a + b\text{sech}^2(c + dx))^3 \sinh^4(c + dx)\, dx$$
$$= \frac{(b + a\cosh^2(c + dx))^3 \text{sech}(c)\text{sech}^5(c + dx)\,(1200a(a^2 - 12ab + 8b^2)\, dx \cosh(dx) + 1200a(a^2 - 12ab + 8b}{}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]
```

output
```
((b + a*Cosh[c + d*x]^2)^3*Sech[c]*Sech[c + d*x]^5*(1200*a*(a^2 - 12*a*b +
 8*b^2)*d*x*Cosh[d*x] + 1200*a*(a^2 - 12*a*b + 8*b^2)*d*x*Cosh[2*c + d*x]
+ 600*a^3*d*x*Cosh[2*c + 3*d*x] - 7200*a^2*b*d*x*Cosh[2*c + 3*d*x] + 4800*
a*b^2*d*x*Cosh[2*c + 3*d*x] + 600*a^3*d*x*Cosh[4*c + 3*d*x] - 7200*a^2*b*d
*x*Cosh[4*c + 3*d*x] + 4800*a*b^2*d*x*Cosh[4*c + 3*d*x] + 120*a^3*d*x*Cosh
[4*c + 5*d*x] - 1440*a^2*b*d*x*Cosh[4*c + 5*d*x] + 960*a*b^2*d*x*Cosh[4*c
+ 5*d*x] + 120*a^3*d*x*Cosh[6*c + 5*d*x] - 1440*a^2*b*d*x*Cosh[6*c + 5*d*x
] + 960*a*b^2*d*x*Cosh[6*c + 5*d*x] - 180*a^3*Sinh[d*x] + 12120*a^2*b*Sinh
[d*x] - 14080*a*b^2*Sinh[d*x] + 1280*b^3*Sinh[d*x] - 180*a^3*Sinh[2*c + d*
x] - 7080*a^2*b*Sinh[2*c + d*x] + 11520*a*b^2*Sinh[2*c + d*x] - 310*a^3*Si
nh[2*c + 3*d*x] + 8760*a^2*b*Sinh[2*c + 3*d*x] - 8960*a*b^2*Sinh[2*c + 3*d
*x] - 310*a^3*Sinh[4*c + 3*d*x] - 840*a^2*b*Sinh[4*c + 3*d*x] + 3840*a*b^2
*Sinh[4*c + 3*d*x] - 640*b^3*Sinh[4*c + 3*d*x] - 150*a^3*Sinh[4*c + 5*d*x]
 + 2520*a^2*b*Sinh[4*c + 5*d*x] - 2560*a*b^2*Sinh[4*c + 5*d*x] + 128*b^3*S
inh[4*c + 5*d*x] - 150*a^3*Sinh[6*c + 5*d*x] + 600*a^2*b*Sinh[6*c + 5*d*x]
 - 15*a^3*Sinh[6*c + 7*d*x] + 120*a^2*b*Sinh[6*c + 7*d*x] - 15*a^3*Sinh[8*
c + 7*d*x] + 120*a^2*b*Sinh[8*c + 7*d*x] + 5*a^3*Sinh[8*c + 9*d*x] + 5*a^3
*Sinh[10*c + 9*d*x]))/(1280*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

3.17.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4620, 369, 27, 439, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3dx$$

$\Big\downarrow$ 3042

$$\int \sin(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^3dx$$

$\Big\downarrow$ 4620

$$\frac{\int \frac{\tanh^4(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 369

$$\frac{\frac{\tanh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{3\tanh^2(c+dx)\left(-3b\tanh^2(c+dx)+a+b\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tanh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\int \frac{\tanh^2(c+dx)\left(-3b\tanh^2(c+dx)+a+b\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 439

$$\frac{\frac{\tanh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{1}{2}\int -\frac{\tanh^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)\left((a-8b)(a+b)-(5a-16b)b\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+ \right.}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\tanh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{(a-2b)\tanh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^2}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{\tanh^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)\left((a-8b)(\right.}{1-\tanh^2(c+dx)}\right.}{d}$$

$\Big\downarrow$ 437

$$\frac{\frac{\tanh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{(a-2b)\tanh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^2}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \left(-\left((5a-16b)b^2\tanh^4(c+dx)\right)+b(\right.\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\tanh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{1}{2}\left(-a\left(a^2-12ab+8b^2\right)\text{arctanh}(\tanh(c+dx))-\frac{1}{3}b\left(6a^2-23ab-8b^2\right)\tanh^3(\right.\right.}{}$$

3.17. $\int \left(a+b\text{sech}^2(c+dx)\right)^3\sinh^4(c+dx)\,dx$

input
```
Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]
```

output
```
((Tanh[c + d*x]^3*(a + b - b*Tanh[c + d*x]^2)^3)/(4*(1 - Tanh[c + d*x]^2)^
2) - (3*(((a - 2*b)*Tanh[c + d*x]^3*(a + b - b*Tanh[c + d*x]^2)^2)/(2*(1 -
 Tanh[c + d*x]^2)) + (-(a*(a^2 - 12*a*b + 8*b^2)*ArcTanh[Tanh[c + d*x]]) +
 a*(a^2 - 12*a*b + 8*b^2)*Tanh[c + d*x] - (b*(6*a^2 - 23*a*b - 8*b^2)*Tanh
[c + d*x]^3)/3 + ((5*a - 16*b)*b^2*Tanh[c + d*x]^5)/5)/2))/4)/d
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.17.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00

$$a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3a\,b^2$$

input
```
int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x)
```

output
```
1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3
*a^2*b*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+3*a*b
^2*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+b^3*(-1/2*sinh(d*x+c)^3/cosh(d*x+
c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d
*x+c)^2)*tanh(d*x+c)))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(176) = 352.

Time = 0.26 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.99

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\sinh^4(c+dx)\,dx$$
$$=\frac{5\,a^3\sinh\left(dx+c\right)^9+15\left(12\,a^3\cosh\left(dx+c\right)^2-a^3+8\,a^2b\right)\sinh\left(dx+c\right)^7-8\left(120\,a^2b-160\,ab^2+8\,b^3-\right.}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/320*(5*a^3*sinh(d*x + c)^9 + 15*(12*a^3*cosh(d*x + c)^2 - a^3 + 8*a^2*b)
*sinh(d*x + c)^7 - 8*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b +
 8*a*b^2)*d*x)*cosh(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a
^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (630*a^3*cos
h(d*x + c)^4 - 150*a^3 + 1560*a^2*b - 1280*a*b^2 + 64*b^3 - 315*(a^3 - 8*a
^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3
 - 15*(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 5*(84*a^3*cosh(d*x
 + c)^6 - 105*(a^3 - 8*a^2*b)*cosh(d*x + c)^4 - 62*a^3 + 792*a^2*b - 512*a
*b^2 - 64*b^3 - 4*(75*a^3 - 780*a^2*b + 640*a*b^2 - 32*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 - 40*(2*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a
^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b - 160*a*b^2 + 8*b^3 -
15*(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 80*(12
0*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x
+ c) + 5*(9*a^3*cosh(d*x + c)^8 - 21*(a^3 - 8*a^2*b)*cosh(d*x + c)^6 - 2*(
75*a^3 - 780*a^2*b + 640*a*b^2 - 32*b^3)*cosh(d*x + c)^4 - 36*a^3 + 504*a^
2*b - 256*a*b^2 + 128*b^3 - 6*(31*a^3 - 396*a^2*b + 256*a*b^2 + 32*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(
d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

3.17.6 Sympy [F(-1)]

Timed out.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \sinh^4(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**4,x)
```

output
```
Timed out
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(176) = 352.

Time = 0.20 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.32

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^4(c+dx)\,dx$$
$$= \frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$+ab^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$-\frac{3}{8}a^2b\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)$$
$$+\frac{2}{5}b^3\left(\frac{10e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{}{d(5e^{(-2dx-2c)}}\right.$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) - 3/8*a^2*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (1
7*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)))) + 2/5*b
^3*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8
*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1)))
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.86

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^4(c+dx)\,dx$$
$$= \frac{5a^3e^{(4dx+4c)}-40a^3e^{(2dx+2c)}+120a^2be^{(2dx+2c)}+120\left(a^3-12a^2b+8ab^2\right)(dx+c)-5\left(18a^3e^{(4dx+4c)}-\right.}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="giac")
```

output
```
1/320*(5*a^3*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) + 120*a^2*b*e^(2*d*x
 + 2*c) + 120*(a^3 - 12*a^2*b + 8*a*b^2)*(d*x + c) - 5*(18*a^3*e^(4*d*x +
4*c) - 216*a^2*b*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*x + 4*c) - 8*a^3*e^(2*
d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-4*d*x - 4*c) - 128*(15*a^
2*b*e^(8*d*x + 8*c) - 30*a*b^2*e^(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 6
0*a^2*b*e^(6*d*x + 6*c) - 90*a*b^2*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4
*c) - 110*a*b^2*e^(4*d*x + 4*c) + 10*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d
*x + 2*c) - 70*a*b^2*e^(2*d*x + 2*c) + 15*a^2*b - 20*a*b^2 + b^3)/(e^(2*d*
x + 2*c) + 1)^5)/d
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 686, normalized size of antiderivative = 3.77

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^3\sinh^4(c+dx)\,dx$$

$$= \frac{\frac{2\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}-\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b-2\,a\,b^2+b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}-\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}$$
$$-\frac{\frac{2\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d}-\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{12\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b-2\,a\,b^2+b^3\right)}{5\,d}-\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{8\,c+8\,\cdots}}{}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}$$
$$+\frac{\frac{2\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}-\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}$$
$$-\frac{\frac{2\,\left(3\,a^2\,b-2\,a\,b^2+b^3\right)}{5\,d}-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}$$
$$+\frac{3\,a\,x\,\left(a^2-12\,a\,b+8\,b^2\right)}{8}-\frac{a^3\,{\mathrm{e}}^{-4\,c-4\,d\,x}}{64\,d}+\frac{a^3\,{\mathrm{e}}^{4\,c+4\,d\,x}}{64\,d}$$
$$-\frac{2\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{a^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(a-3\,b\right)}{8\,d}-\frac{a^2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a-3\,b\right)}{8\,d}$$

input
```
int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
((2*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (6*exp(2*c + 2*d*x)*(3*a^2*b - 2*a*
b^2 + b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) -
 (2*exp(6*c + 6*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(4*exp(2*c + 2*d*x)
 + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(
3*a^2*b - 6*a*b^2 + b^3))/(5*d) - (8*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b +
 b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(3*a^2*b - 2*a*b^2 + b^3))/(5*d) - (8*
exp(6*c + 6*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3
*a^2*b - 6*a*b^2 + b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x)
+ 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) + ((2
*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (2*exp(2*c + 2*d*x)*(3*a^2*b - 6*a*b^2
 + b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(3*a^2*b
 - 2*a*b^2 + b^3))/(5*d) - (4*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/
(5*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(3*exp(2*c +
 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (3*a*x*(a^2 - 12*a*
b + 8*b^2))/8 - (a^3*exp(- 4*c - 4*d*x))/(64*d) + (a^3*exp(4*c + 4*d*x))/(
64*d) - (2*(3*a^2*b - 6*a*b^2 + b^3))/(5*d*(exp(2*c + 2*d*x) + 1)) + (a^2*
exp(- 2*c - 2*d*x)*(a - 3*b))/(8*d) - (a^2*exp(2*c + 2*d*x)*(a - 3*b))/(8*
d)
```

3.17. $\int \left(a+b\text{sech}^2(c+dx)\right)^3\sinh^4(c+dx)\,dx$

3.18 $\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^3(c+dx)\,dx$

3.18.1 Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^3(c+dx)\,dx = -\frac{a^2(a-3b)\cosh(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{3a(a-b)b\text{sech}(c+dx)}{d} + \frac{(3a-b)b^2\text{sech}^3(c+dx)}{3d} + \frac{b^3\text{sech}^5(c+dx)}{5d}$$

output
```
-a^2*(a-3*b)*cosh(d*x+c)/d+1/3*a^3*cosh(d*x+c)^3/d+3*a*(a-b)*b*sech(d*x+c)
/d+1/3*(3*a-b)*b^2*sech(d*x+c)^3/d+1/5*b^3*sech(d*x+c)^5/d
```

3.18.2 Mathematica [A] (verified)

Time = 7.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^3(c+dx)\,dx = \frac{4\left(b+a\cosh^2(c+dx)\right)^3\left(6b^3+10(3a-b)b^2\cosh^2(c+dx)+90a(a-b)b\cosh^4(c+dx)+5a^2\cosh^6(c+d\right.}{15d(a+2b+a\cosh(2(c+dx)))^3}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]
```

output
```
(4*(b + a*Cosh[c + d*x]^2)^3*(6*b^3 + 10*(3*a - b)*b^2*Cosh[c + d*x]^2 + 9
0*a*(a - b)*b*Cosh[c + d*x]^4 + 5*a^2*Cosh[c + d*x]^6*(-5*a + 18*b + a*Cos
h[2*(c + d*x)]))*Sech[c + d*x]^5)/(15*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

3.18.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \left(b\sec(ic+idx)^2+a\right)^3\sin(ic+idx)^3dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^3\text{sech}^6(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(b^3\text{sech}^6(c+dx)+(3a-b)b^2\text{sech}^4(c+dx)+3a(a-b)b\text{sech}^2(c+dx)-a^3\cosh^2(c+dx)+a^2(a-3b)\right)d\cosh}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}a^3\cosh^3(c+dx)+a^2(a-3b)\cosh(c+dx)-\frac{1}{3}b^2(3a-b)\text{sech}^3(c+dx)-3ab(a-b)\text{sech}(c+dx)-\frac{1}{5}b^3\text{sech}^5}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]
```

output
```
-((a^2*(a - 3*b)*Cosh[c + d*x] - (a^3*Cosh[c + d*x]^3)/3 - 3*a*(a - b)*b*S
ech[c + d*x] - ((3*a - b)*b^2*Sech[c + d*x]^3)/3 - (b^3*Sech[c + d*x]^5)/5
)/d)
```

3.18.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.18.4 Maple [A] (verified)

Time = 222.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b^3 \operatorname{sech}(dx+c)^5}{5}+a\,b^2\operatorname{sech}(dx+c)^3-\frac{b^3\operatorname{sech}(dx+c)^3}{3}+3a^2b\operatorname{sech}(dx+c)-3a\,b^2\operatorname{sech}(dx+c)+\frac{a^3}{3\operatorname{sech}(dx+c)^3}-\frac{a^2(a-3b)}{\operatorname{sech}(dx+c)}}{d}$
default	$\frac{\frac{b^3 \operatorname{sech}(dx+c)^5}{5}+a\,b^2\operatorname{sech}(dx+c)^3-\frac{b^3\operatorname{sech}(dx+c)^3}{3}+3a^2b\operatorname{sech}(dx+c)-3a\,b^2\operatorname{sech}(dx+c)+\frac{a^3}{3\operatorname{sech}(dx+c)^3}-\frac{a^2(a-3b)}{\operatorname{sech}(dx+c)}}{d}$
parts	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b^3\left(\frac{\operatorname{sech}(dx+c)^5}{5}-\frac{\operatorname{sech}(dx+c)^3}{3}\right)}{d}+\frac{3a\,b^2\left(\frac{\operatorname{sech}(dx+c)^3}{3}-\operatorname{sech}(dx+c)\right)}{d}+\frac{3a^2b}{}$
risch	$\frac{a^3 e^{3dx+3c}}{24d}-\frac{3a^3 e^{dx+c}}{8d}+\frac{3a^2 e^{dx+c}b}{2d}-\frac{3a^3 e^{-dx-c}}{8d}+\frac{3a^2 e^{-dx-c}b}{2d}+\frac{a^3 e^{-3dx-3c}}{24d}+\frac{2\,e^{dx+c}b(45a^2e^{8dx+8c}-4}{}$

input
```
int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*b^3*sech(d*x+c)^5+a*b^2*sech(d*x+c)^3-1/3*b^3*sech(d*x+c)^3+3*a^2
*b*sech(d*x+c)-3*a*b^2*sech(d*x+c)+1/3*a^3/sech(d*x+c)^3-a^2*(a-3*b)/sech(
d*x+c))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(93) = 186.

Time = 0.25 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.07

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\sinh^3(c+dx)\,dx$$
$$=\frac{5\,a^3\cosh\left(dx+c\right)^8+5\,a^3\sinh\left(dx+c\right)^8-20\left(a^3-9\,a^2b\right)\cosh\left(dx+c\right)^6+20\left(7\,a^3\cosh\left(dx+c\right)^2-a^3+9\right.}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/120*(5*a^3*cosh(d*x + c)^8 + 5*a^3*sinh(d*x + c)^8 - 20*(a^3 - 9*a^2*b)*
cosh(d*x + c)^6 + 20*(7*a^3*cosh(d*x + c)^2 - a^3 + 9*a^2*b)*sinh(d*x + c)
^6 - 20*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^4 + 10*(35*a^3*cosh(d
*x + c)^4 - 22*a^3 + 180*a^2*b - 72*a*b^2 - 30*(a^3 - 9*a^2*b)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 - 425*a^3 + 3960*a^2*b - 1200*a*b^2 + 64*b^3 - 20*(3
1*a^3 - 279*a^2*b + 96*a*b^2 + 16*b^3)*cosh(d*x + c)^2 + 20*(7*a^3*cosh(d*
x + c)^6 - 15*(a^3 - 9*a^2*b)*cosh(d*x + c)^4 - 31*a^3 + 279*a^2*b - 96*a*
b^2 - 16*b^3 - 6*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(
d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 +
 10*d*cosh(d*x + c))
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \sinh^3(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**3,x)
```

output
```
Timed out
```

3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(93) = 186$.

Time = 0.19 (sec) , antiderivative size = 489, normalized size of antiderivative = 4.94

$$\begin{aligned}
&\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \sinh^3(c + dx)\, dx \\
&= \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right) \\
&\quad + \frac{3}{2}a^2b\left(\frac{e^{(-dx-c)}}{d} + \frac{5\,e^{(-2\,dx-2\,c)} + 1}{d(e^{(-dx-c)} + e^{(-3\,dx-3\,c)})}\right) \\
&\quad - 2\,ab^2\left(\frac{3\,e^{(-dx-c)}}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)} + \frac{2\,e^{(-3\,dx-3\,c)}}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right. \\
&\quad - \frac{8}{15}b^3\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)} - \frac{}{d(5\,e^{(-2\,dx-2\,c}}\right.
\end{aligned}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d
*x - 3*c)/d) + 3/2*a^2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^
(-d*x - c) + e^(-3*d*x - 3*c)))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x
 - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)
/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*
e^(-5*d*x - 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x -
 6*c) + 1))) - 8/15*b^3*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 1
0*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*
c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-
6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(93) = 186$.

Time = 0.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.95

$$\int \left(a + b\text{sech}^2(c+dx)\right)^3 \sinh^3(c+dx)\,dx$$

$$= \frac{5\,a^3\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 - 60\,a^3\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 180\,a^2b\left(e^{(dx+c)} + e^{(-dx-c)}\right) + \frac{16\left(45\,a^2b\left(e^{(dx+c)} + e^{(-dx-c)} \cdots\right.\right.}{\cdots}}{120\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="giac")
```

output
```
1/120*(5*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 60*a^3*(e^(d*x + c) + e^(-d*
x - c)) + 180*a^2*b*(e^(d*x + c) + e^(-d*x - c)) + 16*(45*a^2*b*(e^(d*x +
c) + e^(-d*x - c))^4 - 45*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 60*a*b^2*
(e^(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 4
8*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\sinh^3(c+dx)\,dx$$
$$=\frac{a^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}+\frac{a^3\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{3\,a^2\,\mathrm{e}^{-c-d\,x}\,(a-4\,b)}{8\,d}+\frac{8\,\mathrm{e}^{c+d\,x}\,(3\,a\,b^2-b^3)}{3\,d\,(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1)}$$
$$-\frac{64\,b^3\,\mathrm{e}^{c+d\,x}}{5\,d\,(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1)}$$
$$-\frac{8\,\mathrm{e}^{c+d\,x}\,(15\,a\,b^2-17\,b^3)}{15\,d\,(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1)}$$
$$+\frac{32\,b^3\,\mathrm{e}^{c+d\,x}}{5\,d\,(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1)}$$
$$-\frac{6\,\mathrm{e}^{c+d\,x}\,(a\,b^2-a^2\,b)}{d\,(\mathrm{e}^{2\,c+2\,d\,x}+1)}-\frac{3\,a^2\,\mathrm{e}^{c+d\,x}\,(a-4\,b)}{8\,d}$$

input
```
int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(a^3*exp(- 3*c - 3*d*x))/(24*d) + (a^3*exp(3*c + 3*d*x))/(24*d) - (3*a^2*e
xp(- c - d*x)*(a - 4*b))/(8*d) + (8*exp(c + d*x)*(3*a*b^2 - b^3))/(3*d*(2*
exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (64*b^3*exp(c + d*x))/(5*d*(4*
exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1)) - (8*exp(c + d*x)*(15*a*b^2 - 17*b^3))/(15*d*(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*exp(c + d*x))/(5*
d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(
8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (6*exp(c + d*x)*(a*b^2 - a^2*b))
/(d*(exp(2*c + 2*d*x) + 1)) - (3*a^2*exp(c + d*x)*(a - 4*b))/(8*d)
```

3.19 $\int (a+b\text{sech}^2(c+dx))^3 \sinh^2(c+dx)\,dx$

3.19.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int (a+b\text{sech}^2(c+dx))^3 \sinh^2(c+dx)\,dx = -\frac{1}{2}a^2(a-6b)x + \frac{a^3}{4d(1-\tanh(c+dx))} - \frac{3a^2b\tanh(c+dx)}{d} + \frac{b^2(3a+b)\tanh^3(c+dx)}{3d} - \frac{b^3\tanh^5(c+dx)}{5d} - \frac{a^3}{4d(1+\tanh(c+dx))}$$

output
```
-1/2*a^2*(a-6*b)*x+1/4*a^3/d/(1-tanh(d*x+c))-3*a^2*b*tanh(d*x+c)/d+1/3*b^2
*(3*a+b)*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d-1/4*a^3/d/(tanh(d*x+c)+1)
```

3.19.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 480 vs. 2(112) = 224.

Time = 3.72 (sec) , antiderivative size = 480, normalized size of antiderivative = 4.29

$$\int (a+b\text{sech}^2(c+dx))^3 \sinh^2(c+dx)\,dx$$
$$= \frac{\text{sech}(c)\text{sech}^5(c+dx)\left(-600a^2(a-6b)dx\cosh(dx) - 600a^2(a-6b)dx\cosh(2c+dx) - 300a^3dx\cosh(2c+\right.}{}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]
```

output
```
(Sech[c]*Sech[c + d*x]^5*(-600*a^2*(a - 6*b)*d*x*Cosh[d*x] - 600*a^2*(a -
6*b)*d*x*Cosh[2*c + d*x] - 300*a^3*d*x*Cosh[2*c + 3*d*x] + 1800*a^2*b*d*x*
Cosh[2*c + 3*d*x] - 300*a^3*d*x*Cosh[4*c + 3*d*x] + 1800*a^2*b*d*x*Cosh[4*
c + 3*d*x] - 60*a^3*d*x*Cosh[4*c + 5*d*x] + 360*a^2*b*d*x*Cosh[4*c + 5*d*x
] - 60*a^3*d*x*Cosh[6*c + 5*d*x] + 360*a^2*b*d*x*Cosh[6*c + 5*d*x] + 75*a^
3*Sinh[d*x] - 4320*a^2*b*Sinh[d*x] + 960*a*b^2*Sinh[d*x] - 160*b^3*Sinh[d*
x] + 75*a^3*Sinh[2*c + d*x] + 2880*a^2*b*Sinh[2*c + d*x] - 1440*a*b^2*Sinh
[2*c + d*x] - 480*b^3*Sinh[2*c + d*x] + 135*a^3*Sinh[2*c + 3*d*x] - 2880*a
^2*b*Sinh[2*c + 3*d*x] + 480*a*b^2*Sinh[2*c + 3*d*x] + 160*b^3*Sinh[2*c +
3*d*x] + 135*a^3*Sinh[4*c + 3*d*x] + 720*a^2*b*Sinh[4*c + 3*d*x] - 720*a*b
^2*Sinh[4*c + 3*d*x] + 75*a^3*Sinh[4*c + 5*d*x] - 720*a^2*b*Sinh[4*c + 5*d
*x] + 240*a*b^2*Sinh[4*c + 5*d*x] + 32*b^3*Sinh[4*c + 5*d*x] + 75*a^3*Sinh
[6*c + 5*d*x] + 15*a^3*Sinh[6*c + 7*d*x] + 15*a^3*Sinh[8*c + 7*d*x]))/(384
0*d)
```

3.19.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.43, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 25, 4620, 369, 403, 25, 403, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ic+idx)^2\left(-\left(a+b\sec(ic+idx)^2\right)^3\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(b\sec(ic+idx)^2+a\right)^3\sin(ic+idx)^2dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^3}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{369}$$

$$\frac{\frac{\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^3}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int\frac{\left(-7b\tanh^2(c+dx)+a+b\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$\downarrow$ 403

$$\frac{\frac{1}{2}\left(\frac{1}{5}\int-\frac{\left(-b\tanh^2(c+dx)+a+b\right)\left((5a-2b)(a+b)-(33a-2b)b\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{7}{5}b\tanh(c+dx)\left(a-b\tanh^2(c+dx\right.}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\left(-\frac{1}{5}\int\frac{\left(-b\tanh^2(c+dx)+a+b\right)\left((5a-2b)(a+b)-(33a-2b)b\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{7}{5}b\tanh(c+dx)\left(a-b\tanh^2(c+dx\right.}{d}$$

$\downarrow$ 403

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\int-\frac{(a+b)\left(15a^2-24ba-4b^2\right)-b\left(81a^2-28ba-4b^2\right)\tanh^2(c+dx)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{1}{3}b(33a-2b)\tanh(c+dx)\left(a-b\tanh\right.\right.}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(-\frac{1}{3}\int\frac{(a+b)\left(15a^2-24ba-4b^2\right)-b\left(81a^2-28ba-4b^2\right)\tanh^2(c+dx)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{1}{3}b(33a-2b)\tanh(c+dx)\left(a-b\tanh\right.\right.}{d}$$

$\downarrow$ 299

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(-15a^2(a-6b)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-b\left(81a^2-28ab-4b^2\right)\tanh(c+dx)\right)-\frac{1}{3}b(33a-2b)\tanh(\right.\right.}{d}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(-15a^2(a-6b)\text{arctanh}(\tanh(c+dx))-b\left(81a^2-28ab-4b^2\right)\tanh(c+dx)\right)-\frac{1}{3}b(33a-2b)\tanh(c+dx)\right.\right.}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]
```

output
```
((Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2)^3)/(2*(1 - Tanh[c + d*x]^2)) +
 ((-7*b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2)^2)/5 + ((-15*a^2*(a - 6*
b)*ArcTanh[Tanh[c + d*x]] - b*(81*a^2 - 28*a*b - 4*b^2)*Tanh[c + d*x])/3 -
 ((33*a - 2*b)*b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2))/3)/5)/2)/d
```

3.19. $\int\left(a+b\text{sech}^2(c+dx)\right)^3\sinh^2(c+dx)\,dx$

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.19.4 Maple [A] (verified)

Time = 131.69 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b(dx+c-\tanh(dx+c))+3a\,b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b(dx+c-\tanh(dx+c))+3a\,b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
parts	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b^3\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)}{d}+\frac{3a\,b^2\left(\right.}{}$
risch	$-\frac{a^3x}{2}+3b\,a^2x+\frac{a^3\mathrm{e}^{2dx+2c}}{8d}-\frac{a^3\mathrm{e}^{-2dx-2c}}{8d}+\frac{2b\left(45a^2\mathrm{e}^{8dx+8c}-45ab\,\mathrm{e}^{8dx+8c}+180a^2\mathrm{e}^{6dx+6c}-90ab\,\mathrm{e}^{6dx+6c}-3\right.}{}$

input
```
int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(d*x+c-tanh(d
*x+c))+3*a*b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)
*tanh(d*x+c))+b^3*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c
)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(100) = 200.

Time = 0.26 (sec) , antiderivative size = 595, normalized size of antiderivative = 5.31

$$\int\left(a+b\mathrm{sech}^2(c+dx)\right)^3\sinh^2(c+dx)\,dx$$
$$=\frac{15\,a^3\sinh\left(dx+c\right)^7+4\left(90\,a^2b-30\,ab^2-4\,b^3-15\left(a^3-6\,a^2b\right)dx\right)\cosh\left(dx+c\right)^5+20\left(90\,a^2b-30\,ab^2-\right.}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/120*(15*a^3*sinh(d*x + c)^7 + 4*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 -
 6*a^2*b)*d*x)*cosh(d*x + c)^5 + 20*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3
 - 6*a^2*b)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (315*a^3*cosh(d*x + c)^2
+ 75*a^3 - 360*a^2*b + 120*a*b^2 + 16*b^3)*sinh(d*x + c)^5 + 20*(90*a^2*b
- 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c)^3 + 5*(105*a^3*
cosh(d*x + c)^4 + 27*a^3 - 216*a^2*b - 24*a*b^2 + 16*b^3 + 2*(75*a^3 - 360
*a^2*b + 120*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 20*(2*(90*
a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c)^3 + 3*(90
*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*
x + c)^2 + 40*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(
d*x + c) + 5*(21*a^3*cosh(d*x + c)^6 + (75*a^3 - 360*a^2*b + 120*a*b^2 + 1
6*b^3)*cosh(d*x + c)^4 + 15*a^3 - 144*a^2*b - 48*a*b^2 - 64*b^3 + 3*(27*a^
3 - 216*a^2*b - 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh
(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*
(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x
+ c))
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\sinh^2(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**2,x)
```

output
```
Timed out
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(100) = 200$.

Time = 0.19 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.96

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\sinh^2(c+dx)\,dx$$
$$=-\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+3a^2b\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)$$
$$+\frac{4}{15}b^3\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}-\frac{}{d(5e^{(-2dx-2c}}\right.$$
$$+2ab^2\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right.$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 3*a^2*b*(x + c/d
 - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 4/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-
2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8
*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c)
 + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10
*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4
*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c
) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a*b^2*(3*e^(-4*d
*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) +
 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) +
1)))
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(100) = 200$.

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.48

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\sinh^2(c+dx)\,dx$$

$$= \frac{15\,a^3e^{(2\,dx+2\,c)} - 60\,(a^3-6\,a^2b)(dx+c) + 15\left(2\,a^3e^{(2\,dx+2\,c)} - 12\,a^2be^{(2\,dx+2\,c)} - a^3\right)e^{(-2\,dx-2\,c)} + \frac{16\,(45\,a^2be^{(8}}{}}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/120*(15*a^3*e^(2*d*x + 2*c) - 60*(a^3 - 6*a^2*b)*(d*x + c) + 15*(2*a^3*e
^(2*d*x + 2*c) - 12*a^2*b*e^(2*d*x + 2*c) - a^3)*e^(-2*d*x - 2*c) + 16*(45
*a^2*b*e^(8*d*x + 8*c) - 45*a*b^2*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6
*c) - 90*a*b^2*e^(6*d*x + 6*c) - 30*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d
*x + 4*c) - 60*a*b^2*e^(4*d*x + 4*c) + 10*b^3*e^(4*d*x + 4*c) + 180*a^2*b*
e^(2*d*x + 2*c) - 30*a*b^2*e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + 45*a
^2*b - 15*a*b^2 - 2*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.29

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^3\sinh^2(c+dx)\,dx$$
$$=\frac{\frac{2\,(9\,a^2\,b+3\,a\,b^2+4\,b^3)}{15\,d}-\frac{6\,e^{4\,c+4\,d\,x}\,(a\,b^2-a^2\,b)}{5\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b-b^3)}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}$$
$$+\frac{\frac{2\,(3\,a^2\,b-b^3)}{5\,d}-\frac{6\,e^{6\,c+6\,d\,x}\,(a\,b^2-a^2\,b)}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(9\,a^2\,b+3\,a\,b^2+4\,b^3)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(3\,a^2\,b-b^3)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}$$
$$+\frac{\frac{2\,(3\,a^2\,b-b^3)}{5\,d}-\frac{6\,e^{2\,c+2\,d\,x}\,(a\,b^2-a^2\,b)}{5\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}$$
$$+\frac{\frac{4\,e^{4\,c+4\,d\,x}\,(9\,a^2\,b+3\,a\,b^2+4\,b^3)}{5\,d}-\frac{6\,e^{8\,c+8\,d\,x}\,(a\,b^2-a^2\,b)}{5\,d}-\frac{6\,(a\,b^2-a^2\,b)}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b-b^3)}{5\,d}+\frac{8\,e^{6\,c+6\,d\,x}\,(3\,a^2\,b-b^3)}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}$$
$$-\frac{6\,(a\,b^2-a^2\,b)}{5\,d\,(e^{2\,c+2\,d\,x}+1)}-\frac{a^2\,x\,(a-6\,b)}{2}-\frac{a^3\,e^{-2\,c-2\,d\,x}}{8\,d}+\frac{a^3\,e^{2\,c+2\,d\,x}}{8\,d}$$

input
```
int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
((2*(3*a*b^2 + 9*a^2*b + 4*b^3))/(15*d) - (6*exp(4*c + 4*d*x)*(a*b^2 - a^2
*b))/(5*d) + (4*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d))/(3*exp(2*c + 2*d*
x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((2*(3*a^2*b - b^3))/(5*
d) - (6*exp(6*c + 6*d*x)*(a*b^2 - a^2*b))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a
*b^2 + 9*a^2*b + 4*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a^2*b - b^3))/(5*d
))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
 + 8*d*x) + 1) + ((2*(3*a^2*b - b^3))/(5*d) - (6*exp(2*c + 2*d*x)*(a*b^2 -
 a^2*b))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((4*exp(4*c
+ 4*d*x)*(3*a*b^2 + 9*a^2*b + 4*b^3))/(5*d) - (6*exp(8*c + 8*d*x)*(a*b^2 -
 a^2*b))/(5*d) - (6*(a*b^2 - a^2*b))/(5*d) + (8*exp(2*c + 2*d*x)*(3*a^2*b
- b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(3*a^2*b - b^3))/(5*d))/(5*exp(2*c + 2
*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + e
xp(10*c + 10*d*x) + 1) - (6*(a*b^2 - a^2*b))/(5*d*(exp(2*c + 2*d*x) + 1))
- (a^2*x*(a - 6*b))/2 - (a^3*exp(- 2*c - 2*d*x))/(8*d) + (a^3*exp(2*c + 2*
d*x))/(8*d)
```

3.20 $\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx$

3.20.1 Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx=\frac{a^3\cosh(c+dx)}{d}-\frac{3a^2b\text{sech}(c+dx)}{d}-\frac{ab^2\text{sech}^3(c+dx)}{d}-\frac{b^3\text{sech}^5(c+dx)}{5d}$$

output
```
a^3*cosh(d*x+c)/d-3*a^2*b*sech(d*x+c)/d-a*b^2*sech(d*x+c)^3/d-1/5*b^3*sech
(d*x+c)^5/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.45

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx$$
$$=\frac{8\left(b+a\cosh^2(c+dx)\right)^3\left(-b^3-5ab^2\cosh^2(c+dx)-15a^2b\cosh^4(c+dx)+5a^3\cosh^6(c+dx)\right)\text{sech}^5(c+}{5d(a+2b+a\cosh(2(c+dx)))^3}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x],x]
```

output
```
(8*(b + a*Cosh[c + d*x]^2)^3*(-b^3 - 5*a*b^2*Cosh[c + d*x]^2 - 15*a^2*b*Co
sh[c + d*x]^4 + 5*a^3*Cosh[c + d*x]^6)*Sech[c + d*x]^5)/(5*d*(a + 2*b + a*
Cosh[2*(c + d*x)])^3)
```

3.20.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 4621, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a+b\sec(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \left(b\sec(ic+idx)^2+a\right)^3\sin(ic+idx)dx$$

$$\Big\downarrow \text{4621}$$

$$\frac{\int \left(a\cosh^2(c+dx)+b\right)^3\text{sech}^6(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(b^3\text{sech}^6(c+dx)+3ab^2\text{sech}^4(c+dx)+3a^2b\text{sech}^2(c+dx)+a^3\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\cosh(c+dx)-3a^2b\text{sech}(c+dx)-ab^2\text{sech}^3(c+dx)-\frac{1}{5}b^3\text{sech}^5(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x],x]
```

output
```
(a^3*Cosh[c + d*x] - 3*a^2*b*Sech[c + d*x] - a*b^2*Sech[c + d*x]^3 - (b^3*
Sech[c + d*x]^5)/5)/d
```

3.20.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.20.4 Maple [A] (verified)

Time = 72.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

method	result
derivativedivides	$-\frac{\frac{b^3 \operatorname{sech}(dx+c)^5}{5}+a\,b^2 \operatorname{sech}(dx+c)^3+3a^2b \operatorname{sech}(dx+c)-\frac{a^3}{\operatorname{sech}(dx+c)}}{d}$
default	$-\frac{\frac{b^3 \operatorname{sech}(dx+c)^5}{5}+a\,b^2 \operatorname{sech}(dx+c)^3+3a^2b \operatorname{sech}(dx+c)-\frac{a^3}{\operatorname{sech}(dx+c)}}{d}$
parts	$\frac{a^3\cosh(dx+c)}{d}-\frac{3a^2b \operatorname{sech}(dx+c)}{d}-\frac{a\,b^2 \operatorname{sech}(dx+c)^3}{d}-\frac{b^3 \operatorname{sech}(dx+c)^5}{5d}$
risch	$\frac{a^3 e^{dx+c}}{2d}+\frac{a^3 e^{-dx-c}}{2d}-\frac{2\,e^{dx+c} b\left(15a^2 e^{8dx+8c}+60a^2 e^{6dx+6c}+20ab\,e^{6dx+6c}+90a^2 e^{4dx+4c}+40ab\,e^{4dx+4c}+16\,e^{4dx+\cdots}\right.}{5d\left(e^{2dx+2c}+1\right)^5}$

input
```
int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/5*b^3*sech(d*x+c)^5+a*b^2*sech(d*x+c)^3+3*a^2*b*sech(d*x+c)-a^3/se
ch(d*x+c))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 276, normalized size of antiderivative = 4.31

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx$$
$$= \frac{5\,a^3\cosh\left(dx+c\right)^6+5\,a^3\sinh\left(dx+c\right)^6+30\,(a^3-2\,a^2b)\cosh\left(dx+c\right)^4+15\left(5\,a^3\cosh\left(dx+c\right)^2+2\,a^3-\right.}{10\left(d\cosh\left(dx+c\right)^5+5\,d\cosh\right.}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="fricas")
```

output
```
1/10*(5*a^3*cosh(d*x + c)^6 + 5*a^3*sinh(d*x + c)^6 + 30*(a^3 - 2*a^2*b)*c
osh(d*x + c)^4 + 15*(5*a^3*cosh(d*x + c)^2 + 2*a^3 - 4*a^2*b)*sinh(d*x + c
)^4 + 50*a^3 - 180*a^2*b - 80*a*b^2 - 32*b^3 + 5*(15*a^3 - 48*a^2*b - 16*a
*b^2)*cosh(d*x + c)^2 + 5*(15*a^3*cosh(d*x + c)^4 + 15*a^3 - 48*a^2*b - 16
*a*b^2 + 36*(a^3 - 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x
+ c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*
cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

3.20.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx = \int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)^3 \sinh\left(c+dx\right)dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*sinh(c + d*x), x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx = \frac{a^3\cosh(dx+c)}{d} - \frac{6a^2b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)} - \frac{8ab^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3} - \frac{32b^3}{5d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="maxima")
```

output
```
a^3*cosh(d*x + c)/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))) - 8*a*b^2/(
d*(e^(d*x + c) + e^(-d*x - c))^3) - 32/5*b^3/(d*(e^(d*x + c) + e^(-d*x - c
))^5)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.58

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx = \frac{5a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right) - \frac{4\left(15a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+20ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+16b^3\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}}{10d}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="giac")
```

output
```
1/10*(5*a^3*(e^(d*x + c) + e^(-d*x - c)) - 4*(15*a^2*b*(e^(d*x + c) + e^(-
d*x - c))^4 + 20*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 16*b^3)/(e^(d*x +
c) + e^(-d*x - c))^5)/d
```

3.20.9 Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 288, normalized size of antiderivative = 4.50

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^3\sinh(c+dx)\,dx$$
$$=\frac{a^3\,\mathrm{e}^{c+d\,x}}{2\,d}+\frac{a^3\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{64\,b^3\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$
$$+\frac{8\,\mathrm{e}^{c+d\,x}\,\left(5\,a\,b^2-4\,b^3\right)}{5\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}$$
$$-\frac{32\,b^3\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}$$
$$-\frac{6\,a^2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,a\,b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

input
```
int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(a^3*exp(c + d*x))/(2*d) + (a^3*exp(- c - d*x))/(2*d) + (64*b^3*exp(c + d*
x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + e
xp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(5*a*b^2 - 4*b^3))/(5*d*(3*exp(2*c
 + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c +
d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x)
 + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (6*a^2*b*exp(c + d*x))/
(d*(exp(2*c + 2*d*x) + 1)) - (8*a*b^2*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x)
 + exp(4*c + 4*d*x) + 1))
```

3.21 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.21.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=-\frac{(a+b)^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}+\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

output
```
-(a+b)^3*arctanh(cosh(d*x+c))/d+b*(3*a^2+3*a*b+b^2)*sech(d*x+c)/d+1/3*b^2*
(3*a+b)*sech(d*x+c)^3/d+1/5*b^3*sech(d*x+c)^5/d
```

3.21.2 Mathematica [A] (verified)

Time = 6.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=$$

$$-\frac{8\left(b+a\cosh^2(c+dx)\right)^3\left(-3b^3-5b^2(3a+b)\cosh^2(c+dx)-15b(3a^2+3ab+b^2)\cosh^4(c+dx)+15(a \ldots\right.}{15d(a+2b+a\cosh(2(c \ldots}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-8*(b + a*Cosh[c + d*x]^2)^3*(-3*b^3 - 5*b^2*(3*a + b)*Cosh[c + d*x]^2 -
15*b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]^4 + 15*(a + b)^3*Cosh[c + d*x]^5*
(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))*Sech[c + d*x]^5)/(15*d*
(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

3.21.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 4621, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sec(ic+idx)^2\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sec(ic+idx)^2+a\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^3\operatorname{sech}^6(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$-\frac{\int \left(b^3\operatorname{sech}^6(c+dx)+b^2(3a+b)\operatorname{sech}^4(c+dx)+b\left(3a^2+3ba+b^2\right)\operatorname{sech}^2(c+dx)-\frac{(a+b)^3}{\cosh^2(c+dx)-1}\right)d\cosh(c+dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)+(a+b)^3\operatorname{arctanh}(\cosh(c+dx))-\frac{1}{3}b^2(3a+b)\operatorname{sech}^3(c+dx)-\frac{1}{5}b^3\operatorname{sech}^5(c+d}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-(((a + b)^3*ArcTanh[Cosh[c + d*x]] - b*(3*a^2 + 3*a*b + b^2)*Sech[c + d*x
] - (b^2*(3*a + b)*Sech[c + d*x]^3)/3 - (b^3*Sech[c + d*x]^5)/5)/d)
```

3.21.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.21.4 Maple [A] (verified)

Time = 143.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-2a^3\operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a\,b^2\left(\frac{1}{3\cosh(dx+c)^3}+\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)+}{d}$
default	$\frac{-2a^3\operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a\,b^2\left(\frac{1}{3\cosh(dx+c)^3}+\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)+}{d}$
risch	$\frac{2b\,e^{dx+c}\left(45a^2e^{8dx+8c}+45ab\,e^{8dx+8c}+15b^2e^{8dx+8c}+180a^2e^{6dx+6c}+240ab\,e^{6dx+6c}+80b^2e^{6dx+6c}+270a^2e^{4dx+4c}+390a\right.}{15d\left(e^{2dx+2c}+1\right)^5}$

input
```
int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c
)))+3*a*b^2*(1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c)))+b^3*(1
/5/cosh(d*x+c)^5+1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3443 vs. $2(79) = 158$.

Time = 0.29 (sec) , antiderivative size = 3443, normalized size of antiderivative = 41.48

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sinh
(d*x + c)^9 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^7 + 40*(9*a^2*
b + 12*a*b^2 + 4*b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b +
 12*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 195*a*b
^2 + 89*b^3)*cosh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^4 + 135*a^2*b + 195*a*b^2 + 89*b^3 + 210*(9*a^2*b + 12*a*b^2 + 4*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^5 + 70*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + (135*a^2*b
+ 195*a*b^2 + 89*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 12*a*
b^2 + 4*b^3)*cosh(d*x + c)^3 + 40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^6 + 35*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 12*a*b
^2 + 4*b^3 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^3 + 40*(27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 21*(9*a^2*b + 12
*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*
x + c)^3 + 3*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 +
 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c) - 15*((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + ...
```

3.21.6 Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3\operatorname{csch}\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x), x)
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(79) = 158.

Time = 0.19 (sec) , antiderivative size = 358, normalized size of antiderivative = 4.31

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=$$
$$-\frac{1}{15}b^3\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15\,e^{(-dx-c)}+80\,e^{(-3\,dx-3\,c)}+178\,e^{(-5\,dx-5\,c)}\right.}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+}\right.$$
$$-ab^2\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(3\,e^{(-dx-c)}+10\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$-3\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$
$$+\frac{a^3\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/15*b^3*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d - 2*(15
*e^(-d*x - c) + 80*e^(-3*d*x - 3*c) + 178*e^(-5*d*x - 5*c) + 80*e^(-7*d*x
- 7*c) + 15*e^(-9*d*x - 9*c))/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) -
a*b^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d - 2*(3*e^(-d*
x - c) + 10*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(3*e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 3*a^2*b*(log(e^(-d*x - c)
 + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2*d*x - 2*c) +
1))) + a^3*log(tanh(1/2*d*x + 1/2*c))/d
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(79) = 158$.

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.75

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\,(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(e^{(dx+c)}+e^{(-d}\right.}{}$$

input
```
integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/30*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(d*x + c) + e^(-d*x - c) +
 2) - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(d*x + c) + e^(-d*x - c) -
2) - 4*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 45*a*b^2*(e^(d*x + c) +
e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 + 60*a*b^2*(e^(d*x
 + c) + e^(-d*x - c))^2 + 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 48*b^3)/
(e^(d*x + c) + e^(-d*x - c))^5)/d
```

3.21.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.23

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{2\,e^{c+d\,x}\,(3\,a^2\,b+3\,a\,b^2+b^3)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

$$-\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^3\,\sqrt{-d^2}+b^3\,\sqrt{-d^2}+3\,a\,b^2\,\sqrt{-d^2}+3\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+b^6}}\right)\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5}}{\sqrt{-d^2}}$$

$$-\frac{64\,b^3\,e^{c+d\,x}}{5\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$

$$-\frac{8\,e^{c+d\,x}\,(15\,a\,b^2-7\,b^3)}{15\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$

$$+\frac{32\,b^3\,e^{c+d\,x}}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$

$$+\frac{8\,e^{c+d\,x}\,(b^3+3\,a\,b^2)}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x),x)
```

output
```
(2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (2
*atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) + b^3*(-d^2)^(1/2) + 3*a*b^2*(-d^
2)^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a
^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))*(6*a*b^5 + 6*a^5*b + a^6 + b^6 +
 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (64*b^3*exp(c
 + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x
) + exp(8*c + 8*d*x) + 1)) - (8*exp(c + d*x)*(15*a*b^2 - 7*b^3))/(15*d*(3*
exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*e
xp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c +
 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (8*exp(c + d*x)*
(3*a*b^2 + b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.21. $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.22 $\int \operatorname{csch}^2(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx$

3.22.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \operatorname{csch}^2(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = -\frac{(a+b)^3\coth(c+dx)}{d} - \frac{3b(a+b)^2\tanh(c+dx)}{d} + \frac{b^2(a+b)\tanh^3(c+dx)}{d} - \frac{b^3\tanh^5(c+dx)}{5d}$$

output
```
-(a+b)^3*coth(d*x+c)/d-3*b*(a+b)^2*tanh(d*x+c)/d+b^2*(a+b)*tanh(d*x+c)^3/d
-1/5*b^3*tanh(d*x+c)^5/d
```

3.22.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 380 vs. 2(70) = 140.

Time = 5.13 (sec) , antiderivative size = 380, normalized size of antiderivative = 5.43

$$\int \operatorname{csch}^2(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = -\frac{\coth(c+dx)\operatorname{csch}(c)\operatorname{sech}(c) \left(a+b\operatorname{sech}^2(c+dx)\right)^3 (10a(5a^2+12ab+8b^2)\sinh(2c) - 10(5a^3+18a^2b+}{}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/40*(Coth[c + d*x]*Csch[c]*Sech[c]*(a + b*Sech[c + d*x]^2)^3*(10*a*(5*a^
2 + 12*a*b + 8*b^2)*Sinh[2*c] - 10*(5*a^3 + 18*a^2*b + 20*a*b^2 + 8*b^3)*S
inh[2*d*x] - 25*a^3*Sinh[2*(c + d*x)] + 50*a*b^2*Sinh[2*(c + d*x)] + 30*b^
3*Sinh[2*(c + d*x)] - 20*a^3*Sinh[4*(c + d*x)] + 40*a*b^2*Sinh[4*(c + d*x)
] + 24*b^3*Sinh[4*(c + d*x)] - 5*a^3*Sinh[6*(c + d*x)] + 10*a*b^2*Sinh[6*(
c + d*x)] + 6*b^3*Sinh[6*(c + d*x)] - 25*a^3*Sinh[2*(c + 2*d*x)] - 120*a^2
*b*Sinh[2*(c + 2*d*x)] - 160*a*b^2*Sinh[2*(c + 2*d*x)] - 64*b^3*Sinh[2*(c
+ 2*d*x)] + 25*a^3*Sinh[4*c + 2*d*x] + 30*a^2*b*Sinh[4*c + 2*d*x] + 5*a^3*
Sinh[6*c + 4*d*x] - 5*a^3*Sinh[4*c + 6*d*x] - 30*a^2*b*Sinh[4*c + 6*d*x] -
 40*a*b^2*Sinh[4*c + 6*d*x] - 16*b^3*Sinh[4*c + 6*d*x]))/(d*(a + 2*b + a*C
osh[2*(c + d*x)])^3)
```

3.22.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4620, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\left(a+b\sec(ic+idx)^2\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\left(b\sec(ic+idx)^2+a\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \coth^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^3d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 244$$

$$\frac{\int \left(-b^3\tanh^4(c+dx)+3b^2(a+b)\tanh^2(c+dx)-3b(a+b)^2+(a+b)^3\coth^2(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b^2(a+b)\tanh^3(c+dx) - 3b(a+b)^2\tanh(c+dx) - (a+b)^3\coth(c+dx) - \frac{1}{5}b^3\tanh^5(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-((a + b)^3*Coth[c + d*x]) - 3*b*(a + b)^2*Tanh[c + d*x] + b^2*(a + b)*Ta
nh[c + d*x]^3 - (b^3*Tanh[c + d*x]^5)/5)/d
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(68) = 136.

Time = 241.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{-\coth(dx+c)a^3+3a^2b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+3a\,b^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\right.}{d}$
default	$\frac{-\coth(dx+c)a^3+3a^2b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+3a\,b^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\right.}{d}$
risch	$-\frac{2(5a^3e^{10dx+10c}+25a^3e^{8dx+8c}+30a^2b\,e^{8dx+8c}+50a^3e^{6dx+6c}+120a^2b\,e^{6dx+6c}+80a\,b^2e^{6dx+6c}+50a^3e^{4dx+4c}+180a^2}{5d(e^{2d}}$

input
```
int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^3+3*a^2*b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c))+3
*a*b^2*(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)
)+b^3*(-1/sinh(d*x+c)/cosh(d*x+c)^5-6*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*
x+c)^2)*tanh(d*x+c)))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(68) = 136.

Time = 0.27 (sec) , antiderivative size = 622, normalized size of antiderivative = 8.89

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{4\left((5\,a^3+15\,a^2b+20\,ab^2+8\,b^3)\cosh\left(dx+c\right)^5+5\left(5\,a^3+15\,a^2b+20\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx\right.\right.}{5\left(d\cosh\left(dx+c\right)^7+7\,d\cosh\left(dx+c\right)\sinh\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-4/5*((5*a^3 + 15*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 1
5*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2*b + 20
*a*b^2 + 8*b^3)*sinh(d*x + c)^5 + (25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*
cosh(d*x + c)^3 - (45*a^2*b + 80*a*b^2 + 32*b^3 + 10*(15*a^2*b + 20*a*b^2
+ 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (10*(5*a^3 + 15*a^2*b + 20*a*b
^2 + 8*b^3)*cosh(d*x + c)^3 + 3*(25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*co
sh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 15*a^2*b + 14*a*b^2 + 4*b^3)*co
sh(d*x + c) - (5*(15*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 30*a^2*b
+ 60*a*b^2 + 40*b^3 + 3*(45*a^2*b + 80*a*b^2 + 32*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*si
nh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*
x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d
*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sin
h(d*x + c)^3 + 3*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x
+ c))*sinh(d*x + c)^2 - 5*d*cosh(d*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*co
sh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c))
```

3.22.6 Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\operatorname{csch}^2(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**2, x)
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(68) = 136.

Time = 0.19 (sec) , antiderivative size = 358, normalized size of antiderivative = 5.11

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=$$

$$-\frac{32}{5}b^3\left(\frac{4\,e^{(-2\,dx-2\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+5\,e^{(-4\,dx-4\,c)}-5\,e^{(-8\,dx-8\,c)}-4\,e^{(-10\,dx-10\,c)}-e^{(-12\,dx-12\,c)}+1)}+\frac{}{d(4\,e^{(-2\,dx-2\,c)}}\right.$$

$$-16\,ab^2\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1)}+\frac{1}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1}\right.$$

$$+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}-1)}+\frac{12\,a^2b}{d(e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-32/5*b^3*(4*e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c)
- 5*e^(-8*d*x - 8*c) - 4*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) + 1)) + 5
*e^(-4*d*x - 4*c)/(d*(4*e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) - 5*e^(-8*d*
x - 8*c) - 4*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) + 1)) + 1/(d*(4*e^(-2
*d*x - 2*c) + 5*e^(-4*d*x - 4*c) - 5*e^(-8*d*x - 8*c) - 4*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) + 1))) - 16*a*b^2*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*
d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1)) + 1/(d*(2*e^(-2*d
*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1))) + 2*a^3/(d*(e^(-2
*d*x - 2*c) - 1)) + 12*a^2*b/(d*(e^(-4*d*x - 4*c) - 1))
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(68) = 136$.

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.56

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{2\left(\frac{5\,(a^3+3\,a^2b+3\,ab^2+b^3)}{e^{(2\,dx+2\,c)}-1}-\frac{15\,a^2be^{(8\,dx+8\,c)}+15\,ab^2e^{(8\,dx+8\,c)}+5\,b^3e^{(8\,dx+8\,c)}+60\,a^2be^{(6\,dx+6\,c)}+90\,ab^2e^{(6\,dx+6\,c)}+30\,b^3e^{(6\,dx+6\,c)}+90}{}\right.}{5\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-2/5*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(e^(2*d*x + 2*c) - 1) - (15*a^2*b*
e^(8*d*x + 8*c) + 15*a*b^2*e^(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^
2*b*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 30*b^3*e^(6*d*x + 6*c) +
90*a^2*b*e^(4*d*x + 4*c) + 160*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4
*c) + 60*a^2*b*e^(2*d*x + 2*c) + 110*a*b^2*e^(2*d*x + 2*c) + 50*b^3*e^(2*d
*x + 2*c) + 15*a^2*b + 25*a*b^2 + 11*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.22.9 Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 644, normalized size of antiderivative = 9.20

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\frac{\frac{2\,\left(3\,a^2\,b+6\,a\,b^2+2\,b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}$$
$$+\frac{\frac{2\,\left(3\,a^2\,b+6\,a\,b^2+2\,b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b+7\,a\,b^2+5\,b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b+6\,a\,b^2+2\,b^3\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}$$
$$+\frac{\frac{2\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b+6\,a\,b^2+2\,b^3\right)}{5\,d}+\frac{12\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b+7\,a\,b^2+5\,b^3\right)}{5\,d}+8\,{\mathrm{e}}^{6\,c+6\,d}\cdots}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}$$
$$+\frac{\frac{2\,\left(3\,a^2\,b+7\,a\,b^2+5\,b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b+6\,a\,b^2+2\,b^3\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}$$
$$-\frac{2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

output
```
((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 3*
a^2*b + b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(6*
a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(6*c + 6*d*x)*(3*a*b^2 + 3*a^2*b +
 b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6
*exp(4*c + 4*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(4*exp(2*c + 2*d*x)
+ 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + ((2*(3
*a*b^2 + 3*a^2*b + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b +
b^3))/(5*d) + (8*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (12
*exp(4*c + 4*d*x)*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (8*exp(6*c + 6*d*x)
*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*
d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)
+ ((2*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(3*a*b^2 +
3*a^2*b + b^3))/(5*d) + (4*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(
5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (
2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(exp(2*c + 2*d*x) - 1)) + (2*(3*a*b^
2 + 3*a^2*b + b^3))/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.23 $\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.23.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx &= \frac{(a+b)^2(a+7b)\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{(a+b)^2(a+7b)\operatorname{sech}(c+dx)}{2d}\\&\quad-\frac{b(6a^2+15ab+7b^2)\operatorname{sech}^3(c+dx)}{6d}-\frac{b^2(5a+7b)\operatorname{sech}^5(c+dx)}{10d}\\&\quad-\frac{(a+b)\left(b+a\cosh^2(c+dx)\right)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^5(c+dx)}{2d}\end{aligned}$$

output
```
1/2*(a+b)^2*(a+7*b)*arctanh(cosh(d*x+c))/d-1/2*(a+b)^2*(a+7*b)*sech(d*x+c)
/d-1/6*b*(6*a^2+15*a*b+7*b^2)*sech(d*x+c)^3/d-1/10*b^2*(5*a+7*b)*sech(d*x+
c)^5/d-1/2*(a+b)*(b+a*cosh(d*x+c)^2)^2*csch(d*x+c)^2*sech(d*x+c)^5/d
```

3.23.2 Mathematica [A] (verified)

Time = 9.97 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.56

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{\left(b+a\cosh^2(c+dx)\right)^3\operatorname{sech}^6(c+dx)\left(\left(150a^3+270a^2b-30ab^2-206b^3+10(9a^3+45a^2b+75ab^2+35\right.\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/120*((b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^6*((150*a^3 + 270*a^2*b -
30*a*b^2 - 206*b^3 + 10*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*Cosh[4*(c +
 d*x)] + 15*(a + b)^2*(a + 7*b)*Cosh[6*(c + d*x)])*Coth[c + d*x]*Csch[c +
d*x] - 480*(a + b)^2*(a + 7*b)*Cosh[c + d*x]^6*(Log[Cosh[(c + d*x)/2]] - L
og[Sinh[(c + d*x)/2]]) + (3*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*Csch
[c + d*x]^3*Sinh[4*(c + d*x)])/4))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

3.23.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4621, 370, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a+b\sec(ic+idx)^2\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(b\sec(ic+idx)^2+a\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{4621}$$

$$\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^3\operatorname{sech}^6(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{370}$$

$$\frac{\frac{1}{2}\int \frac{\left(a\cosh^2(c+dx)+b\right)\left(a(a+3b)\cosh^2(c+dx)+b(5a+7b)\right)\operatorname{sech}^6(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{(a+b)\operatorname{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^2}{2\left(1-\cosh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{437}$$

$$\frac{\frac{1}{2}\int \left(b^2(5a+7b)\text{sech}^6(c+dx)+b\left(6a^2+15ba+7b^2\right)\text{sech}^4(c+dx)+(a+b)^2(a+7b)\text{sech}^2(c+dx)-\frac{(a+b)^2(a+7b}{\cosh^2(c+dx)-}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}\left(-\frac{1}{3}b\left(6a^2+15ab+7b^2\right)\text{sech}^3(c+dx)+(a+b)^2(a+7b)\text{arctanh}(\cosh(c+dx))-\frac{1}{5}b^2(5a+7b)\text{sech}^5(c+dx)-\right.}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(((a + b)*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^5)/(2*(1 - Cosh[c + d*x]
^2)) + ((a + b)^2*(a + 7*b)*ArcTanh[Cosh[c + d*x]] - (a + b)^2*(a + 7*b)*S
ech[c + d*x] - (b*(6*a^2 + 15*a*b + 7*b^2)*Sech[c + d*x]^3)/3 - (b^2*(5*a
+ 7*b)*Sech[c + d*x]^5)/5)/2)/d
```

3.23.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.23. $\int \text{csch}^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.23.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.33

$$a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(-\frac{1}{2\sinh(dx+c)^2\cosh(dx+c)}-\frac{3}{2\cosh(dx+c)}+3\operatorname{arctanh}\left(e^{dx+c}\right)\right.$$

input
```
int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(-1/2/
sinh(d*x+c)^2/cosh(d*x+c)-3/2/cosh(d*x+c)+3*arctanh(exp(d*x+c)))+3*a*b^2*(
-1/2/sinh(d*x+c)^2/cosh(d*x+c)^3-5/6/cosh(d*x+c)^3-5/2/cosh(d*x+c)+5*arcta
nh(exp(d*x+c)))+b^3*(-1/2/sinh(d*x+c)^2/cosh(d*x+c)^5-7/10/cosh(d*x+c)^5-7
/6/cosh(d*x+c)^3-7/2/cosh(d*x+c)+7*arctanh(exp(d*x+c))))
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6717 vs. 2(134) = 268.

Time = 0.32 (sec) , antiderivative size = 6717, normalized size of antiderivative = 46.65

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.23.6 Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3\operatorname{csch}^3\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**3, x)
```

3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(134) = 268$.

Time = 0.20 (sec) , antiderivative size = 556, normalized size of antiderivative = 3.86

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{30}b^3\left(\frac{105\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{105\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(105\,e^{(-dx-c)}+350\,e^{(-3\,dx-3\,c)}+231\,e^{(-5\,dx-5\,c}\right.}{d(3\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}-5\,e^{(-6\,dx-6\,c}}\right.$$
$$+\frac{1}{2}ab^2\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15\,e^{(-dx-c)}+20\,e^{(-3\,dx-3\,c)}-22\,e^{(-5\,dx-5\,c)}+\right.}{d(e^{(-2\,dx-2\,c)}-2\,e^{(-4\,dx-4\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(}}\right.$$
$$+\frac{3}{2}a^2b\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-2\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}-e^{(-6\,dx-6\,c)}-1)}\right)$$
$$+\frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/30*b^3*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*x - c) - 1)/d - 2*(1
05*e^(-d*x - c) + 350*e^(-3*d*x - 3*c) + 231*e^(-5*d*x - 5*c) - 412*e^(-7*
d*x - 7*c) + 231*e^(-9*d*x - 9*c) + 350*e^(-11*d*x - 11*c) + 105*e^(-13*d*
x - 13*c))/(d*(3*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) - 5*e^(-6*d*x - 6*c)
- 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 3*e^(-12*d*x - 12*c) + e^(-14*
d*x - 14*c) + 1))) + 1/2*a*b^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*
x - c) - 1)/d - 2*(15*e^(-d*x - c) + 20*e^(-3*d*x - 3*c) - 22*e^(-5*d*x -
5*c) + 20*e^(-7*d*x - 7*c) + 15*e^(-9*d*x - 9*c))/(d*(e^(-2*d*x - 2*c) - 2
*e^(-4*d*x - 4*c) - 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + e^(-10*d*x - 1
0*c) + 1))) + 3/2*a^2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) -
1)/d + 2*(3*e^(-d*x - c) - 2*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^
(-2*d*x - 2*c) + e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c) - 1))) + 1/2*a^3*(log
(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d
*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(134) = 268.

Time = 0.31 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.37

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$

$$= \frac{15\,(a^3+9\,a^2b+15\,ab^2+7\,b^3)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\,(a^3+9\,a^2b+15\,ab^2+7\,b^3)\log\left(e^{(dx+c)}+e\right.}{}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/60*(15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*log(e^(d*x + c) + e^(-d*x - c)
 + 2) - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*log(e^(d*x + c) + e^(-d*x -
c) - 2) - 60*(a^3*(e^(d*x + c) + e^(-d*x - c)) + 3*a^2*b*(e^(d*x + c) + e^
(-d*x - c)) + 3*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + b^3*(e^(d*x + c) + e^
(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) - 8*(45*a^2*b*(e^(d*x +
c) + e^(-d*x - c))^4 + 90*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 45*b^3*(e
^(d*x + c) + e^(-d*x - c))^4 + 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4
0*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c)
)^5)/d
```

3.23.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.72

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(a^3\sqrt{-d^2}+7\,b^3\sqrt{-d^2}+15\,a\,b^2\sqrt{-d^2}+9\,a^2\,b\sqrt{-d^2}\right)}{d\sqrt{a^6+18\,a^5\,b+111\,a^4\,b^2+284\,a^3\,b^3+351\,a^2\,b^4+210\,a\,b^5+49\,b^6}}\right)\sqrt{a^6+18\,a^5\,b+111\,a^4\,b^2+284\,a^3\,b^3+351\,a^2\,b^4+}}{\sqrt{-d^2}}$$

$$-\frac{2\,e^{c+dx}\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}-\frac{8\,e^{c+dx}\left(2\,b^3+3\,a\,b^2\right)}{3\,d\left(2\,e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

$$-\frac{6\,e^{c+dx}\left(a^2\,b+2\,a\,b^2+b^3\right)}{d\left(e^{2c+2dx}+1\right)}+\frac{64\,b^3\,e^{c+dx}}{5\,d\left(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}$$

$$+\frac{8\,e^{c+dx}\left(15\,a\,b^2-2\,b^3\right)}{15\,d\left(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1\right)}$$

$$-\frac{32\,b^3\,e^{c+dx}}{5\,d\left(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1\right)}$$

$$-\frac{e^{c+dx}\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\left(e^{2c+2dx}-1\right)}$$

input

```
int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

output

```
(atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) + 7*b^3*(-d^2)^(1/2) + 15*a*b^2*(
-d^2)^(1/2) + 9*a^2*b*(-d^2)^(1/2)))/(d*(210*a*b^5 + 18*a^5*b + a^6 + 49*b
^6 + 351*a^2*b^4 + 284*a^3*b^3 + 111*a^4*b^2)^(1/2)))*(210*a*b^5 + 18*a^5*
b + a^6 + 49*b^6 + 351*a^2*b^4 + 284*a^3*b^3 + 111*a^4*b^2)^(1/2))/(-d^2)^
(1/2) - (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(exp(4*c + 4*d
*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*exp(c + d*x)*(3*a*b^2 + 2*b^3))/(3*d*(
2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (6*exp(c + d*x)*(2*a*b^2 + a
^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*ex
p(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x
) + 1)) + (8*exp(c + d*x)*(15*a*b^2 - 2*b^3))/(15*d*(3*exp(2*c + 2*d*x) +
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(
5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c
 + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (exp(c + d*x)*(3*a*b^2 + 3*a^2*b +
a^3 + b^3))/(d*(exp(2*c + 2*d*x) - 1))
```

3.24 $\int \text{csch}^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

3.24.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \text{csch}^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\frac{(a+b)^2(a+4b)\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{3b(a+b)(a+2b)\tanh(c+dx)}{d}-\frac{b^2(3a+4b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

output
```
(a+b)^2*(a+4*b)*coth(d*x+c)/d-1/3*(a+b)^3*coth(d*x+c)^3/d+3*b*(a+b)*(a+2*b
)*tanh(d*x+c)/d-1/3*b^2*(3*a+4*b)*tanh(d*x+c)^3/d+1/5*b^3*tanh(d*x+c)^5/d
```

3.24.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 235 vs. 2(104) = 208.

Time = 4.31 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.26

$$\int \text{csch}^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=$$

$$-\frac{8\left(b+a\cosh^2(c+dx)\right)^3\text{sech}^5(c+dx)\left(-3b^3\cosh(c+dx)+\cosh^3(c+dx)\left(-b^2(15a+14b)+5(a+b)^3\right.\right.}{}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-8*(b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^5*(-3*b^3*Cosh[c + d*x] + Cosh
[c + d*x]^3*(-(b^2*(15*a + 14*b)) + 5*(a + b)^3*Coth[c]^2*Coth[c + d*x]^2)
 - 3*b^3*Csch[c]*Sinh[d*x] - Cosh[c + d*x]^2*(b^2*(15*a + 14*b) + 5*(a + b
)^3*Coth[c]*Coth[c + d*x]^3)*Csch[c]*Sinh[d*x] + Cosh[c + d*x]^3*((5*a^3 +
 60*a^2*b + 120*a*b^2 + 64*b^3)*Cosh[d*x] + (5*a^3 + 15*a^2*b - 9*b^3)*Cos
h[2*c + d*x])*Coth[c + d*x]*Csch[c]^2*Sinh[d*x])*Tanh[c])/(15*d*(a + 2*b +
 a*Cosh[2*(c + d*x)])^3)
```

3.24.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^3}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^3d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left((a+b)^3\coth^4(c+dx)-(a+b)^2(a+4b)\coth^2(c+dx)+b^3\tanh^4(c+dx)-b^2(3a+4b)\tanh^2(c+dx)+3b(a\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}b^2(3a+4b)\tanh^3(c+dx)+3b(a+b)(a+2b)\tanh(c+dx)-\frac{1}{3}(a+b)^3\coth^3(c+dx)+(a+b)^2(a+4b)\coth(}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + b)^2*(a + 4*b)*Coth[c + d*x] - ((a + b)^3*Coth[c + d*x]^3)/3 + 3*b*(
a + b)*(a + 2*b)*Tanh[c + d*x] - (b^2*(3*a + 4*b)*Tanh[c + d*x]^3)/3 + (b^
3*Tanh[c + d*x]^5)/5)/d
```

3.24.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(98) = 196$.

Time = 0.85 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.05

$$a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(-\frac{1}{3\sinh(dx+c)^3\cosh(dx+c)}+\frac{4}{3\sinh(dx+c)\cosh(dx+c)}+\frac{8\tanh(dx+c)}{3}\right)+3a\,b^2\left(\right.$$

input
```
int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-1/3/sinh(d*x+c)^3/c
osh(d*x+c)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c))+3*a*b^2*(-1/3/sinh
(d*x+c)^3/cosh(d*x+c)^3+2/sinh(d*x+c)/cosh(d*x+c)^3+8*(2/3+1/3*sech(d*x+c)
^2)*tanh(d*x+c))+b^3*(-1/3/sinh(d*x+c)^3/cosh(d*x+c)^5+8/3/sinh(d*x+c)/cos
h(d*x+c)^5+16*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(98) = 196$.

Time = 0.25 (sec) , antiderivative size = 955, normalized size of antiderivative = 9.18

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-8/15*((5*a^3 - 30*a^2*b - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^6 + 12*(5*a^3
+ 15*a^2*b + 30*a*b^2 + 16*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^3 - 3
0*a^2*b - 60*a*b^2 - 32*b^3)*sinh(d*x + c)^6 + 2*(15*a^3 - 60*a*b^2 - 32*b
^3)*cosh(d*x + c)^4 + (30*a^3 - 120*a*b^2 - 64*b^3 + 15*(5*a^3 - 30*a^2*b
- 60*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^3 + 15*a
^2*b + 30*a*b^2 + 16*b^3)*cosh(d*x + c)^3 + 4*(5*a^3 + 15*a^2*b + 15*a*b^2
 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 50*a^3 + 240*a^2*b + 360*a*b^2
+ 192*b^3 + (75*a^3 + 270*a^2*b + 300*a*b^2 + 64*b^3)*cosh(d*x + c)^2 + (1
5*(5*a^3 - 30*a^2*b - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^4 + 75*a^3 + 270*a^
2*b + 300*a*b^2 + 64*b^3 + 12*(15*a^3 - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 4*(3*(5*a^3 + 15*a^2*b + 30*a*b^2 + 16*b^3)*cosh(d*x +
 c)^5 + 8*(5*a^3 + 15*a^2*b + 15*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (25*a^3
+ 75*a^2*b + 30*a*b^2 - 32*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x
+ c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 2*d*co
sh(d*x + c)^8 + (45*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(15*d*cos
h(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^7 - 3*d*cosh(d*x + c)^6 +
(210*d*cosh(d*x + c)^4 + 56*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(
126*d*cosh(d*x + c)^5 + 56*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x
 + c)^5 - 8*d*cosh(d*x + c)^4 + (210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x +
c)^4 - 45*d*cosh(d*x + c)^2 - 8*d)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x +...
```

3.24.6 Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3\operatorname{csch}^4\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. 2(98) = 196.

Time = 0.19 (sec) , antiderivative size = 664, normalized size of antiderivative = 6.38

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$=\frac{4}{3}a^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$
$$+\frac{256}{15}b^3\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-4\,dx-4\,c)}-6\,e^{(-6\,dx-6\,c)}+6\,e^{(-10\,dx-10\,c)}+2\,e^{(-12\,dx-12\,c)}-2\,e^{(-14\,dx-14\,c)}-\right.$$
$$+16\,a^2b\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1)}-\frac{1}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1\right.$$
$$+32\,ab^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-4\,dx-4\,c)}-3\,e^{(-8\,dx-8\,c)}+e^{(-12\,dx-12\,c)}-1)}-\frac{1}{d(3\,e^{(-4\,dx-4\,c)}-3\,e^{(-8\,dx-8\,c)}+e^{(-12\,dx-12\,c)}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) - 1))) + 256/15*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x -
2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*
e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) - 2*e
^(-4*d*x - 4*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x
- 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c
) - e^(-16*d*x - 16*c) + 1)) - 6*e^(-6*d*x - 6*c)/(d*(2*e^(-2*d*x - 2*c) -
 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-12
*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) + 1/(d*(2*e
^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x -
 10*c) + 2*e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c)
+ 1))) + 16*a^2*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x
 - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x
- 6*c) + e^(-8*d*x - 8*c) - 1))) + 32*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-
4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)) - 1/(d*(3*e^(
-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)))
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(98) = 196$.

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.41

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{2\left(\frac{5\left(9\,a^2be^{(4\,dx+4\,c)}+18\,ab^2e^{(4\,dx+4\,c)}+9\,b^3e^{(4\,dx+4\,c)}-6\,a^3e^{(2\,dx+2\,c)}-36\,a^2be^{(2\,dx+2\,c)}-54\,ab^2e^{(2\,dx+2\,c)}-24\,b^3e^{(2\,dx+2\,c)}+2\,a^3+15\,a^2b+24\,\cdots}{\left(e^{(2\,dx+2\,c)}-1\right)^3}\cdots\right.}{\cdots}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
2/15*(5*(9*a^2*b*e^(4*d*x + 4*c) + 18*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d
*x + 4*c) - 6*a^3*e^(2*d*x + 2*c) - 36*a^2*b*e^(2*d*x + 2*c) - 54*a*b^2*e^
(2*d*x + 2*c) - 24*b^3*e^(2*d*x + 2*c) + 2*a^3 + 15*a^2*b + 24*a*b^2 + 11*
b^3)/(e^(2*d*x + 2*c) - 1)^3 - (45*a^2*b*e^(8*d*x + 8*c) + 90*a*b^2*e^(8*d
*x + 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 450*a*b^2
*e^(6*d*x + 6*c) + 240*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 7
50*a*b^2*e^(4*d*x + 4*c) + 490*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x +
2*c) + 510*a*b^2*e^(2*d*x + 2*c) + 320*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 12
0*a*b^2 + 73*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 745, normalized size of antiderivative = 7.16

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\frac{6\left(a^2b+2ab^2+b^3\right)}{d\left(e^{2c+2dx}-1\right)}$$
$$-\frac{\frac{2\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{6e^{6c+6dx}\left(a^2b+2ab^2+b^3\right)}{5d}+\frac{6e^{4c+4dx}\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{2e^{2c+2dx}\left(9a^2b+30ab^2+25b^3\right)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$-\frac{\frac{6\left(a^2b+2ab^2+b^3\right)}{5d}+\frac{6e^{8c+8dx}\left(a^2b+2ab^2+b^3\right)}{5d}+\frac{8e^{2c+2dx}\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{8e^{6c+6dx}\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{4e^{4c+4dx}(9}{}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$-\frac{\frac{2\left(9a^2b+30ab^2+25b^3\right)}{15d}+\frac{6e^{4c+4dx}\left(a^2b+2ab^2+b^3\right)}{5d}+\frac{4e^{2c+2dx}\left(3a^2b+9ab^2+5b^3\right)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$-\frac{4\left(a^3+3a^2b+3ab^2+b^3\right)}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}-\frac{\frac{2\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{6e^{2c+2dx}\left(a^2b+2ab^2+b^3\right)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$
$$-\frac{6\left(a^2b+2ab^2+b^3\right)}{5d\left(e^{2c+2dx}+1\right)}-\frac{8\left(a^3+3a^2b+3ab^2+b^3\right)}{3d\left(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

output
```
(6*(2*a*b^2 + a^2*b + b^3))/(d*(exp(2*c + 2*d*x) - 1)) - ((2*(9*a*b^2 + 3*
a^2*b + 5*b^3))/(5*d) + (6*exp(6*c + 6*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d)
 + (6*exp(4*c + 4*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(2*c + 2
*d*x)*(30*a*b^2 + 9*a^2*b + 25*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*
c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((6*(2*a*b^2 + a
^2*b + b^3))/(5*d) + (6*exp(8*c + 8*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) +
(8*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (8*exp(6*c + 6*d*
x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (4*exp(4*c + 4*d*x)*(30*a*b^2 + 9*
a^2*b + 25*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp
(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(30*a*b
^2 + 9*a^2*b + 25*b^3))/(15*d) + (6*exp(4*c + 4*d*x)*(2*a*b^2 + a^2*b + b^
3))/(5*d) + (4*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d))/(3*exp
(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (4*(3*a*b^2 +
 3*a^2*b + a^3 + b^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (
(2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(2*a*b^2 + a^2
*b + b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (6*(2*a*b^
2 + a^2*b + b^3))/(5*d*(exp(2*c + 2*d*x) + 1)) - (8*(3*a*b^2 + 3*a^2*b + a
^3 + b^3))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x
) - 1))
```

3.25 $\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.25.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{(3a^2+12ab+8b^2)\,x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d} - \frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad}$$

output
```
1/8*(3*a^2+12*a*b+8*b^2)*x/a^3-1/8*(5*a+4*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d
+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d-(a+b)^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)
/(a+b)^(1/2))*b^(1/2)/a^3/d
```

3.25.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 294 vs. 2(117) = 234.

Time = 3.21 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.51

$$\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(\sqrt{b}(3a^3+34a^2b+64ab^2+32b^3)\,\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\text{s}}{2\sqrt{a-}}\right.\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/64*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[b]*(3*a^3 + 3
4*a^2*b + 64*a*b^2 + 32*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((
a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]) - Sqrt[b*(Cosh[c] - Sinh[c])^4]*(a^2
*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a
 + b]*(-2*a^2*c + 12*a^2*d*x + 48*a*b*d*x + 32*b^2*d*x - 8*a*(a + b)*Sinh[
2*(c + d*x)] + a^2*Sinh[4*(c + d*x)]))))/(a^3*Sqrt[b]*Sqrt[a + b]*d*(a + b
*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

3.25.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4620, 372, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(ic+idx)^4}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2}-\frac{\int \frac{(4a+3b)\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)^2\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{4a}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{\int -\frac{b(5a+4b)\tanh^2(c+dx)+(a+b)(3a+4b)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a}+\frac{(5a+4b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}}{4a}}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+4b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}-\frac{\int\frac{b(5a+4b)\tanh^2(c+dx)+(a+b)(3a+4b)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a}}{4a}}{d}$$

$\big\downarrow$ 397

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+4b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{8b(a+b)^2\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a}}{4a}}{d}$$

$\big\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+4b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\text{arctanh}(\tanh(c+dx))}{a}-\frac{8b(a+b)^2\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a}}{4a}}{d}$$

$\big\downarrow$ 221

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+4b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\text{arctanh}(\tanh(c+dx))}{a}-\frac{8\sqrt{b}(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a}}{2a}}{4a}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
(Tanh[c + d*x]/(4*a*(1 - Tanh[c + d*x]^2)^2) - (-1/2*(((3*a^2 + 12*a*b + 8
*b^2)*ArcTanh[Tanh[c + d*x]])/a - (8*Sqrt[b]*(a + b)^(3/2)*ArcTanh[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a + b]])/a)/a + ((5*a + 4*b)*Tanh[c + d*x])/(2*a*(1
- Tanh[c + d*x]^2)))/(4*a))/d
```

3.25. $\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(103) = 206$.

Time = 235.07 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.79

method	result
risch	$\frac{3x}{8a}+\frac{3bx}{2a^2}+\frac{x\,b^2}{a^3}+\frac{e^{4dx+4c}}{64da}-\frac{e^{2dx+2c}}{8ad}-\frac{e^{2dx+2c}b}{8a^2d}+\frac{e^{-2dx-2c}}{8ad}+\frac{e^{-2dx-2c}b}{8a^2d}-\frac{e^{-4dx-4c}}{64da}+\frac{\sqrt{ab+b^2}\,\ln\left(\right.}{}$
derivativedivides	$\frac{\frac{1}{4a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+4b}{8a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a+4b}{8a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-3a^2-12ab-8b^2\right)\ln\left(\tanh\right.}{8a^3}}{}$
default	$\frac{\frac{1}{4a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+4b}{8a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a+4b}{8a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-3a^2-12ab-8b^2\right)\ln\left(\tanh\right.}{8a^3}}{}$

input
```
int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/a+3/2*b*x/a^2+x/a^3*b^2+1/64/d/a*exp(4*d*x+4*c)-1/8/a/d*exp(2*d*x+2*
c)-1/8/a^2/d*exp(2*d*x+2*c)*b+1/8/a/d*exp(-2*d*x-2*c)+1/8/a^2/d*exp(-2*d*x
-2*c)*b-1/64/d/a*exp(-4*d*x-4*c)+1/2*(a*b+b^2)^(1/2)/d/a^2*ln(exp(2*d*x+2*
c)+(a+2*(a*b+b^2)^(1/2)+2*b)/a)+1/2*(a*b+b^2)^(1/2)/d/a^3*ln(exp(2*d*x+2*c
)+(a+2*(a*b+b^2)^(1/2)+2*b)/a)*b-1/2*(a*b+b^2)^(1/2)/d/a^2*ln(exp(2*d*x+2*
c)-(-a+2*(a*b+b^2)^(1/2)-2*b)/a)-1/2*(a*b+b^2)^(1/2)/d/a^3*ln(exp(2*d*x+2*
c)-(-a+2*(a*b+b^2)^(1/2)-2*b)/a)*b
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(103) = 206$.

Time = 0.29 (sec) , antiderivative size = 1681, normalized size of antiderivative = 14.37

$$\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sin
h(d*x + c)^8 + 8*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a
*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x +
 c)^6 + 8*(7*a^2*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x - 60*(a
^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*
(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^
3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c
)^6 + 12*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cos
h(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*((a + b)*cosh(d*x + c)^
4 + 4*(a + b)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a + b)*cosh(d*x + c)^2*si
nh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x
 + c)^4)*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*si
nh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*
(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^
2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2
 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d
*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b...
```

3.25.6 Sympy [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\sinh^4(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(103) = 206$.

Time = 0.30 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.50

$$\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{3\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}ad} + \frac{3\,(dx+c)}{8\,ad} - \frac{\left(8\,be^{(-2\,dx-2\,c)}-a\right)e^{(4\,dx+4\,c)}}{64\,a^2d} - \frac{e^{(2\,dx+2\,c)}}{8\,ad} + \frac{e^{(-2\,dx-2\,c)}}{8\,ad} + \frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^2d} - \frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,a^2d} - \frac{(ab+2\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}a^2d} + \frac{(ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}a^2d} + \frac{(ab+2\,b^2)(dx+c)}{2\,a^3d} + \frac{8\,be^{(-2\,dx-2\,c)}-ae^{(-4\,dx-4\,c)}}{64\,a^2d} + \frac{(a^2b+8\,ab^2+8\,b^3)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}a^3d}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
3/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 3/8*(d*x +
 c)/(a*d) - 1/64*(8*b*e^(-2*d*x - 2*c) - a)*e^(4*d*x + 4*c)/(a^2*d) - 1/8*
e^(2*d*x + 2*c)/(a*d) + 1/8*e^(-2*d*x - 2*c)/(a*d) + 1/4*b*log(a*e^(4*d*x
+ 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) - 1/4*b*log(2*(a + 2*b)*
e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) - 1/8*(a*b + 2*b^2)*log
((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a
+ 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) + 1/8*(a*b + 2*b^2)*lo
g((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
 a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) + 1/2*(a*b + 2*b^2)
*(d*x + c)/(a^3*d) + 1/64*(8*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x - 4*c))/(a^2
*d) + 1/16*(a^2*b + 8*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2
*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqr
t((a + b)*b)*a^3*d)
```

3.25.8 Giac [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\sinh\left(dx+c\right)^4}{b\,\text{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.25.9 Mupad [B] (verification not implemented)

Time = 3.09 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.80

$$\begin{aligned}
&\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx \\
&= \frac{x\left(3a^2+12ab+8b^2\right)}{8a^3} - \frac{e^{-4c-4dx}}{64ad} + \frac{e^{4c+4dx}}{64ad} + \frac{e^{-2c-2dx}\left(a+b\right)}{8a^2d} - \frac{e^{2c+2dx}\left(a+b\right)}{8a^2d} \\
&\quad + \frac{\sqrt{b}\,\ln\left(\frac{4b(a+b)^3\left(2ab+a^2+a^2e^{2c+2dx}+8b^2e^{2c+2dx}+8abe^{2c+2dx}\right)}{a^8} - \frac{8b^{3/2}(a+b)^{7/2}\left(a+2ae^{2c+2dx}+4be^{2c+2dx}\right)}{a^8}\right)\left(a+b\right)^3}{2a^3d} \\
&\quad - \frac{\sqrt{b}\,\ln\left(\frac{8b^{3/2}(a+b)^{7/2}\left(a+2ae^{2c+2dx}+4be^{2c+2dx}\right)}{a^8} + \frac{4b(a+b)^3\left(2ab+a^2+a^2e^{2c+2dx}+8b^2e^{2c+2dx}+8abe^{2c+2dx}\right)}{a^8}\right)\left(a+b\right)^3}{2a^3d}
\end{aligned}$$

input
```
int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

output
```
(x*(12*a*b + 3*a^2 + 8*b^2))/(8*a^3) - exp(- 4*c - 4*d*x)/(64*a*d) + exp(4
*c + 4*d*x)/(64*a*d) + (exp(- 2*c - 2*d*x)*(a + b))/(8*a^2*d) - (exp(2*c +
 2*d*x)*(a + b))/(8*a^2*d) + (b^(1/2)*log((4*b*(a + b)^3*(2*a*b + a^2 + a^
2*exp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*x)))/a^8
 - (8*b^(3/2)*(a + b)^(7/2)*(a + 2*a*exp(2*c + 2*d*x) + 4*b*exp(2*c + 2*d*
x)))/a^8)*(a + b)^(3/2))/(2*a^3*d) - (b^(1/2)*log((8*b^(3/2)*(a + b)^(7/2)
*(a + 2*a*exp(2*c + 2*d*x) + 4*b*exp(2*c + 2*d*x)))/a^8 + (4*b*(a + b)^3*(
2*a*b + a^2 + a^2*exp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*
c + 2*d*x)))/a^8)*(a + b)^(3/2))/(2*a^3*d)
```

3.26 $\int \frac{\sinh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.26.1 Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{\sinh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d} - \frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad}$$

output
```
-(a+b)*cosh(d*x+c)/a^2/d+1/3*cosh(d*x+c)^3/a/d+(a+b)*arctan(cosh(d*x+c)*a^
(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/d
```

3.26.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.24

$$\int \frac{\sinh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\left(3(a^2+8ab+8b^2)\arctan\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\right.}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh
[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/S
qrt[b]] + 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(C
osh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
+ b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - 3*a^2*(ArcTan[
(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I
*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]]) - 6*Sqrt[a]*Sqrt[b]*(3*a + 4*b)*
Cosh[c + d*x] + 2*a^(3/2)*Sqrt[b]*Cosh[3*(c + d*x)]))/(48*a^(5/2)*Sqrt[b]*
d*(b + a*Cosh[c + d*x]^2))
```

3.26.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4621, 363, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin(ic+idx)^3}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\sin(ic+idx)^3}{b\sec(ic+idx)^2+a}dx$$

$$\Big\downarrow \text{ 4621}$$

$$-\frac{\int \frac{\cosh^2(c+dx)\left(1-\cosh^2(c+dx)\right)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 363}$$

$$-\frac{\frac{(a+b)\int \frac{\cosh^2(c+dx)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a}-\frac{\cosh^3(c+dx)}{3a}}{d}$$

$$\Big\downarrow \; 262$$

$$-\frac{\frac{(a+b)\left(\frac{\cosh(c+dx)}{a}-\frac{b\int\frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a}\right)}{a}-\frac{\cosh^3(c+dx)}{3a}}{d}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{(a+b)\left(\frac{\cosh(c+dx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}-\frac{\cosh^3(c+dx)}{3a}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
-((-1/3*Cosh[c + d*x]^3/a + ((a + b)*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c +
d*x])/Sqrt[b]])/a^(3/2)) + Cosh[c + d*x]/a))/a)/d)
```

3.26.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(61) = 122$.

Time = 37.00 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.39

method	result
derivativedivides	$\frac{\frac{1}{3a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a+2b}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+b)b\arctan\left(\frac{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2a-2b}{4\sqrt{ab}}\right)}{a^2\sqrt{ab}}-\frac{}{3a(}}{d}$
default	$\frac{\frac{1}{3a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a+2b}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+b)b\arctan\left(\frac{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2a-2b}{4\sqrt{ab}}\right)}{a^2\sqrt{ab}}-\frac{}{3a(}}{d}$
risch	$\frac{e^{3dx+3c}}{24da}-\frac{3\,e^{dx+c}}{8ad}-\frac{e^{dx+c}b}{2a^2d}-\frac{3\,e^{-dx-c}}{8ad}-\frac{e^{-dx-c}b}{2a^2d}+\frac{e^{-3dx-3c}}{24da}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}\,e^{dx+c}}{a}+1\right)}{2a^2d}+$

input
```
int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3/a/(1+tanh(1/2*d*x+1/2*c))^3-1/2/a/(1+tanh(1/2*d*x+1/2*c))^2-1/2*(
a+2*b)/a^2/(1+tanh(1/2*d*x+1/2*c))+(a+b)*b/a^2/(a*b)^(1/2)*arctan(1/4*(2*(
a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))-1/3/a/(tanh(1/2*d*x+1/2*c
)-1)^3-1/2/a/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/a^2*(-a-2*b)/(tanh(1/2*d*x+1/2*
c)-1))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(61) = 122$.

Time = 0.28 (sec) , antiderivative size = 1246, normalized size of antiderivative = 17.55

$$\int \frac{\sinh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x
+ c)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - 3*a - 4*
b)*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c))
*sinh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4
- 6*(3*a + 4*b)*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^2 + 12*((a + b)
*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a + b)*cos
h(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt(-b/a)*log((a*co
sh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*
(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c
)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a
*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 +
 a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a
)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c)
 + a)) + 6*(a*cosh(d*x + c)^5 - 2*(3*a + 4*b)*cosh(d*x + c)^3 - (3*a + 4*b
)*cosh(d*x + c))*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^3 + 3*a^2*d*cosh(
d*x + c)^2*sinh(d*x + c) + 3*a^2*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*d*s
inh(d*x + c)^3), 1/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)
^5 + a*sinh(d*x + c)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*...
```

3.26.6 Sympy [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\sinh^3\left(c+dx\right)}{a+b\,\text{sech}^2\left(c+dx\right)}\,dx$$

input
```
integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.26.7 Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)^3}{b\,\text{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/24*(3*(3*a*e^(4*c) + 4*b*e^(4*c))*e^(4*d*x) + 3*(3*a*e^(2*c) + 4*b*e^(2
*c))*e^(2*d*x) - a*e^(6*d*x + 6*c) - a)*e^(-3*d*x - 3*c)/(a^2*d) + 1/8*int
egrate(16*((a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(
d*x))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*
d*x)), x)
```

3.26.8 Giac [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)^3}{b\,\text{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.26.9 Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 473, normalized size of antiderivative = 6.66

$$\int \frac{\sinh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{e^{-3c-3dx}}{24\,a\,d} - \frac{\left(2\,\mathrm{atan}\left(\frac{a^6\,e^{d\,x}\,e^c\left(\frac{4\left(2\,a^4\,b\,d\,\sqrt{a^2\,b+2\,a\,b^2+b^3}+2\,a^2\,b^3\,d\,\sqrt{a^2\,b+2\,a\,b^2+b^3}+4\,a^3\,b^2\,d\,\sqrt{a^2\,b+2\,a\,b^2+b^3}\right)}{a^{11}\,d^2\,(a+b)}+\frac{2\left(b^4\,\sqrt{a^5\,d^2}+3\,a^2\,b^2\,\sqrt{a^5\,d^2}+3\,a\,b^3\,\sqrt{a^5 \cdots}\right.}{a^8\,d\,\sqrt{b\,(a+b)^2}\,\sqrt{a^5\,d^2 \cdots}}\cdots\right.}{4\,a^2\,b+8\,a\,b^2+4\,b^3}\cdots\right.\right.}{\cdots} + \frac{e^{3c+3dx}}{24\,a\,d} - \frac{e^{c+dx}\,(3\,a+4\,b)}{8\,a^2\,d} - \frac{e^{-c-dx}\,(3\,a+4\,b)}{8\,a^2\,d}$$

input
```
int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

output
```
exp(- 3*c - 3*d*x)/(24*a*d) - ((2*atan((a^6*exp(d*x)*exp(c)*((4*(2*a^4*b*d
*(2*a*b^2 + a^2*b + b^3)^(1/2) + 2*a^2*b^3*d*(2*a*b^2 + a^2*b + b^3)^(1/2)
 + 4*a^3*b^2*d*(2*a*b^2 + a^2*b + b^3)^(1/2)))/(a^11*d^2*(a + b)) + (2*(b^
4*(a^5*d^2)^(1/2) + 3*a^2*b^2*(a^5*d^2)^(1/2) + 3*a*b^3*(a^5*d^2)^(1/2) +
a^3*b*(a^5*d^2)^(1/2)))/(a^8*d*(b*(a + b)^2)^(1/2)*(a^5*d^2)^(1/2)))*(a^5*
d^2)^(1/2))/(8*a*b^2 + 4*a^2*b + 4*b^3) + (2*exp(3*c)*exp(3*d*x)*(b^4*(a^5
*d^2)^(1/2) + 3*a^2*b^2*(a^5*d^2)^(1/2) + 3*a*b^3*(a^5*d^2)^(1/2) + a^3*b*
(a^5*d^2)^(1/2)))/(a^2*d*(b*(a + b)^2)^(1/2)*(8*a*b^2 + 4*a^2*b + 4*b^3)))
 - 2*atan((exp(d*x)*exp(c)*(a + b)*(a^5*d^2)^(1/2))/(2*a^2*d*(b*(a + b)^2)
^(1/2))))*(2*a*b^2 + a^2*b + b^3)^(1/2))/(2*(a^5*d^2)^(1/2)) + exp(3*c + 3
*d*x)/(24*a*d) - (exp(c + d*x)*(3*a + 4*b))/(8*a^2*d) - (exp(- c - d*x)*(3
*a + 4*b))/(8*a^2*d)
```

3.26. $\int \frac{\sinh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.27 $\int \frac{\sinh^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

3.27.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\sinh^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx = -\frac{(a+2b)x}{2a^2}+\frac{\sqrt{b}\sqrt{a+b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2ad}$$

output
```
-1/2*(a+2*b)*x/a^2+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d+arctanh(b^(1/2)*tanh(d*
x+c)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/d
```

3.27.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 236 vs. $2(75) = 150$.

Time = 0.97 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.15

$$\int \frac{\sinh^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\mathrm{sech}^2(c+dx)\left(-\frac{\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}d}+\frac{-4(a+2b)x+\frac{(a^2+8ab+8b^2)\mathrm{arctanh}\left(\frac{\mathrm{sech}(d}{}\right.}{}}{}\right.}{16\left(a+b\,\mathrm{sech}^2(c+dx)\right.}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(-(ArcTanh[(Sqrt[b]*Tanh[
c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)) + (-4*(a + 2*b)*x + ((a^2
+ 8*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sin
h[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])
]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) +
 (2*a*Cosh[2*d*x]*Sinh[2*c])/d + (2*a*Cosh[2*c]*Sinh[2*d*x])/d)/a^2))/(16*
(a + b*Sech[c + d*x]^2))
```

3.27.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4620, 373, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{b\sec(ic+idx)^2+a}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))^2(-b\tanh^2(c+dx)+a+b)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))}-\frac{\int \frac{b\tanh^2(c+dx)+a+b}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)}d\tanh(c+dx)}{2a}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))}-\frac{\frac{(a+2b)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{2b(a+b)\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))}-\frac{\frac{(a+2b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{2b(a+b)\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))}-\frac{\frac{(a+2b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{2\sqrt{b}\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a}}{2a}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-1/2*(((a + 2*b)*ArcTanh[Tanh[c + d*x]])/a - (2*Sqrt[b]*Sqrt[a + b]*ArcTa
nh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/a)/a + Tanh[c + d*x]/(2*a*(1 - Ta
nh[c + d*x]^2)))/d
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.27.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(63) = 126$.

Time = 8.79 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.97

method	result
risch	$-\frac{x}{2a}-\frac{bx}{a^2}+\frac{e^{2dx+2c}}{8ad}-\frac{e^{-2dx-2c}}{8ad}+\frac{\sqrt{ab+b^2}\,\ln\left(e^{2dx+2c}-\frac{-a+2\sqrt{ab+b^2}-2b}{a}\right)}{2d\,a^2}-\frac{\sqrt{ab+b^2}\,\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{a\cdots}}{\cdots}\right.}{2d\,a^2}$
derivativedivides	$\frac{-\frac{1}{2a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a-2b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{2b(a+b)\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\cdots\right.\right.}{4\sqrt{b}\,\sqrt{a+b}}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{1}{2a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a-2b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{2b(a+b)\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\cdots\right.\right.}{4\sqrt{b}\,\sqrt{a+b}}\cdots\right.}{\cdots}}{\cdots}$

3.27. $\int \frac{\sinh^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

input
```
int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/a-b*x/a^2+1/8/a/d*exp(2*d*x+2*c)-1/8/a/d*exp(-2*d*x-2*c)+1/2*(a*b+b
^2)^(1/2)/d/a^2*ln(exp(2*d*x+2*c)-(-a+2*(a*b+b^2)^(1/2)-2*b)/a)-1/2*(a*b+b
^2)^(1/2)/d/a^2*ln(exp(2*d*x+2*c)+(a+2*(a*b+b^2)^(1/2)+2*b)/a)
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(63) = 126$.

Time = 0.28 (sec) , antiderivative size = 805, normalized size of antiderivative = 10.73

$$\int \frac{\sinh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(a + 2*b)*d*x*cosh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x
+ c)*sinh(d*x + c)^3 - a*sinh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d
*x + c)^2)*sinh(d*x + c)^2 - 4*sqrt(a*b + b^2)*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*
cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh
(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^
2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*
sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a
*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh
(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^
2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)
^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(2*(a + 2*b)*d*x*cos
h(d*x + c) - a*cosh(d*x + c)^3)*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^2
+ 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2), -1/8*(4*(a
 + 2*b)*d*x*cosh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x + c)*sinh(d
*x + c)^3 - a*sinh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d*x + c)^2)*
sinh(d*x + c)^2 - 8*sqrt(-a*b - b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*
x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b
+ b^2)) + 4*(2*(a + 2*b)*d*x*cosh(d*x + c) - a*cosh(d*x + c)^3)*sinh(d*...
```

3.27.6 Sympy [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \int \frac{\sinh^2{(c+dx)}}{a+b\,\mathrm{sech}^2{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.27.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(63) = 126$.

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.69

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = &-\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}ad} - \frac{dx+c}{2\,ad} + \frac{e^{(2\,dx+2\,c)}}{8\,ad} \\ &- \frac{e^{(-2\,dx-2\,c)}}{8\,ad} - \frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^2d} \\ &+ \frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,a^2d} \\ &+ \frac{(ab+2\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}a^2d} \\ &- \frac{(ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}a^2d}\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) - 1/2*(d*x +
 c)/(a*d) + 1/8*e^(2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b
*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) + 1/4*b*
log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) + 1/8*(
a*b + 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2
*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) - 1/8*
(a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^
(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d)
```

3.27.8 Giac [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)^2}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.27.9 Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.68

$$\int \frac{\sinh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{e^{2c+2dx}}{8ad} - \frac{e^{-2c-2dx}}{8ad} - \frac{x(a+2b)}{2a^2}$$
$$- \frac{\sqrt{b}\ln\left(2ab+a^2+a^2e^{2c+2dx}+8b^2e^{2c+2dx}-2a\sqrt{b}\sqrt{a+b}-8b^{3/2}e^{2c+2dx}\sqrt{a+b}+8abe^{2c+2dx}-\right.}{2a^2d}$$
$$+ \frac{\sqrt{b}\ln\left(2ab+a^2+a^2e^{2c+2dx}+8b^2e^{2c+2dx}+2a\sqrt{b}\sqrt{a+b}+8b^{3/2}e^{2c+2dx}\sqrt{a+b}+8abe^{2c+2dx}+\right.}{2a^2d}$$

input
```
int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

output
```
exp(2*c + 2*d*x)/(8*a*d) - exp(- 2*c - 2*d*x)/(8*a*d) - (x*(a + 2*b))/(2*a
^2) - (b^(1/2)*log(2*a*b + a^2 + a^2*exp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*
d*x) - 2*a*b^(1/2)*(a + b)^(1/2) - 8*b^(3/2)*exp(2*c + 2*d*x)*(a + b)^(1/2
) + 8*a*b*exp(2*c + 2*d*x) - 4*a*b^(1/2)*exp(2*c + 2*d*x)*(a + b)^(1/2))*(
a + b)^(1/2))/(2*a^2*d) + (b^(1/2)*log(2*a*b + a^2 + a^2*exp(2*c + 2*d*x)
+ 8*b^2*exp(2*c + 2*d*x) + 2*a*b^(1/2)*(a + b)^(1/2) + 8*b^(3/2)*exp(2*c +
 2*d*x)*(a + b)^(1/2) + 8*a*b*exp(2*c + 2*d*x) + 4*a*b^(1/2)*exp(2*c + 2*d
*x)*(a + b)^(1/2))*(a + b)^(1/2))/(2*a^2*d)
```

3.28 $\int \frac{\sinh(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

3.28.1 Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{\sinh(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx=-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d}+\frac{\cosh(c+dx)}{ad}$$

output
```
cosh(d*x+c)/a/d-arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/d
```

3.28.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 328, normalized size of antiderivative = 6.98

$$\int \frac{\sinh(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx = \frac{\left(-\frac{(a+4b)\left(\arctan\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{\left(\sqrt{a}+i\sqrt{a+\cdots}\right.}{\cdots}\right.\right.}{\sqrt{b}}\cdots\right.}{\cdots}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
((-(((a + 4*b)*(ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^
2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c]
 - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b]
*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I
*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]))/Sqrt[b
]) + (a*(ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + Arc
Tan[(Sqrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]]))/Sqrt[b] + 4*Sqr
t[a]*Cosh[c + d*x])*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2)/(8*a^
(3/2)*d*(a + b*Sech[c + d*x]^2))
```

3.28.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = $\{3042, 26, 4621, 262, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{b\sec(ic+idx)^2+a}dx$$

$$\Big\downarrow \text{4621}$$

$$\frac{\int \frac{\cosh^2(c+dx)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\cosh(c+dx)}{a}-\frac{b\int \frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\cosh(c+dx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-((Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/a^(3/2)) + Cosh[c + d
*x]/a)/d
```

3.28.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.28.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{1}{da\ \mathrm{sech}(dx+c)}+\frac{b\arctan\left(\frac{b\ \mathrm{sech}(dx+c)}{\sqrt{ab}}\right)}{da\sqrt{ab}}$	44
default	$\frac{1}{da\ \mathrm{sech}(dx+c)}+\frac{b\arctan\left(\frac{b\ \mathrm{sech}(dx+c)}{\sqrt{ab}}\right)}{da\sqrt{ab}}$	44
risch	$\frac{e^{dx+c}}{2ad}+\frac{e^{-dx-c}}{2ad}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}\,e^{dx+c}}{a}+1\right)}{2a^2d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}\,e^{dx+c}}{a}+1\right)}{2a^2d}$	119

input
```
int(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a/sech(d*x+c)+1/d*b/a/(a*b)^(1/2)*arctan(b*sech(d*x+c)/(a*b)^(1/2))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(39) = 78.

Time = 0.27 (sec) , antiderivative size = 595, normalized size of antiderivative = 12.66

$$\int \frac{\sinh(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{b}{a}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)\log\left(\frac{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(a-2\,b)\cosh(dx+c)^2 \cdots}{a\cosh(dx+c)^4+4\,a\cos \cdots}\right. \cdots}{\cdots}\right.$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*(cosh(d*x + c) + sinh(d*x + c))*log((a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d
*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(
d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^3
 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c)
 + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x +
c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)
*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(
a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)/(a*d*cosh(
d*x + c) + a*d*sinh(d*x + c)), 1/2*(2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x
+ c))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 +
a*sinh(d*x + c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4
*b)*sinh(d*x + c))*sqrt(b/a)/b) - 2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x +
c))*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)/(a*d*cosh(d
*x + c) + a*d*sinh(d*x + c))]
```

3.28.6 Sympy [F]

$$\int \frac{\sinh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\sinh(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.28.7 Maxima [F]

$$\int \frac{\sinh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\,\text{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x
 + 3*c) - b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a
*b*e^(2*c))*e^(2*d*x)), x)
```

3.28.8 Giac [F]

$$\int \frac{\sinh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\,\text{sech}\,(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\sinh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\cosh(c+d\,x)}{a\,d} - \frac{b\,\text{atan}\left(\frac{a\cosh(c+d\,x)}{\sqrt{a\,b}}\right)}{a\,d\,\sqrt{a\,b}}$$

input
```
int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

output
```
cosh(c + d*x)/(a*d) - (b*atan((a*cosh(c + d*x))/(a*b)^(1/2)))/(a*d*(a*b)^(
1/2))
```

3.29 $\int \frac{\text{csch}(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.29.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\text{csch}(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)d}-\frac{\text{arctanh}(\cosh(c+dx))}{(a+b)d}$$

output
```
-arctanh(cosh(d*x+c))/(a+b)/d+arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/
(a+b)/d/a^(1/2)
```

3.29.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.22

$$\int \frac{\text{csch}(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}}+\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(c\cdots}{\cdots}\right.}{\cdots}}{(\cdots}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
((Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Si
nh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sin
h[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((Sqrt[a] + I
*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*
(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[
b]])/Sqrt[a] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]])/((a + b)*d
)
```

3.29.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4621, 383, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a+b\sec(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{383}$$

$$-\frac{\frac{\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}-\frac{b\int \frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a+b}}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\operatorname{arctanh}(\cosh(c+dx))}{a+b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
-((-((Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(Sqrt[a]*(a + b)))
+ ArcTanh[Cosh[c + d*x]]/(a + b))/d)
```

3.29.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.29.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}+\frac{b\arctan\left(\frac{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2a-2b}{4\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}+\frac{b\arctan\left(\frac{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2a-2b}{4\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}}{d}$
risch	$-\frac{\ln(\mathrm{e}^{dx+c}+1)}{d(a+b)}+\frac{\ln(\mathrm{e}^{dx+c}-1)}{d(a+b)}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-ab}\,\mathrm{e}^{dx+c}}{a}+1\right)}{2a(a+b)d}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{-ab}\,\mathrm{e}^{dx+c}}{a}+1\right)}{2a(a+b)d}$

input
```
int(csch(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a+b)*ln(tanh(1/2*d*x+1/2*c))+b/(a+b)/(a*b)^(1/2)*arctan(1/4*(2*(a+
b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2)))
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(47) = 94.

Time = 0.28 (sec) , antiderivative size = 533, normalized size of antiderivative = 9.69

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a-2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2+a-2b\right)\sinh(dx+c)^2}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(}\right.}{}\right.$$
$$-\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(a\cosh(dx+c)^3+3a\cosh(dx+c)\sinh(dx+c)^2+a\sinh(dx+c)^3+(a+4b)\cosh(dx+c)+\left(3a\cosh(dx+c)^2+a+4b\right)\sinh(dx+c)\right.}{2b}\right.}{}$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^
3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x
+ c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x +
c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sin
h(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*
x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d
*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*co
sh(d*x + c))*sinh(d*x + c) + a)) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1
) + 2*log(cosh(d*x + c) + sinh(d*x + c) - 1))/((a + b)*d), -(sqrt(b/a)*arc
tan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*
x + c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(
d*x + c))*sqrt(b/a)/b) - sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*
x + c))*sqrt(b/a)/b) + log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d
*x + c) + sinh(d*x + c) - 1))/((a + b)*d)]
```

3.29.6 Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.29.7 Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-log((e^(d*x + c) + 1)*e^(-c))/(a*d + b*d) + log((e^(d*x + c) - 1)*e^(-c))
/(a*d + b*d) + 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b
+ (a^2*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) + 3*a*b*e^(2*c) +
 2*b^2*e^(2*c))*e^(2*d*x)), x)
```

3.29.8 Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{b\operatorname{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.29.9 Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 616, normalized size of antiderivative = 11.20

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$
$$= -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^4\,\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}+16\,a^2\,b^2\,\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}+8\,a\,b^3\,\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}\right)}{16\,d\,a^3\,b^2+24\,d\,a^2\,b^3+9\,d\,a\,b^4+d\,b^5}\right)}{\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}}$$
$$-\frac{\sqrt{b}\left(2\,\mathrm{atan}\left(\frac{\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(\frac{64\,\left(2\,b^{7/2}\,d+8\,a^2\,b^{3/2}\,d+10\,a\,b^{5/2}\,d\right)}{a^5\,\left(a+b\right)\,\sqrt{a\,d^2\,{\left(a+b\right)}^2}\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}}+\frac{32\,\left(b^2\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}+4\,a\,b\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}\right)}{a^5\,\sqrt{b}\,d\,{\left(a+b\right)}^2\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}}\right)+}{6}\right.\right.}{2}$$

input
```
int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)),x)
```

output
```
- (2*atan((exp(d*x)*exp(c)*(b^4*(- a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) +
16*a^2*b^2*(- a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) + 8*a*b^3*(- a^2*d^2 -
b^2*d^2 - 2*a*b*d^2)^(1/2)))/(b^5*d + 24*a^2*b^3*d + 16*a^3*b^2*d + 9*a*b^
4*d)))/(- a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) - (b^(1/2)*(2*atan(((exp(d*
x)*exp(c)*((64*(2*b^(7/2)*d + 8*a^2*b^(3/2)*d + 10*a*b^(5/2)*d))/(a^5*(a +
 b)*(a*d^2*(a + b)^2)^(1/2)*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2)) + (
32*(b^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2) + 4*a*b*(a^3*d^2 + a*b^2
*d^2 + 2*a^2*b*d^2)^(1/2)))/(a^5*b^(1/2)*d*(a + b)^2*(a^3*d^2 + a*b^2*d^2
+ 2*a^2*b*d^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(b^2*(a^3*d^2 + a*b^2*d^2
 + 2*a^2*b*d^2)^(1/2) + 4*a*b*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2)))/
(a^5*b^(1/2)*d*(a + b)^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2)))*(a^6*
(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2) + a^5*b*(a^3*d^2 + a*b^2*d^2 + 2
*a^2*b*d^2)^(1/2)))/(256*a*b + 64*b^2)) - 2*atan((exp(d*x)*exp(c)*(a*d^2*(
a + b)^2)^(1/2))/(2*b^(1/2)*d*(a + b)))))/(2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*
b*d^2)^(1/2))
```

3.29. $\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.30 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.30.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d}-\frac{\coth(c+dx)}{(a+b)d}$$

output
```
-coth(d*x+c)/(a+b)/d+arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b
)^(3/2)/d
```

3.30.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(53) = 106.

Time = 1.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.38

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$
$$=\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(b\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(c\cdots\right.}{2(a+b)^{3/2}d\left(a+b\operatorname{sech}^2(c+dx)\right)\sqrt{b(c\cdots}}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*ArcTanh[(Sech[d*x]*(Co
sh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]) + Sqrt[a + b]
*Csch[c]*Csch[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x]))/(2*(a + b
)^(3/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

3.30.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4620, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\coth^2(c+dx)}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{\frac{b\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a+b}-\frac{\coth(c+dx)}{a+b}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\coth(c+dx)}{a+b}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
((Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2) - Co
th[c + d*x]/(a + b))/d
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.30.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(45) = 90$.

Time = 0.98 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

method	result
risch	$-\frac{2}{d(a+b)(e^{2dx+2c}-1)}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+b)b}-a-2b}{a}\right)}{2(a+b)^2 d}-\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}+a+2b}{a}\right)}{2(a+b)^2 d}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}-\frac{1}{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\right.}{4\sqrt{b}}\right.}{a+b}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}-\frac{1}{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\right.}{4\sqrt{b}}\right.}{a+b}}{d}$

input
```
int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a+b)/(exp(2*d*x+2*c)-1)+1/2*((a+b)*b)^(1/2)/(a+b)^2/d*ln(exp(2*d*x+2
*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a)-1/2*((a+b)*b)^(1/2)/(a+b)^2/d*ln(exp(2*d*
x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/a)
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 588, normalized size of antiderivative = 11.09

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

$$= \left[\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{a^2\cosh(dx+c)^4+4a^2\cosh(dx+\right.}{}}{}\right.$$

input
```
integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
 1)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*
x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^
2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4
*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a
^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a
^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(
d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
+ 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4
)/((a + b)*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d*x + c)*sinh(d*x + c) + (
a + b)*d*sinh(d*x + c)^2 - (a + b)*d), ((cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(
d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b
)*sqrt(-b/(a + b))/b) - 2)/((a + b)*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d
*x + c)*sinh(d*x + c) + (a + b)*d*sinh(d*x + c)^2 - (a + b)*d)]
```

3.30.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}(a+b)d} + \frac{2}{((a+b)e^{(-2\,dx-2\,c)}-a-b)d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a + b)*d) + 2/((
(a + b)*e^(-2*d*x - 2*c) - a - b)*d)
```

3.30.8 Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2}{b\operatorname{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.30.9 Mupad [B] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 847, normalized size of antiderivative = 15.98

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{2}{\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,\left(a\,d+b\,d\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(\frac{2\,\left(8\,b^{5/2}\,\sqrt{-a^3\,d^2-3\,a^2\,b\,d^2-3\,a\,b^2\,d^2-b^3\,d^2}+8\,a\,b^{3/2}\,\sqrt{-a^3\,d^2-3\,a^2\,b\,d^2-3\,a\,b^2\,d^2-b^3\,d^2}+a^2\,\sqrt{b}\,\sqrt{-a^3\,d^2-3\,a^2\,b\,d^2-3\,a\,b^2 \cdots}{a^5\,d\,(a+b)^3\,\left(a^2+2\,a\,b+b^2\right)\,\sqrt{-a^3\,d^2-3\,a^2\,b\,d^2-3\,a\,b^2\,d^2-b^3\,d^2}} \cdots}{\cdots}\right)}{\cdots}$$

input
```
int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)),x)
```

output
```
- 2/((exp(2*c + 2*d*x) - 1)*(a*d + b*d)) - (b^(1/2)*atan(((exp(2*c)*exp(2*
d*x)*((2*(8*b^(5/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2
) + 8*a*b^(3/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) +
a^2*b^(1/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))*(8*a*
b + a^2 + 8*b^2))/(a^5*d*(a + b)^3*(2*a*b + a^2 + b^2)*(- a^3*d^2 - b^3*d^
2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(8*b^4*d +
16*a^2*b^2*d + 20*a*b^3*d + 4*a^3*b*d))/(a^5*(a + b)*(-d^2*(a + b)^3)^(1/2
)*(2*a*b + a^2 + b^2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1
/2))) + (2*(2*a*b^(3/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^
(1/2) + a^2*b^(1/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2
))*(8*a*b + a^2 + 8*b^2))/(a^5*d*(a + b)^3*(2*a*b + a^2 + b^2)*(- a^3*d^2
- b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(4*
a^2*b^2*d + 2*a*b^3*d + 2*a^3*b*d))/(a^5*(a + b)*(-d^2*(a + b)^3)^(1/2)*(2
*a*b + a^2 + b^2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))
)*(a^5*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 3*a^4*b*(
- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a^2*b^3*(- a^3*d^
2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 3*a^3*b^2*(- a^3*d^2 - b^
3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))/(4*b)))/(- a^3*d^2 - b^3*d^2 -
3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)
```

3.30. $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.31 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.31.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{(a+b)^2d} + \frac{(a-b)\operatorname{arctanh}(\cosh(c+dx))}{2(a+b)^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d}$$

output
```
1/2*(a-b)*arctanh(cosh(d*x+c))/(a+b)^2/d-1/2*coth(d*x+c)*csch(d*x+c)/(a+b)
/d-arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/d
```

3.31.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.89

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{(a+2b+a\cosh(2(c+dx)))\left(8\sqrt{a}\sqrt{b}\arctan\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+}\right.}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/16*((a + 2*b + a*Cosh[2*(c + d*x)])*(8*Sqrt[a]*Sqrt[b]*ArcTan[((Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh
[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/S
qrt[b]] + 8*Sqrt[a]*Sqrt[b]*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c]
 - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*S
qrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + (a + b)*Csch[(c + d*
x)/2]^2 - 4*a*Log[Cosh[(c + d*x)/2]] + 4*b*Log[Cosh[(c + d*x)/2]] + 4*a*Lo
g[Sinh[(c + d*x)/2]] - 4*b*Log[Sinh[(c + d*x)/2]] + (a + b)*Sech[(c + d*x)
/2]^2)*Sech[c + d*x]^2)/((a + b)^2*d*(a + b*Sech[c + d*x]^2))
```

3.31.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4621, 373, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{4621}$$

$$\frac{\int \frac{\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2\left(a\cosh^2(c+dx)+b\right)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\cosh(c+dx)}{2(a+b)\left(1-\cosh^2(c+dx)\right)}-\frac{\int \frac{b-a\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}d\cosh(c+dx)}{2(a+b)}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\cosh(c+dx)}{2(a+b)(1-\cosh^2(c+dx))}-\frac{\frac{2ab\int \frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a+b}-\frac{(a-b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{2(a+b)}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\cosh(c+dx)}{2(a+b)(1-\cosh^2(c+dx))}-\frac{\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a+b}-\frac{(a-b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{2(a+b)}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\cosh(c+dx)}{2(a+b)(1-\cosh^2(c+dx))}-\frac{\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a+b}-\frac{(a-b)\text{arctanh}(\cosh(c+dx))}{a+b}}{2(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-1/2*((2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a + b)
 - ((a - b)*ArcTanh[Cosh[c + d*x]])/(a + b))/(a + b) + Cosh[c + d*x]/(2*(a
 + b)*(1 - Cosh[c + d*x]^2)))/d
```

3.31.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.31.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a+8b}-\frac{1}{8(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a+2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}-\frac{ab\arctan\left(\frac{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2a-2b}{4\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a+8b}-\frac{1}{8(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a+2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}-\frac{ab\arctan\left(\frac{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2a-2b}{4\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}+1\right)}{d(a+b)\left(e^{2dx+2c}-1\right)^2}-\frac{\ln\left(e^{dx+c}-1\right)a}{2d(a^2+2ab+b^2)}+\frac{\ln\left(e^{dx+c}-1\right)b}{2d(a^2+2ab+b^2)}+\frac{\ln\left(e^{dx+c}+1\right)a}{2d(a^2+2ab+b^2)}-\frac{\ln\left(e^{dx+c}+1\right)b}{2d(a^2+2ab+b^2)}+\frac{\sqrt{-ab}\ \mathrm{l}}{}$

input
```
int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/(a+b)-1/8/(a+b)/tanh(1/2*d*x+1/2*c)^2+1/4/(
a+b)^2*(-2*a+2*b)*ln(tanh(1/2*d*x+1/2*c))-a*b/(a+b)^2/(a*b)^(1/2)*arctan(1
/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2)))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(75) = 150$.

Time = 0.32 (sec) , antiderivative size = 1881, normalized size of antiderivative = 21.62

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(2*(a + b)*cosh(d*x + c)^3 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2
 + 2*(a + b)*sinh(d*x + c)^3 - (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
 + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*
cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*s
qrt(-a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*s
inh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a
- 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*s
inh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh
(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(
-a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh
(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2
*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh
(d*x + c) + a)) + 2*(a + b)*cosh(d*x + c) - ((a - b)*cosh(d*x + c)^4 + 4*(
a - b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)
*cosh(d*x + c)^2 + 2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 +
 4*((a - b)*cosh(d*x + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b
)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a - b)*cosh(d*x + c)^4 + 4*(a
 - b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 +
4*((a - b)*cosh(d*x + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a -...
```

3.31.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.31.7 Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(a - b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) - 1/2*
(a - b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) - (e^(3*d*
x + 3*c) + e^(d*x + c))/(a*d + b*d + (a*d*e^(4*c) + b*d*e^(4*c))*e^(4*d*x)
 - 2*(a*d*e^(2*c) + b*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/4*(a*b*e^(3*d*
x + 3*c) - a*b*e^(d*x + c))/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^(4*c) + 2*a^2*
b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) + 4*a^2*b*e^(2*c) +
5*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

3.31.8 Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.31.9 Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 1586, normalized size of antiderivative = 18.23

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)),x)
```

output
```
((a*b)^(1/2)*(2*atan(((exp(d*x)*exp(c)*((64*(2*b^5*d*(a*b)^(1/2) + 2*a*b^4
*d*(a*b)^(1/2) + 2*a^4*b*d*(a*b)^(1/2) + 2*a^3*b^2*d*(a*b)^(1/2)))/(a^4*(a
 + b)^3*(d^2*(a + b)^4)^(1/2)*(2*a*b + a^2 + b^2)*(a^4*d^2 + b^4*d^2 + 4*a
*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)) + (32*(a*b^3*(a^4*d^2 + b^4
*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + a^3*b*(a^4*d^2 +
 b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) - a^2*b^2*(a^4
*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))/(a^3*d
*(a*b)^(1/2)*(a + b)^5*(2*a*b + a^2 + b^2)*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^
2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(a*b^3*
(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + a^
3*b*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)
- a^2*b^2*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^
(1/2)))/(a^3*d*(a*b)^(1/2)*(a + b)^5*(2*a*b + a^2 + b^2)*(a^4*d^2 + b^4*d^
2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))*(a^8*(a^4*d^2 + b^4
*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 5*a^7*b*(a^4*d^2
 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + a^3*b^5*(a
^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 5*a^
4*b^4*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2
) + 10*a^5*b^3*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*
d^2)^(1/2) + 10*a^6*b^2*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 ...
```

3.32 $\int \frac{\text{csch}^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.32.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\text{csch}^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{a\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} + \frac{a\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d}$$

output
```
a*coth(d*x+c)/(a+b)^2/d-1/3*coth(d*x+c)^3/(a+b)/d-a*arctanh(b^(1/2)*tanh(d
*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d
```

3.32.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(75) = 150.

Time = 2.93 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.88

$$\int \frac{\text{csch}^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$= \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(3ab\,\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)\right.}{6(a+b)^{5/}}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a*b*ArcTanh[(Sech[d*x]
*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqr
t[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + (Sqrt[
a + b]*Csch[c]*Csch[c + d*x]^3*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(6*a*Sinh[d*x
] - 3*b*Sinh[2*c + d*x] + (-2*a + b)*Sinh[2*c + 3*d*x]))/4))/(6*(a + b)^(5
/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

3.32.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4620, 359, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^4(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)}dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 359$$

$$\frac{-\frac{a\int \frac{\coth^2(c+dx)}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a+b}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{a\left(\frac{b\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a+b}-\frac{\coth(c+dx)}{a+b}\right)}{a+b}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{-\frac{a\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\coth(c+dx)}{a+b}\right)}{a+b}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-1/3*Coth[c + d*x]^3/(a + b) - (a*((Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a + b]])/(a + b)^(3/2) - Coth[c + d*x]/(a + b)))/(a + b))/d
```

3.32.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(65) = 130$.

Time = 2.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.01

method	result
risch	$-\frac{2\left(3b\,e^{4dx+4c}+6\,e^{2dx+2c}a-2a+b\right)}{3d(a+b)^{2}\left(e^{2dx+2c}-1\right)^{3}}+\frac{\sqrt{(a+b)b}\,a\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}+a+2b}{a}\right)}{2(a+b)^{3}d}-\frac{\sqrt{(a+b)b}\,a\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+ ...}}{...}\right)}{2(a+b)^{3}d}$
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)^{2}}+2ab\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{...}\right)}{4\sqrt{b}\,\sqrt{a+b}} ...\right)}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)^{2}}+2ab\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{...}\right)}{4\sqrt{b}\,\sqrt{a+b}} ...\right)}{d}$

input
```
int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*b*exp(4*d*x+4*c)+6*exp(2*d*x+2*c)*a-2*a+b)/d/(a+b)^2/(exp(2*d*x+2*
c)-1)^3+1/2*((a+b)*b)^(1/2)/(a+b)^3*a/d*ln(exp(2*d*x+2*c)+(2*((a+b)*b)^(1/
2)+a+2*b)/a)-1/2*((a+b)*b)^(1/2)/(a+b)^3*a/d*ln(exp(2*d*x+2*c)-(2*((a+b)*b
)^(1/2)-a-2*b)/a)
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(65) = 130$.

Time = 0.28 (sec) , antiderivative size = 1753, normalized size of antiderivative = 23.37

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*si
nh(d*x + c)^4 + 24*a*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 + a)*sinh(d
*x + c)^2 - 3*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*s
inh(d*x + c)^6 - 3*a*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - a)*sinh(d*
x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 3
*a*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sin
h(d*x + c)^2 + 6*(a*cosh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + a*cosh(d*x + c
))*sinh(d*x + c) - a)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cos
h(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*
x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 +
 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sin
h(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/
(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*
b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(
d*x + c) + a)) + 48*(b*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) -
8*a + 4*b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*
d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^6 -
3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*d*co...
```

3.32.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(65) = 130.

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.60

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{ab\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,(a^2+2\,ab+b^2)\sqrt{(a+b)b}d} - \frac{2\left(6\,ae^{(-2\,dx-2\,c)}+3\,be^{(-4\,dx-4\,c)}-2\,a+b\right)}{3\,(a^2+2\,ab+b^2-3\,(a^2+2\,ab+b^2)e^{(-2\,dx-2\,c)}+3\,(a^2+2\,ab+b^2)e^{(-4\,dx-4\,c)}-(a^2+2\,ab+b^2)e^{(-6\,dx-}}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*a*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*
x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)
*b)*d) - 2/3*(6*a*e^(-2*d*x - 2*c) + 3*b*e^(-4*d*x - 4*c) - 2*a + b)/((a^2
 + 2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c) + 3*(a^2 + 2*a*b +
 b^2)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d*x - 6*c))*d)
```

3.32.8 Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^4}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.32.9 Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.31

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{a\sqrt{b}\,\ln\left(\frac{4\,b\,e^{2\,c+2\,d\,x}}{(a+b)^2}-\frac{2\sqrt{b}\,\left(a+a\,e^{2\,c+2\,d\,x}+2\,b\,e^{2\,c+2\,d\,x}\right)}{(a+b)^{5/2}}\right)}{2\,d\,(a+b)^{5/2}} - \frac{8}{3\,(a\,d+b\,d)\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)} - \frac{2\,b}{\left(e^{2\,c+2\,d\,x}-1\right)\,(a+b)\,(a\,d+b\,d)} - \frac{4}{(a\,d+b\,d)\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} - \frac{a\sqrt{b}\,\ln\left(\frac{4\,b\,e^{2\,c+2\,d\,x}}{(a+b)^2}+\frac{2\sqrt{b}\,\left(a+a\,e^{2\,c+2\,d\,x}+2\,b\,e^{2\,c+2\,d\,x}\right)}{(a+b)^{5/2}}\right)}{2\,d\,(a+b)^{5/2}}$$

input
```
int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)),x)
```

output
```
(a*b^(1/2)*log((4*b*exp(2*c + 2*d*x))/(a + b)^2 - (2*b^(1/2)*(a + a*exp(2*
c + 2*d*x) + 2*b*exp(2*c + 2*d*x)))/(a + b)^(5/2)))/(2*d*(a + b)^(5/2)) -
8/(3*(a*d + b*d)*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) - 1)) - (2*b)/((exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d)) - 4/((a*d +
b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (a*b^(1/2)*log((4*b*ex
p(2*c + 2*d*x))/(a + b)^2 + (2*b^(1/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2
*c + 2*d*x)))/(a + b)^(5/2)))/(2*d*(a + b)^(5/2))
```

3.33 $\int \frac{\sinh^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

3.33.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{3(a^2+8ab+8b^2)\,x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d} - \frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{3b(3a+4b)\tanh(c+dx)}{8a^3d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
3/8*(a^2+8*a*b+8*b^2)*x/a^4-3/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^
(1/2))*b^(1/2)*(a+b)^(1/2)/a^4/d-1/8*(5*a+6*b)*cosh(d*x+c)*sinh(d*x+c)/a^2
/d/(a+b-b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x
+c)^2)-3/8*b*(3*a+4*b)*tanh(d*x+c)/a^3/d/(a+b-b*tanh(d*x+c)^2)
```

3.33.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.74 (sec) , antiderivative size = 1330, normalized size of antiderivative = 6.86

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx =$$

$$-\frac{(a+2b+a\cosh(2c+2dx))^2\text{sech}^4(c+dx)\left(16x+\frac{(a^3-6a^2b-24ab^2-16b^3)\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+}{2\sqrt{a+b}\sqrt{b(\cosh(c)}}\right.}{b(a+b)^{3/2}d\sqrt{b(\cosh(c)-\sinh}}\right.}{256a^2\left(a+b\text{sech}^2(c+dx\right.}$$

$$+\frac{3(a+2b+a\cosh(2c+2dx))^2\text{sech}^4(c+dx)\left(\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{3/2}d}-\frac{a\sinh(2(c+dx))}{8b(a+b)d(a+2b+a\cosh(2(c+dx)))}\right)}{128\left(a+b\text{sech}^2(c+dx)\right)^2}$$

$$+\frac{(a+2b+a\cosh(2c+2dx))^2\text{sech}^4(c+dx)\left(\frac{(a^5-30a^4b-480a^3b^2-1600a^2b^3-1920ab^4-768b^5)\left(-\frac{i\arctan\left(\text{sech}(dx)\left(-\frac{}{2\sqrt{c}}\right.\right.}{}\right.}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-1/256*((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(16*x + ((a^3 -
6*a^2*b - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((
a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c]
- Sinh[c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a
*Sinh[2*d*x]))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(a^2*(a + b
*Sech[c + d*x]^2)^2) + (3*(a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^
4*(((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a
+ b)^(3/2)*d) - (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(
c + d*x)]))))/(128*(a + b*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2
*d*x])^2*Sech[c + d*x]^4*(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b^3 -
1920*a*b^4 - 768*b^5)*(((-1/8*I)*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(S
qrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a +
b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*S
inh[2*c + d*x])]*Cosh[2*c])/(a^4*b*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh
[4*c]]) + ((I/8)*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[
b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4
*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]
*Sinh[2*c])/(a^4*b*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/(a + b
) + (Sech[2*c]*(160*a^4*b*d*x*Cosh[2*c] + 1248*a^3*b^2*d*x*Cosh[2*c] + ...
```

3.33.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4620, 372, 402, 27, 402, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ic+idx)^4}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\int \frac{(4a+5b)\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)^2\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4a}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\int -\frac{3\left(b(5a+6b)\tanh^2(c+dx)+(a+b)(a+2b)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{2a}+\frac{(5a+6b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}}{4a}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{(5a+6b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{3\int \frac{b(5a+6b)\tanh^2(c+dx)+(a+b)(a+2b)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{2a}}{4a}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{(5a+6b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{3\left(-\frac{\int -\frac{2(a+b)\left(b(3a+4b)\tanh^2(c+dx)+(a+b)(a+4b)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh\cdots}{2a(a+b)}\cdots\right.}{2a}}{4a}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{(5a+6b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{3\left(\frac{\int \frac{b(3a+4b)\tanh^2(c+dx)+(a+b)(a+4b)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a}\cdots\right.}{2a}}{4a}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{(5a+6b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{3\left(\frac{\left(a^2+8ab+8b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{4b(a+b)\cdots}{a}\cdots\right.}{\cdots}}{4a}}{d}$$

3.33. $\displaystyle\int \frac{\sinh^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

$\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{(5a+6b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{3\left(\frac{\left(a^2+8ab+8b^2\right)\text{arctanh}(\tanh(c+dx))}{a}-\frac{4b(a+b)(a+2b)}{a}\right.}{}}{4a}}{d}$$

$\downarrow$ 221

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{(5a+6b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{3\left(\frac{\left(a^2+8ab+8b^2\right)\text{arctanh}(\tanh(c+dx))}{a}-\frac{4\sqrt{b}\sqrt{a+b}(a+2b)}{a}\right.}{2a}}{4a}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]/(4*a*(1 - Tanh[c + d*x]^2)^2*(a + b - b*Tanh[c + d*x]^2)) -
 (((5*a + 6*b)*Tanh[c + d*x])/(2*a*(1 - Tanh[c + d*x]^2)*(a + b - b*Tanh[c
 + d*x]^2)) - (3*((((a^2 + 8*a*b + 8*b^2)*ArcTanh[Tanh[c + d*x]])/a - (4*S
qrt[b]*Sqrt[a + b]*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])
/a)/a - (b*(3*a + 4*b)*Tanh[c + d*x])/(a*(a + b - b*Tanh[c + d*x]^2))))/(2
*a))/(4*a))/d
```

3.33.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.33. $\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(176) = 352$.

Time = 0.23 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.38

$$\frac{1}{4a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+8b}{8a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a+8b}{8a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-3a^2-24ab-24b^2)\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{8a^4}$$

input
```
int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

output
```
1/d*(1/4/a^2/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1
/8*(a+8*b)/a^3/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(3*a+8*b)/a^3/(tanh(1/2*d*x+1
/2*c)-1)+1/8/a^4*(-3*a^2-24*a*b-24*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/a^2/
(1+tanh(1/2*d*x+1/2*c))^4+1/2/a^2/(1+tanh(1/2*d*x+1/2*c))^3-1/8*(-a-8*b)/a
^3/(1+tanh(1/2*d*x+1/2*c))^2-1/8*(3*a+8*b)/a^3/(1+tanh(1/2*d*x+1/2*c))+1/8
/a^4*(3*a^2+24*a*b+24*b^2)*ln(1+tanh(1/2*d*x+1/2*c))+2*b/a^4*(((-1/2*a^2-1
/2*a*b)*tanh(1/2*d*x+1/2*c)^3-1/2*a*(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d
*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)+1/2*(3*a^2+9*a*b+6*b^2)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln(
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2
))+1/4/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/
2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2)))))
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2464 vs. $2(185) = 370$.

Time = 0.33 (sec) , antiderivative size = 5169, normalized size of antiderivative = 26.64

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.33.6 Sympy [F]

$$\int \frac{\sinh^4(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\sinh^4(c+dx)}{(a+b\,\text{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sinh(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(185) = 370.

Time = 0.33 (sec) , antiderivative size = 1299, normalized size of antiderivative = 6.70

$$\int \frac{\sinh^4(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/64*(3*a^3*b + 42*a^2*b^2 + 88*a*b^3 + 48*b^4)*log((a*e^(2*d*x + 2*c) +
a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)
*b)))/((a^5 + a^4*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3
)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c)
 + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 1/64*
(3*a^3*b + 42*a^2*b^2 + 88*a*b^3 + 48*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2
*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b))
)/((a^5 + a^4*b)*sqrt((a + b)*b)*d) + 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*lo
g((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
 a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 3/32*(3
*a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^
(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)
*b)*d) + 1/16*(a^3*b + 8*a^2*b^2 + 8*a*b^3 + (a^3*b + 18*a^2*b^2 + 48*a*b^
3 + 32*b^4)*e^(2*d*x + 2*c))/((a^6 + a^5*b + (a^6 + a^5*b)*e^(4*d*x + 4*c)
 + 2*(a^6 + 3*a^5*b + 2*a^4*b^2)*e^(2*d*x + 2*c))*d) - 1/16*(a^3*b + 8*a^2
*b^2 + 8*a*b^3 + (a^3*b + 18*a^2*b^2 + 48*a*b^3 + 32*b^4)*e^(-2*d*x - 2*c)
)/((a^6 + a^5*b + 2*(a^6 + 3*a^5*b + 2*a^4*b^2)*e^(-2*d*x - 2*c) + (a^6 +
a^5*b)*e^(-4*d*x - 4*c))*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*
b^3)*e^(2*d*x + 2*c))/((a^5 + a^4*b + (a^5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a
^5 + 3*a^4*b + 2*a^3*b^2)*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b + 2*a*b^2 + ...
```

3.33.8 Giac [F]

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\sinh\left(dx+c\right)^4}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\cosh(c+d\,x)^4\,\sinh(c+d\,x)^4}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^4*sinh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.34 $\int \frac{\sinh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

3.34.1 Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx=\frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d}-\frac{(a+2b)\cosh(c+dx)}{a^3d}+\frac{\cosh^3(c+dx)}{3a^2d}-\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)}$$

output
```
-(a+2*b)*cosh(d*x+c)/a^3/d+1/3*cosh(d*x+c)^3/a^2/d-1/2*b*(a+b)*cosh(d*x+c)
/a^3/d/(b+a*cosh(d*x+c)^2)+1/2*(3*a+5*b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2
))*b^(1/2)/a^(7/2)/d
```

3.34.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 861, normalized size of antiderivative = 7.55

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$$

$$=\frac{(a+2b+a\cosh(2(c+dx)))^2\text{sech}^4(c+dx)\left(\frac{9a^3\arctan\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\cdots\right.}{\sqrt{b}}\right)}{b^{3/2}}\cdots\right.}{\cdots}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((9*a^3*ArcTan[((Sqrt[a
] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cos
h[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/
Sqrt[b]])/b^(3/2) + 576*a*Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(C
osh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a
+ b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 960*b^(3/2)*Ar
cTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(
d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tan
h[(d*x)/2]))/Sqrt[b]] + (9*a^3*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh
[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b
]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/b^(3/2) + 576*a*Sq
rt[b]*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c
]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c]
)^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 960*b^(3/2)*ArcTan[((Sqrt[a] + I*Sqrt[a +
b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] +
 I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - (9*a
^3*ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2) -
(9*a^3*ArcTan[(Sqrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2
) - 96*Sqrt[a]*(3*a + 8*b)*Cosh[c]*Cosh[d*x] + 32*a^(3/2)*Cosh[3*c]*Cosh[3
*d*x] - (384*a^(3/2)*b*Cosh[c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]) -...
```

3.34.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4621, 360, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{\left(b\sec(ic+idx)^2+a\right)^2}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cosh^4(c+dx)\left(1-\cosh^2(c+dx)\right)}{\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{360}$$

$$-\frac{\frac{b(a+b)\cosh(c+dx)}{2a^3\left(a\cosh^2(c+dx)+b\right)}-\frac{\int \frac{2a^2\cosh^4(c+dx)-2a(a+b)\cosh^2(c+dx)+b(a+b)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{2a^3}}{d}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{\frac{b(a+b)\cosh(c+dx)}{2a^3\left(a\cosh^2(c+dx)+b\right)}-\frac{\int \left(2a\cosh^2(c+dx)-2(a+2b)+\frac{5b^2+3ab}{a\cosh^2(c+dx)+b}\right)d\cosh(c+dx)}{2a^3}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b(a+b)\cosh(c+dx)}{2a^3\left(a\cosh^2(c+dx)+b\right)}-\frac{\frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}}-2(a+2b)\cosh(c+dx)+\frac{2}{3}a\cosh^3(c+dx)}{2a^3}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-(((b*(a + b)*Cosh[c + d*x])/(2*a^3*(b + a*Cosh[c + d*x]^2)) - ((Sqrt[b]*(
3*a + 5*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/Sqrt[a] - 2*(a + 2*b)*
Cosh[c + d*x] + (2*a*Cosh[c + d*x]^3)/3)/(2*a^3))/d)
```

3.34.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.34.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(100) = 200$.

Time = 286.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.32

method	result
derivativedivides	$\frac{-\frac{1}{3a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a-4b}{2a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4b\left(\frac{\left(-\frac{a}{4}+\frac{b}{4}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{d}{2}\right.}\right.}{}}{}$
default	$\frac{-\frac{1}{3a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a-4b}{2a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4b\left(\frac{\left(-\frac{a}{4}+\frac{b}{4}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{d}{2}\right.}\right.}{}}{}$
risch	$\frac{e^{3dx+3c}}{24a^2d}-\frac{3\,e^{dx+c}}{8a^2d}-\frac{e^{dx+c}b}{a^3d}-\frac{3\,e^{-dx-c}}{8a^2d}-\frac{e^{-dx-c}b}{a^3d}+\frac{e^{-3dx-3c}}{24a^2d}-\frac{b(a+b)e^{dx+c}\left(e^{2dx+2c}+1\right)}{a^3d\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+\right.}$

input
```
int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/a^2/(tanh(1/2*d*x+1/2*c)-1)^2-
1/2/a^3*(-a-4*b)/(tanh(1/2*d*x+1/2*c)-1)+4*b/a^3*(((-1/4*a+1/4*b)*tanh(1/2
*d*x+1/2*c)^2-1/4*a-1/4*b)/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*
b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/8*(3*a+5*b)/(
a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))
)+1/3/a^2/(1+tanh(1/2*d*x+1/2*c))^3-1/2/a^2/(1+tanh(1/2*d*x+1/2*c))^2-1/2*
(a+4*b)/a^3/(1+tanh(1/2*d*x+1/2*c)))
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2021 vs. 2(100) = 200.

Time = 0.31 (sec) , antiderivative size = 3804, normalized size of antiderivative = 33.37

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/24*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*s
inh(d*x + c)^10 - (7*a^2 + 20*a*b)*cosh(d*x + c)^8 + (45*a^2*cosh(d*x + c)
^2 - 7*a^2 - 20*a*b)*sinh(d*x + c)^8 + 8*(15*a^2*cosh(d*x + c)^3 - (7*a^2
+ 20*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 + 66*a*b + 60*b^2)*co
sh(d*x + c)^6 + 2*(105*a^2*cosh(d*x + c)^4 - 14*(7*a^2 + 20*a*b)*cosh(d*x
+ c)^2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x + c)^6 + 4*(63*a^2*cosh(d*x +
c)^5 - 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^3 - 3*(13*a^2 + 66*a*b + 60*b^2)*
cosh(d*x + c))*sinh(d*x + c)^5 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c
)^4 + 2*(105*a^2*cosh(d*x + c)^6 - 35*(7*a^2 + 20*a*b)*cosh(d*x + c)^4 - 1
5*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 13*a^2 - 66*a*b - 60*b^2)*s
inh(d*x + c)^4 + 8*(15*a^2*cosh(d*x + c)^7 - 7*(7*a^2 + 20*a*b)*cosh(d*x +
 c)^5 - 5*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^3 - (13*a^2 + 66*a*b +
60*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c)^2
+ (45*a^2*cosh(d*x + c)^8 - 28*(7*a^2 + 20*a*b)*cosh(d*x + c)^6 - 30*(13*a
^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^4 - 12*(13*a^2 + 66*a*b + 60*b^2)*cosh
(d*x + c)^2 - 7*a^2 - 20*a*b)*sinh(d*x + c)^2 + 6*((3*a^2 + 5*a*b)*cosh(d*
x + c)^7 + 7*(3*a^2 + 5*a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 5*a*
b)*sinh(d*x + c)^7 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^5 + (21*(3*
a^2 + 5*a*b)*cosh(d*x + c)^2 + 6*a^2 + 22*a*b + 20*b^2)*sinh(d*x + c)^5 +
5*(7*(3*a^2 + 5*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh...
```

3.34.6 Sympy [F]

$$\int \frac{\sinh^3(c+dx)}{(a+b\mathrm{sech}^2(c+dx))^2}\,dx = \int \frac{\sinh^3(c+dx)}{(a+b\,\mathrm{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sinh(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

3.34.7 Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^3}{(b\,\text{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/24*(a^2*e^(10*d*x + 10*c) + a^2 - (7*a^2*e^(8*c) + 20*a*b*e^(8*c))*e^(8*
d*x) - 2*(13*a^2*e^(6*c) + 66*a*b*e^(6*c) + 60*b^2*e^(6*c))*e^(6*d*x) - 2*
(13*a^2*e^(4*c) + 66*a*b*e^(4*c) + 60*b^2*e^(4*c))*e^(4*d*x) - (7*a^2*e^(2
*c) + 20*a*b*e^(2*c))*e^(2*d*x))/(a^4*d*e^(7*d*x + 7*c) + a^4*d*e^(3*d*x +
 3*c) + 2*(a^4*d*e^(5*c) + 2*a^3*b*d*e^(5*c))*e^(5*d*x)) + 1/8*integrate(8
*((3*a*b*e^(3*c) + 5*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 5*b^2*e^c)*e^(d
*x))/(a^4*e^(4*d*x + 4*c) + a^4 + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c))*e^(2*d
*x)), x)
```

3.34.8 Giac [F]

$$\int \frac{\sinh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^3}{(b\,\text{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)^4\sinh(c+d\,x)^3}{(a\cosh(c+d\,x)^2+b)^2}\,dx$$

input
```
int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^4*sinh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.35 $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

3.35.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^2}\,dx = -\frac{(a+4b)x}{2a^3}+\frac{\sqrt{b}(3a+4b)\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b\tanh(c+dx)}{a^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
-1/2*(a+4*b)*x/a^3+1/2*(3*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*
b^(1/2)/a^3/d/(a+b)^(1/2)+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+
c)^2)+b*tanh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)
```

3.35.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 791 vs. 2(131) = 262.

Time = 11.99 (sec) , antiderivative size = 791, normalized size of antiderivative = 6.04

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cosh(2c+2dx))^2\text{sech}^4(c+dx)\left(16x+\frac{(a^3-6a^2b-24ab^2-16b^3)\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\text{s}}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sin}}\right)}{b(a+b)^{3/2}d\sqrt{b(\cosh(c)-\sinh(c)}}\right.}{128a^2\left(a+b\text{sech}^2(c+dx)\right)^2}$$

$$+ \frac{(a+2b+a\cosh(2c+2dx))^2\text{sech}^4(c+dx)\left(-64(a+2b)x+\frac{(-a^4+16a^3b+144a^2b^2+256ab^3+128b^4)\text{arctanh}\left(\text{sec}\right.}{b(a+b)}\right.}{}$$

$$- \frac{(a+2b+a\cosh(2c+2dx))^2\text{sech}^4(c+dx)\left(-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{\sqrt{b}(a+2b)\sinh(2(c+dx))}{(a+b)(a+2b+a\cosh(2(c+dx)))}\right)}{256b^{3/2}d\left(a+b\text{sech}^2(c+dx)\right)^2}$$

$$+ \frac{(a+2b+a\cosh(2c+2dx))^2\text{sech}^4(c+dx)\left(-\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{3/2}d}+\frac{a\sinh(2(c+dx))}{8b(a+b)d(a+2b+a\cosh(2(c+dx)))}\right)}{16\left(a+b\text{sech}^2(c+dx)\right)^2}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

3.35. $\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

output
```
((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(16*x + ((a^3 - 6*a^2*b
 - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b
)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c]
)^4])]*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[
c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2
*d*x]))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(128*a^2*(a + b*Se
ch[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-6
4*(a + 2*b)*x + ((-a^4 + 16*a^3*b + 144*a^2*b^2 + 256*a*b^3 + 128*b^4)*Arc
Tanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c
+ d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[
2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (16*a*Cosh[2*d*
x]*Sinh[2*c])/d + (16*a*Cosh[2*c]*Sinh[2*d*x])/d - ((a^3 + 18*a^2*b + 48*a
*b^2 + 32*b^3)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*(a + b)
*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(256*a^3*(a + b*Sech[c + d*x]^2)^2)
- ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-((a*ArcTanh[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2)) + (Sqrt[b]*(a + 2*b)*Sinh[2*
(c + d*x)])/((a + b)*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(256*b^(3/2)*d*(a
+ b*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]
^4*(-1/8*((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)
*(a + b)^(3/2)*d) + (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*C...
```

3.35.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 4620, 373, 402, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{\left(b\sec(ic+idx)^2+a\right)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 373

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\int \frac{3b\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{2a}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{-\frac{\int -\frac{2(a+b)\left(2b\tanh^2(c+dx)+a+2b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{2b\tanh(c+dx)}{a\left(a-b\tanh^2(c+dx)+b\right)}}{2a}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\int \frac{2b\tanh^2(c+dx)+a+2b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a}-\frac{2b\tanh(c+dx)}{a\left(a-b\tanh^2(c+dx)+b\right)}}{2a}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\frac{(a+4b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b(3a+4b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a}-\frac{2b\tanh(c+dx)}{a\left(a-b\tanh^2(c+dx)+b\right)}}{2a}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\frac{(a+4b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{b(3a+4b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a}-\frac{2b\tanh(c+dx)}{a\left(a-b\tanh^2(c+dx)+b\right)}}{2a}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\frac{(a+4b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{\sqrt{b}(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a}-\frac{2b\tanh(c+dx)}{a\left(a-b\tanh^2(c+dx)+b\right)}}{2a}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

3.35. $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}dx$

output
```
(Tanh[c + d*x]/(2*a*(1 - Tanh[c + d*x]^2)*(a + b - b*Tanh[c + d*x]^2)) - (
(((a + 4*b)*ArcTanh[Tanh[c + d*x]])/a - (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/a - (2*b*Tanh[c + d*x])/
(a*(a + b - b*Tanh[c + d*x]^2)))/(2*a))/d
```

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.35. $\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(117) = 234$.

Time = 44.87 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.48

method	result
risch	$-\frac{x}{2a^2}-\frac{2xb}{a^3}+\frac{e^{2dx+2c}}{8a^2d}-\frac{e^{-2dx-2c}}{8a^2d}-\frac{b(e^{2dx+2c}a+2b\,e^{2dx+2c}+a)}{a^3d(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a)}+\frac{3\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\right.}{4(a+b)d\,a}$
derivativedivides	$\frac{\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a+4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^3}-\frac{1}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$
default	$\frac{\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a+4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^3}-\frac{1}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$

input
```
int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/a^2-2*x/a^3*b+1/8/a^2/d*exp(2*d*x+2*c)-1/8/a^2/d*exp(-2*d*x-2*c)-b*
(exp(2*d*x+2*c)*a+2*b*exp(2*d*x+2*c)+a)/a^3/d/(a*exp(4*d*x+4*c)+2*exp(2*d*
x+2*c)*a+4*b*exp(2*d*x+2*c)+a)+3/4*((a+b)*b)^(1/2)/(a+b)/d/a^2*ln(exp(2*d*
x+2*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a)+((a+b)*b)^(1/2)/(a+b)/d/a^3*ln(exp(2*d
*x+2*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a)*b-3/4*((a+b)*b)^(1/2)/(a+b)/d/a^2*ln(
exp(2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/a)-((a+b)*b)^(1/2)/(a+b)/d/a^3*ln
(exp(2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/a)*b
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(124) = 248$.

Time = 0.32 (sec) , antiderivative size = 2925, normalized size of antiderivative = 22.33

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} \, dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh
(d*x + c)^8 - 2*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^6 + 2*(1
4*a^2*cosh(d*x + c)^2 - 2*(a^2 + 4*a*b)*d*x + a^2 + 2*a*b)*sinh(d*x + c)^6
 + 4*(14*a^2*cosh(d*x + c)^3 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(
d*x + c))*sinh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*co
sh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 - 4*(a^2 + 6*a*b + 8*b^2)*d*x -
15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 - 4*a*b - 8*b^2)*si
nh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 - 5*(2*(a^2 + 4*a*b)*d*x - a^2 -
2*a*b)*cosh(d*x + c)^3 - 4*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x
 + c)^2 + 2*(14*a^2*cosh(d*x + c)^6 - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*
b)*cosh(d*x + c)^4 - 2*(a^2 + 4*a*b)*d*x - 24*((a^2 + 6*a*b + 8*b^2)*d*x +
 a*b + 2*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b)*sinh(d*x + c)^2 + 2*((3*a^2 +
 4*a*b)*cosh(d*x + c)^6 + 6*(3*a^2 + 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^5
+ (3*a^2 + 4*a*b)*sinh(d*x + c)^6 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x +
c)^4 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 + 6*a^2 + 20*a*b + 16*b^2)*sinh
(d*x + c)^4 + 4*(5*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 10*a*b + 8
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c)^2 + (
15*(3*a^2 + 4*a*b)*cosh(d*x + c)^4 + 12*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x
+ c)^2 + 3*a^2 + 4*a*b)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b)*cosh(d*x...
```

3.35.6 Sympy [F]

$$\int \frac{\sinh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\sinh^2(c+dx)}{(a+b\,\text{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sinh(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(124) = 248$.

Time = 0.29 (sec) , antiderivative size = 696, normalized size of antiderivative = 5.31

$$\begin{aligned}
&\int \frac{\sinh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx \\
&= \frac{(3\,a^2b+12\,ab^2+8\,b^3)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^4+a^3b)\sqrt{(a+b)b}d} \\
&\quad - \frac{(3\,a^2b+12\,ab^2+8\,b^3)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^4+a^3b)\sqrt{(a+b)b}d} \\
&\quad - \frac{(3\,ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,(a^3+a^2b)\sqrt{(a+b)b}d} \\
&\quad - \frac{a^2b+2\,ab^2+(a^2b+8\,ab^2+8\,b^3)e^{(2\,dx+2\,c)}}{4\,(a^5+a^4b+(a^5+a^4b)e^{(4\,dx+4\,c)}+2\,(a^5+3\,a^4b+2\,a^3b^2)e^{(2\,dx+2\,c)})d} \\
&\quad + \frac{a^2b+2\,ab^2+(a^2b+8\,ab^2+8\,b^3)e^{(-2\,dx-2\,c)}}{4\,(a^5+a^4b+2\,(a^5+3\,a^4b+2\,a^3b^2)e^{(-2\,dx-2\,c)}+(a^5+a^4b)e^{(-4\,dx-4\,c)})d} \\
&\quad + \frac{ab+(ab+2\,b^2)e^{(-2\,dx-2\,c)}}{2\,(a^4+a^3b+2\,(a^4+3\,a^3b+2\,a^2b^2)e^{(-2\,dx-2\,c)}+(a^4+a^3b)e^{(-4\,dx-4\,c)})d} - \frac{dx+c}{2\,a^2d} \\
&\quad + \frac{e^{(2\,dx+2\,c)}}{8\,a^2d} - \frac{e^{(-2\,dx-2\,c)}}{8\,a^2d} - \frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{2\,a^3d} \\
&\quad + \frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^3d}
\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqr
t((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a
^3*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(-2*
d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) - 1/8*(3*a*b + 2*b^2
)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*
c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) - 1/4
*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(2*d*x + 2*c))/((a^5 + a^4
*b + (a^5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(2*d*
x + 2*c))*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(-2*d*x
- 2*c))/((a^5 + a^4*b + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(-2*d*x - 2*c) + (
a^5 + a^4*b)*e^(-4*d*x - 4*c))*d) + 1/2*(a*b + (a*b + 2*b^2)*e^(-2*d*x - 2
*c))/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4
 + a^3*b)*e^(-4*d*x - 4*c))*d) - 1/2*(d*x + c)/(a^2*d) + 1/8*e^(2*d*x + 2*
c)/(a^2*d) - 1/8*e^(-2*d*x - 2*c)/(a^2*d) - 1/2*b*log(a*e^(4*d*x + 4*c) +
2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3*d) + 1/2*b*log(2*(a + 2*b)*e^(-2*d*x
 - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

3.35.8 Giac [F]

$$\int \frac{\sinh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^2}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^4\,\sinh(c+d\,x)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^4*sinh(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.36 $\int \frac{\sinh(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2}\,dx$

3.36.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d} + \frac{3\cosh(c+dx)}{2a^2d} - \frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)}$$

output
```
3/2*cosh(d*x+c)/a^2/d-1/2*cosh(d*x+c)^3/a/d/(b+a*cosh(d*x+c)^2)-3/2*arctan
(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/d
```

3.36.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.03 (sec) , antiderivative size = 479, normalized size of antiderivative = 5.70

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{(a+2b+a\cosh(2(c+dx)))^2\text{sech}^4(c+dx)\left(\frac{32\cosh(c)\cosh(dx)}{a^2} + \frac{32b\cosh(c+dx)}{a^2(a+2b+a\cosh(2(c+dx)))} + \frac{2\left(-\left((a^2+24b^2)\arctan\right.\right.}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((32*Cosh[c]*Cosh[d*x])
/a^2 + (32*b*Cosh[c + d*x])/(a^2*(a + 2*b + a*Cosh[2*(c + d*x)])) + (2*(-(
(a^2 + 24*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2
])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c]
- Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]) - a^2*ArcTan[((Sqrt[a] + I*Sqrt[a
+ b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a]
 + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - 24
*b^2*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]
*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])
^2]*Tanh[(d*x)/2]))/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c
 + d*x)/2])/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/
2])/Sqrt[b]] + 16*Sqrt[a]*b^(3/2)*Sinh[c]*Sinh[d*x]))/(a^(5/2)*b^(3/2))))/
(128*d*(a + b*Sech[c + d*x]^2)^2)
```

3.36.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4621, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{\left(b\sec(ic+idx)^2+a\right)^2}dx$$

$$\Big\downarrow \text{4621}$$

$$\frac{\int \frac{\cosh^4(c+dx)}{\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{3\int \frac{\cosh^2(c+dx)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{2a}-\frac{\cosh^3(c+dx)}{2a(a\cosh^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{3\left(\frac{\cosh(c+dx)}{a}-\frac{b\int \frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a}\right)}{2a}-\frac{\cosh^3(c+dx)}{2a(a\cosh^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{\cosh(c+dx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{\cosh^3(c+dx)}{2a(a\cosh^2(c+dx)+b)}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((3*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/a^(3/2)) + Cosh[c
 + d*x]/a))/(2*a) - Cosh[c + d*x]^3/(2*a*(b + a*Cosh[c + d*x]^2)))/d
```

3.36.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.36.4 Maple [A] (verified)

Time = 17.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{-\frac{1}{a^2\,\mathrm{sech}(dx+c)}-\frac{b\left(\frac{\mathrm{sech}(dx+c)}{2a+2b\,\mathrm{sech}(dx+c)^2}+\frac{3\arctan\left(\frac{b\,\mathrm{sech}(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}}{d}$
default	$-\frac{-\frac{1}{a^2\,\mathrm{sech}(dx+c)}-\frac{b\left(\frac{\mathrm{sech}(dx+c)}{2a+2b\,\mathrm{sech}(dx+c)^2}+\frac{3\arctan\left(\frac{b\,\mathrm{sech}(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}}{d}$
risch	$\frac{e^{dx+c}}{2a^2d}+\frac{e^{-dx-c}}{2a^2d}+\frac{e^{dx+c}b\left(e^{2dx+2c}+1\right)}{d\,a^2\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}\,e^{dx+c}}{a}+1\right)}{4a^3d}-\frac{3\sqrt{-}}{}$

input
```
int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-1/a^2/sech(d*x+c)-1/a^2*b*(1/2*sech(d*x+c)/(a+b*sech(d*x+c)^2)+3/2/
(a*b)^(1/2)*arctan(b*sech(d*x+c)/(a*b)^(1/2))))
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 1780, normalized size of antiderivative = 21.19

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*cosh(d*x + c)^6 + 12*a*cosh(d*x + c)*sinh(d*x + c)^5 + 2*a*sinh(
d*x + c)^6 + 6*(a + 2*b)*cosh(d*x + c)^4 + 6*(5*a*cosh(d*x + c)^2 + a + 2*
b)*sinh(d*x + c)^4 + 8*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*s
inh(d*x + c)^3 + 6*(a + 2*b)*cosh(d*x + c)^2 + 6*(5*a*cosh(d*x + c)^4 + 6*
(a + 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^
5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*co
sh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a
*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x
 + c) + (5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a)*sinh(d*x +
 c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^
2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x +
 c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c
)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh
(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*
x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cos
h(d*x + c))*sinh(d*x + c) + a)) + 12*(a*cosh(d*x + c)^5 + 2*(a + 2*b)*cosh
(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a)/(a^3*d*cosh(d*
x + c)^5 + 5*a^3*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^3*d*sinh(d*x + c)^...
```

3.36.6 Sympy [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(c+dx\right)}{\left(a+b\,\text{sech}^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sinh(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

3.36.7 Maxima [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(3*(a*e^(4*c) + 2*b*e^(4*c))*e^(4*d*x) + 3*(a*e^(2*c) + 2*b*e^(2*c))*e
^(2*d*x) + a*e^(6*d*x + 6*c) + a)/(a^3*d*e^(5*d*x + 5*c) + a^3*d*e^(d*x +
c) + 2*(a^3*d*e^(3*c) + 2*a^2*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(6*(b
*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2
*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)), x)
```

3.36.8 Giac [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.36.9 Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{b\cosh(c+d\,x)}{2\,\left(d\,a^3\cosh\left(c+d\,x\right)^2+b\,d\,a^2\right)} + \frac{\cosh(c+d\,x)}{a^2\,d} - \frac{3\,\sqrt{b}\,\text{atan}\left(\frac{\sqrt{a}\cosh(c+d\,x)}{\sqrt{b}}\right)}{2\,a^{5/2}\,d}$$

input
```
int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(b*cosh(c + d*x))/(2*(a^3*d*cosh(c + d*x)^2 + a^2*b*d)) + cosh(c + d*x)/(a
^2*d) - (3*b^(1/2)*atan((a^(1/2)*cosh(c + d*x))/b^(1/2)))/(2*a^(5/2)*d)
```

3.37 $\int \frac{\text{csch}(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2} \, dx$

3.37.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\text{csch}(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2} \, dx = \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2d} - \frac{\text{arctanh}(\cosh(c+dx))}{(a+b)^2d} - \frac{b\cosh(c+dx)}{2a(a+b)d\left(b+a\cosh^2(c+dx)\right)}$$

output
```
-arctanh(cosh(d*x+c))/(a+b)^2/d-1/2*b*cosh(d*x+c)/a/(a+b)/d/(b+a*cosh(d*x+
c)^2)+1/2*(3*a+b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+b
)^2/d
```

3.37.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.81

$$\int \frac{\text{csch}(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2} \, dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^3(c+dx)\left(-\frac{2b(a+b)}{a} + \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*((-2*b*(a + b))/a + (Sqrt
[b]*(3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]
)*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] -
 Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech
[c + d*x])/a^(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*S
qrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b +
 a*Cosh[2*(c + d*x)])*Sech[c + d*x])/a^(3/2) - 2*(a + 2*b + a*Cosh[2*(c +
d*x)])*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] + 2*(a + 2*b + a*Cosh[2*(c + d
*x)])*Log[Sinh[(c + d*x)/2]]*Sech[c + d*x]))/(8*(a + b)^2*d*(a + b*Sech[c
+ d*x]^2)^2)
```

3.37.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4621, 372, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^2\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cosh^4(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$-\frac{\frac{b\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{\int\frac{b-(2a+b)\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}d\cosh(c+dx)}{2a(a+b)}}{d}$$

$$\Big\downarrow \; 397$$

$$-\frac{\frac{b\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{\frac{b(3a+b)\int\frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a+b}-\frac{2a\int\frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{2a(a+b)}}{d}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{b\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{2a\int\frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{2a(a+b)}}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{b\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{2a\,\text{arctanh}(\cosh(c+dx))}{a+b}}{2a(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-((-1/2*((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(Sqrt
[a]*(a + b)) - (2*a*ArcTanh[Cosh[c + d*x]])/(a + b))/(a*(a + b)) + (b*Cosh
[c + d*x])/(2*a*(a + b)*(b + a*Cosh[c + d*x]^2)))/d)
```

3.37.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.37.4 Maple [A] (verified)

Time = 26.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{4b\left(\frac{-\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a}-\frac{a+b}{4a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}+\frac{(3a+b)\arctan\left(\frac{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2a-2b}{4\sqrt{ab}}\right.}{8a\sqrt{ab}}\right.}{(a+b)^2}$ / d
default	$\frac{4b\left(\frac{-\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a}-\frac{a+b}{4a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}+\frac{(3a+b)\arctan\left(\frac{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2a-2b}{4\sqrt{ab}}\right.}{8a\sqrt{ab}}\right.}{(a+b)^2}$ / d
risch	$-\frac{b\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{da(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}-\frac{\ln\left(e^{dx+c}+1\right)}{d(a^2+2ab+b^2)}+\frac{\ln\left(e^{dx+c}-1\right)}{d(a^2+2ab+b^2)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{}}{}\right.}{4a(a+b)^2a}$

input
```
int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(4*b/(a+b)^2*((-1/4*(a-b)/a*tanh(1/2*d*x+1/2*c)^2-1/4*(a+b)/a)/(tanh(1
/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)+1/8*(3*a+b)/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tan
h(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2)))+1/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)
))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. 2(87) = 174.

Time = 0.32 (sec) , antiderivative size = 2376, normalized size of antiderivative = 24.00

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d
*x + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((3*a^2 + a*b)*cosh(d*x + c)^4
 + 4*(3*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x
+ c)^4 + 2*(3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*co
sh(d*x + c)^2 + 3*a^2 + 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*(
(3*a^2 + a*b)*cosh(d*x + c)^3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*
x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cos
h(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(
d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 +
a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*s
inh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*
cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2
*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(a*b + b^2)*cosh(d*x + c) + 4*(
a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x +
 c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cos
h(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 4*(a^2
*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x +...
```

3.37.6 Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

3.37.7 Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + a^2*b*d + (a^3*d*e^(4*c) + a
^2*b*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) + 3*a^2*b*d*e^(2*c) + 2*a*b^2
*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d + 2*a*b*d +
b^2*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) + 2*integ
rate(1/2*((3*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + b^2*e^c)*
e^(d*x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b
^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 5*a^2*b^2*e^(2*
c) + 2*a*b^3*e^(2*c))*e^(2*d*x)), x)
```

3.37.8 Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\cosh(c+d\,x)^4}{\sinh\left(c+d\,x\right)\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(cosh(c + d*x)^4/(sinh(c + d*x)*(b + a*cosh(c + d*x)^2)^2), x)
```

3.38 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

3.38.1 Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\frac{3\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d}-\frac{3\coth(c+dx)}{2(a+b)^2d}+\frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
-3/2*coth(d*x+c)/(a+b)^2/d+3/2*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^
(1/2)/(a+b)^(5/2)/d+1/2*coth(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

3.38.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 220 vs. $2(92) = 184$.

Time = 3.87 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.39

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^4(c+dx)\left(\frac{3b\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+\cdots}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\cdots\right.}{8(a+\cdots}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((3*b*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(C
osh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + 2*(a
+ 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x] + b*Sech[2*c]
*Sinh[2*d*x] - (b*(a + 2*b)*Tanh[2*c])/a))/(8*(a + b)^2*d*(a + b*Sech[c +
d*x]^2)^2)
```

3.38.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4620, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^2\sin(ic+idx)^2}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 253$$

$$\frac{\frac{3\int \frac{\coth^2(c+dx)}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{2(a+b)}+\frac{\coth(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \; 264$$

$$\frac{\frac{3\left(\frac{b\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a+b}-\frac{\coth(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\coth(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{3\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\coth(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\coth(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((3*((Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2)
- Coth[c + d*x]/(a + b)))/(2*(a + b)) + Coth[c + d*x]/(2*(a + b)*(a + b -
b*Tanh[c + d*x]^2)))/d
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.38.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(78) = 156$.

Time = 25.41 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.59

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{2}+2ab+b^{2}\right)}-\frac{1}{2(a+b)^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\cdots}{\cdots}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{2}+2ab+b^{2}\right)}-\frac{1}{2(a+b)^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\cdots}{\cdots}}{d}$
risch	$-\frac{2a^{2}e^{4dx+4c}+ab\,e^{4dx+4c}+2\,e^{4dx+4c}b^{2}+4a^{2}e^{2dx+2c}+8ab\,e^{2dx+2c}-2\,e^{2dx+2c}b^{2}+2a^{2}-ab}{d(a+b)^{2}\left(e^{2dx+2c}-1\right)a\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}+\frac{3\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\cdots\right.}{4(a+b)^{3}\cdots}$

input
```
int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/2/(a+b)^2/tanh(1/2*d*x+1/2
*c)-4*b/(a+b)^2*((-1/4*tanh(1/2*d*x+1/2*c)^3-1/4*tanh(1/2*d*x+1/2*c))/(tan
h(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)+3/16/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1
/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-3/16/b^(1/2)/(a
+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/
2)+(a+b)^(1/2))))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(81) = 162$.

Time = 0.32 (sec) , antiderivative size = 2407, normalized size of antiderivative = 26.16

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + a*b + 2*b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + a*b + 2*b^2)*sinh(d*x + c)^4 +
8*(2*a^2 + 4*a*b - b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + a*b + 2*b^2)*cosh(
d*x + c)^2 + 2*a^2 + 4*a*b - b^2)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^6
 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2 + 4*a*
b)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 4*a*b)*sinh(d*x + c)^
4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^
3 - (a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 + 4*a
*b)*cosh(d*x + c)^2 - a^2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*a^2*cosh(d
*x + c)^5 + 2*(a^2 + 4*a*b)*cosh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x
+ c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)
^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b
 + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x
+ c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d
*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b
)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x +
c) + a)) + 8*a^2 - 4*a*b + 16*((2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^3 + ...
```

3.38.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(81) = 162$.

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.85

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = -\frac{3\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{(a+b)b}d} - \frac{2\,a^2-ab+2\,(2\,a^2+4\,ab-b^2)e^{(-2\,dx-2\,c)}+(2\,a^2+ab+2\,b^2)e^{(-4\,dx-4\,c)}}{(a^4+2\,a^3b+a^2b^2+(a^4+6\,a^3b+9\,a^2b^2+4\,ab^3)e^{(-2\,dx-2\,c)}-(a^4+6\,a^3b+9\,a^2b^2+4\,ab^3)e^{(-4\,dx-4\,c)}-}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-3/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*
b)*d) - (2*a^2 - a*b + 2*(2*a^2 + 4*a*b - b^2)*e^(-2*d*x - 2*c) + (2*a^2 +
 a*b + 2*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 6*a^3*b
 + 9*a^2*b^2 + 4*a*b^3)*e^(-2*d*x - 2*c) - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*
a*b^3)*e^(-4*d*x - 4*c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d)
```

3.38.8 Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)^4}{\sinh\left(c+d\,x\right)^2\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(cosh(c + d*x)^4/(sinh(c + d*x)^2*(b + a*cosh(c + d*x)^2)^2), x)
```

3.39 $\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$

3.39.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = -\frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}(a+b)^3d} + \frac{(a-3b)\operatorname{arctanh}(\cosh(c+dx))}{2(a+b)^3d} - \frac{(a-b)\cosh(c+dx)}{2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)}$$

output
```
1/2*(a-3*b)*arctanh(cosh(d*x+c))/(a+b)^3/d-1/2*(a-b)*cosh(d*x+c)/(a+b)^2/d
/(b+a*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/(a+b)/d/(b+a*cosh(d*x+c)^
2)-1/2*(3*a-b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/d/a^(1/
2)
```

3.39.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.74 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.14

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^3(c+dx)\left(8b(a+b)+\frac{4\sqrt{b}(-3a+b)\arctan\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\right.}{}\right.}{}\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(8*b*(a + b) + (4*Sqrt[b]
*(-3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*
Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - S
inh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c
 + d*x])/Sqrt[a] + (4*Sqrt[b]*(-3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*
Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b
+ a*Cosh[2*(c + d*x)])*Sech[c + d*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cosh[
2*(c + d*x)])*Csch[(c + d*x)/2]^2*Sech[c + d*x] + 4*(a - 3*b)*(a + 2*b + a
*Cosh[2*(c + d*x)])*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] - 4*(a - 3*b)*(a
+ 2*b + a*Cosh[2*(c + d*x)])*Log[Sinh[(c + d*x)/2]]*Sech[c + d*x] - (a + b
)*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[(c + d*x)/2]^2*Sech[c + d*x]))/(32*
(a + b)^3*d*(a + b*Sech[c + d*x]^2)^2)
```

3.39.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 26, 4621, 372, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^2\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{ 4621}$$

$$\frac{\int \frac{\cosh^4(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\frac{\cosh(c+dx)}{2(a+b)\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}-\frac{\int \frac{b-(a-2b)\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{2(a+b)}}{d}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\cosh(c+dx)}{2(a+b)\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}-\frac{\frac{(a-b)\cosh(c+dx)}{(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{\int -\frac{2b\left(2b-(a-b)\cosh^2(c+dx)\right)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}d\cosh(c+dx)}{2b(a+b)}}{2(a+b)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\cosh(c+dx)}{2(a+b)\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}-\frac{\frac{\int \frac{2b-(a-b)\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}d\cosh(c+dx)}{a+b}+\frac{(a-b)\cosh(c+dx)}{(a+b)\left(a\cosh^2(c+dx)+b\right)}}{2(a+b)}}{d}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\cosh(c+dx)}{2(a+b)\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}-\frac{\frac{\frac{b(3a-b)\int \frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a+b}-\frac{(a-3b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{a+b}+\frac{(a-b)\cosh(c+dx)}{(a+b)\left(a\cosh^2(c+dx)+\right.}}{2(a+b)}}{d}$$

$$\Big\downarrow \text{ 218}$$

3.39. $\int \frac{\text{csch}^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$

$$\frac{\frac{\cosh(c+dx)}{2(a+b)(1-\cosh^2(c+dx))(a\cosh^2(c+dx)+b)}-\frac{\frac{\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{(a-3b)\int\frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{a+b}+\frac{(a-b)\cosh(c+dx)}{(a+b)\left(a\cosh^2(c+dx)+b\right)}}{2(a+b)}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\cosh(c+dx)}{2(a+b)(1-\cosh^2(c+dx))(a\cosh^2(c+dx)+b)}-\frac{\frac{\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{(a-3b)\text{arctanh}(\cosh(c+dx))}{a+b}}{a+b}+\frac{(a-b)\cosh(c+dx)}{(a+b)\left(a\cosh^2(c+dx)+b\right)}}{2(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(Cosh[c + d*x]/(2*(a + b)*(1 - Cosh[c + d*x]^2)*(b + a*Cosh[c + d*x]^2)) -
 ((((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(Sqrt[a]*(a
 + b)) - ((a - 3*b)*ArcTanh[Cosh[c + d*x]])/(a + b))/(a + b) + ((a - b)*Co
sh[c + d*x])/((a + b)*(b + a*Cosh[c + d*x]^2)))/(2*(a + b)))/d
```

3.39.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.39. $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

3.39.4 Maple [A] (verified)

Time = 14.42 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^2+16ab+8b^2}-\frac{2b\left(\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{b}{2}-\frac{a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}+\frac{(3a-b)\arctan\left(\frac{2(a+b)\tanh}{}\right.}{4\sqrt{ab}}\right.}{(a+b)^3}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^2+16ab+8b^2}-\frac{2b\left(\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{b}{2}-\frac{a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}+\frac{(3a-b)\arctan\left(\frac{2(a+b)\tanh}{}\right.}{4\sqrt{ab}}\right.}{(a+b)^3}}{d}$
risch	$-\frac{e^{dx+c}\left(a\,e^{6dx+6c}-b\,e^{6dx+6c}+3a\,e^{4dx+4c}+5b\,e^{4dx+4c}+3\,e^{2dx+2c}a+5b\,e^{2dx+2c}+a-b\right)}{d(a+b)^2\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{dx+c}+1\right)a}{2d\left(a^3+3a^2b+3a\,b^2+b^3\right)}-$

input
```
int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)-2*b/(a+b)^3*(((-1/2*a+1/2*b
)*tanh(1/2*d*x+1/2*c)^2-1/2*b-1/2*a)/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x
+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/4*(
3*a-b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b
)^(1/2)))-1/8/(a+b)^2/tanh(1/2*d*x+1/2*c)^2+1/4/(a+b)^3*(-2*a+6*b)*ln(tanh
(1/2*d*x+1/2*c)))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3620 vs. 2(131) = 262.

Time = 0.38 (sec) , antiderivative size = 6878, normalized size of antiderivative = 46.79

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.39.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

3.39.7 Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a - 3*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d + 3*a^2*b*d + 3*a*b^2*d
 + b^3*d) - 1/2*(a - 3*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d + 3*a^2*b*d
 + 3*a*b^2*d + b^3*d) - ((a*e^(7*c) - b*e^(7*c))*e^(7*d*x) + (3*a*e^(5*c)
+ 5*b*e^(5*c))*e^(5*d*x) + (3*a*e^(3*c) + 5*b*e^(3*c))*e^(3*d*x) + (a*e^c
- b*e^c)*e^(d*x))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e^(8*c) + 2*a^2*b*
d*e^(8*c) + a*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^2*b*d*e^(6*c) + 2*a*b^2*d*e^
(6*c) + b^3*d*e^(6*c))*e^(6*d*x) - 2*(a^3*d*e^(4*c) + 6*a^2*b*d*e^(4*c) +
9*a*b^2*d*e^(4*c) + 4*b^3*d*e^(4*c))*e^(4*d*x) + 4*(a^2*b*d*e^(2*c) + 2*a*
b^2*d*e^(2*c) + b^3*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/8*((3*a*b*e^(3*c
) - b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c - b^2*e^c)*e^(d*x))/(a^4 + 3*a^3*b
 + 3*a^2*b^2 + a*b^3 + (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c)
+ a*b^3*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 5*a^3*b*e^(2*c) + 9*a^2*b^2*
e^(2*c) + 7*a*b^3*e^(2*c) + 2*b^4*e^(2*c))*e^(2*d*x)), x)
```

3.39.8 Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)^4}{\sinh\left(c+d\,x\right)^3\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(cosh(c + d*x)^4/(sinh(c + d*x)^3*(b + a*cosh(c + d*x)^2)^2), x)
```

3.40 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

3.40.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = -\frac{(3a-2b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d} + \frac{(a-b)\coth(c+dx)}{(a+b)^3d} - \frac{\coth^3(c+dx)}{3(a+b)^2d} - \frac{ab\tanh(c+dx)}{2(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
(a-b)*coth(d*x+c)/(a+b)^3/d-1/3*coth(d*x+c)^3/(a+b)^2/d-1/2*(3*a-2*b)*arct
anh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d-1/2*a*b*tanh(d*
x+c)/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)
```

3.40.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.42 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.04

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

$$= -\frac{(a+2b+a\cosh(2c+2dx))^2\coth(c)\operatorname{csch}^2(c+dx)\operatorname{sech}^4(c+dx)}{12(a+b)^2d\,(a+b\operatorname{sech}^2(c+dx))^2}$$

$$+ \frac{(3a-2b)(a+2b+a\cosh(2c+2dx))^2\operatorname{sech}^4(c+dx)\left(\frac{ib\arctan\left(\operatorname{sech}(dx)\left(-\frac{i\cosh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4c)}}+\frac{i\sin\ldots}{2\sqrt{a+b}\sqrt{b\cosh\ldots}}\right.\right.}{8\sqrt{a+b}d\sqrt{b\cos\ldots}}\right.}{\ldots}$$

$$+ \frac{(a+2b+a\cosh(2c+2dx))^2\operatorname{csch}(c)\operatorname{csch}^3(c+dx)\operatorname{sech}^4(c+dx)\sinh(dx)}{12(a+b)^2d\,(a+b\operatorname{sech}^2(c+dx))^2}$$

$$+ \frac{(a+2b+a\cosh(2c+2dx))^2\operatorname{csch}(c)\operatorname{csch}(c+dx)\operatorname{sech}^4(c+dx)(-a\sinh(dx)+2b\sinh(dx))}{6(a+b)^3d\,(a+b\operatorname{sech}^2(c+dx))^2}$$

$$+ \frac{(a+2b+a\cosh(2c+2dx))\operatorname{sech}(2c)\operatorname{sech}^4(c+dx)\,(ab\sinh(2c)+2b^2\sinh(2c)-ab\sinh(2dx))}{8(a+b)^3d\,(a+b\operatorname{sech}^2(c+dx))^2}$$

input

```
Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output

```
-1/12*((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Coth[c]*Csch[c + d*x]^2*Sech[c +
d*x]^4)/((a + b)^2*d*(a + b*Sech[c + d*x]^2)^2) + ((3*a - 2*b)*(a + 2*b +
a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(((I/8)*b*ArcTan[Sech[d*x]*(((-1/2*
I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[
2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b
*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cosh[2*c])/(Sqrt[a + b]*d*Sqrt[b*Cosh[4*c
] - b*Sinh[4*c]]) - ((I/8)*b*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[
a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*S
qrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[
2*c + d*x])]*Sinh[2*c])/(Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/
((a + b)^3*(a + b*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2
*Csch[c]*Csch[c + d*x]^3*Sech[c + d*x]^4*Sinh[d*x])/(12*(a + b)^2*d*(a + b
*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Csch[c]*Csch[c +
 d*x]*Sech[c + d*x]^4*(-(a*Sinh[d*x]) + 2*b*Sinh[d*x]))/(6*(a + b)^3*d*(a
+ b*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Sech[2*c]*Sech[
c + d*x]^4*(a*b*Sinh[2*c] + 2*b^2*Sinh[2*c] - a*b*Sinh[2*d*x]))/(8*(a + b)
^3*d*(a + b*Sech[c + d*x]^2)^2)
```

3.40.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4620, 361, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{1}{2}b\int \frac{\coth^4(c+dx)\left(-\frac{a\tanh^4(c+dx)}{(a+b)^3}-\frac{2a\tanh^2(c+dx)}{b(a+b)^2}+\frac{2}{b(a+b)}\right)}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)-\frac{ab\tanh(c+dx)}{2(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\frac{1}{2}b\int \left(\frac{2\coth^4(c+dx)}{b(a+b)^2}-\frac{2(a-b)\coth^2(c+dx)}{b(a+b)^3}+\frac{2b-3a}{(a+b)^3\left(-b\tanh^2(c+dx)+a+b\right)}\right)d\tanh(c+dx)-\frac{ab\tanh(c+dx)}{2(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b\left(-\frac{(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}(a+b)^{7/2}}-\frac{2\coth^3(c+dx)}{3b(a+b)^2}+\frac{2(a-b)\coth(c+dx)}{b(a+b)^3}\right)-\frac{ab\tanh(c+dx)}{2(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((b*(-(((3*a - 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(Sqrt[b]
*(a + b)^(7/2))) + (2*(a - b)*Coth[c + d*x])/(b*(a + b)^3) - (2*Coth[c + d
*x]^3)/(3*b*(a + b)^2)))/2 - (a*b*Tanh[c + d*x])/(2*(a + b)^3*(a + b - b*T
anh[c + d*x]^2)))/d
```

3.40.3.1 Defintions of rubi rules used

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.40. $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(109) = 218$.

Time = 14.55 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.66

method	result
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)(a+b)}+ 2b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{2}-\frac{\tanh\left(\frac{d}{2}\right.}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)(a+b)}+ 2b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{2}-\frac{\tanh\left(\frac{d}{2}\right.}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.$
risch	$-\frac{9ab\,e^{8dx+8c}-6b^2e^{8dx+8c}+12a^2e^{6dx+6c}+18ab\,e^{6dx+6c}+66b^2e^{6dx+6c}+20a^2e^{4dx+4c}+44ab\,e^{4dx+4c}-66\,e^{4dx+4c}b^2+4a}{3d(a+b)^3\left(e^{2dx+2c}-1\right)^3\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}$

input

```
int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/8/(a^2+2*a*b+b^2)/(a+b)*(1/3*tanh(1/2*d*x+1/2*c)^3*a+1/3*tanh(1/2*
d*x+1/2*c)^3*b-3*tanh(1/2*d*x+1/2*c)*a+5*b*tanh(1/2*d*x+1/2*c))+2*b/(a+b)^
3*((-1/2*tanh(1/2*d*x+1/2*c)^3*a-1/2*tanh(1/2*d*x+1/2*c)*a)/(tanh(1/2*d*x+
1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)+1/2*(3*a-2*b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*t
anh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2
)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*
b^(1/2)-(a+b)^(1/2))))-1/24/(a+b)^2/tanh(1/2*d*x+1/2*c)^3-1/8/(a+b)^3*(-3*
a+5*b)/tanh(1/2*d*x+1/2*c))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2933 vs. 2(112) = 224.

Time = 0.34 (sec) , antiderivative size = 6143, normalized size of antiderivative = 49.94

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.40.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(112) = 224.

Time = 0.34 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.50

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{(3\,ab-2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}d} + \frac{4\,a^2-11\,ab-2\,(2\,a^2-9\,ab+19\,b^2)e^{(-2\,d}}{3\,(a^4+3\,a^3b+3\,a^2b^2+ab^3-(a^4-a^3b-9\,a^2b^2-11\,ab^3-4\,b^4)e^{(-2\,dx-2\,c)}-2\,(a^4+9\,a^3b+21\,a^2b^2+}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(3*a*b - 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))
/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/3*(4*a^2 - 11*a*b - 2*(2*a^2 - 9*a*b +
19*b^2)*e^(-2*d*x - 2*c) - 2*(10*a^2 + 22*a*b - 33*b^2)*e^(-4*d*x - 4*c) -
 6*(2*a^2 + 3*a*b + 11*b^2)*e^(-6*d*x - 6*c) - 3*(3*a*b - 2*b^2)*e^(-8*d*x
 - 8*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 - a^3*b - 9*a^2*b^2 -
11*a*b^3 - 4*b^4)*e^(-2*d*x - 2*c) - 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*
b^3 + 6*b^4)*e^(-4*d*x - 4*c) + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 +
 6*b^4)*e^(-6*d*x - 6*c) + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*e^
(-8*d*x - 8*c) - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-10*d*x - 10*c))*d
)
```

3.40.8 Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^4}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\cosh(c+d\,x)^4}{\sinh\left(c+d\,x\right)^4\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(cosh(c + d*x)^4/(sinh(c + d*x)^4*(b + a*cosh(c + d*x)^2)^2), x)
```

3.41 $\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.41.1 Optimal result

Integrand size = 23, antiderivative size = 242

$$\begin{aligned}\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx &= \frac{3(a^2+12ab+16b^2)\,x}{8a^5} \\ &\quad - \frac{3\sqrt{b}(5a^2+20ab+16b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}d} \\ &\quad - \frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} \\ &\quad + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} \\ &\quad - \frac{b(7a+12b)\tanh(c+dx)}{8a^3d\left(a+b-b\tanh^2(c+dx)\right)^2} \\ &\quad - \frac{3b(a+2b)\tanh(c+dx)}{2a^4d\left(a+b-b\tanh^2(c+dx)\right)}\end{aligned}$$

output
```
3/8*(a^2+12*a*b+16*b^2)*x/a^5-3/8*(5*a^2+20*a*b+16*b^2)*arctanh(b^(1/2)*ta
nh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^5/d/(a+b)^(1/2)-1/8*(5*a+8*b)*cosh(d*x+c)
*sinh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)^2+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a
/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(7*a+12*b)*tanh(d*x+c)/a^3/d/(a+b-b*tanh(
d*x+c)^2)^2-3/2*b*(a+2*b)*tanh(d*x+c)/a^4/d/(a+b-b*tanh(d*x+c)^2)
```

3.41.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 31.09 (sec) , antiderivative size = 3457, normalized size of antiderivative = 14.29

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(3*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*
b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt
[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c
+ d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(16384*b^(5/2)*d*
(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d
*x]^6*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a +
b)^(5/2) + (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a
*b + 4*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*
Cosh[2*(c + d*x)])^2)))/(16384*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) - (3*(
a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b +
80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2
*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]
*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqr
t[b*(Cosh[c] - Sinh[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b
 + 8*b^2)*d*x*Cosh[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] +
128*a^4*b^2*d*x*Cosh[2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)]
+ 128*a^2*b^4*d*x*Cosh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x]
+ 2048*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x]
+ 1024*a*b^5*d*x*Cosh[4*c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 2
56*a^3*b^3*d*x*Cosh[6*c + 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - ...
```

3.41.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4620, 372, 402, 25, 402, 27, 402, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.41. $\int \frac{\sinh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(ic+idx)^4}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\int \frac{(4a+7b)\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)^2\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{4a}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\int -\frac{5b(5a+8b)\tanh^2(c+dx)+(a+b)(3a+8b)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{2a}+\frac{(5a+8b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right.}}{4a}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(5a+8b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\int \frac{5b(5a+8b)\tanh^2(c+dx)+(a+b)(3a+8b)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx}{2a}}{4a}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(5a+8b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{-\frac{\int -\frac{12(a+b)\left(b(7a+12b)\tanh^2(c+dx)+(a+b)(a+4b)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tan}{4a(a+b)}}{2a}}{4a}}{d}$$

$$\Big\downarrow \; 27$$

3.41. $\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(5a+8b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{3\int\frac{b(7a+12b)\tanh^2(c+dx)+(a+b)(a+4b)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+d}{a}}{2a}}{4a}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(5a+8b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{3\left(-\frac{\int-\frac{2(a+b)\left(a^2+8ba+8b^2+4b(a+2b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\,ta}{2a(a+b)}\right.}{a}}{4a}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(5a+8b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{3\left(\frac{\int\frac{a^2+8ba+8b^2+4b(a+2b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+}{a}\right.}{a}}{4a}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(5a+8b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-3\left(\frac{\left(a^2+12ab+16b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b(5}{a}\right.}{}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(5a+8b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-3\left(\frac{\left(a^2+12ab+16b^2\right)\text{arctanh}(\tanh(c+dx))}{a}-\frac{b\left(5a^2+20\right.}{a}\right.}{4a}}{d}$$

$\Big\downarrow$ 221

3.41. $$\int\frac{\sinh^4(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$$

$$\frac{\frac{\tanh(c+dx)}{4a(1-\tanh^2(c+dx))^2(a-b\tanh^2(c+dx)+b)^2} - \frac{\frac{(5a+8b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2} - \frac{3\left(\frac{(a^2+12ab+16b^2)\text{arctanh}(\tanh(c+dx))}{a} - \frac{\sqrt{b}\left(5a^2+2\cdots\right.}{a}\right.}{\cdots}}{4a}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(4*a*(1 - Tanh[c + d*x]^2)^2*(a + b - b*Tanh[c + d*x]^2)^2)
 - (((5*a + 8*b)*Tanh[c + d*x])/(2*a*(1 - Tanh[c + d*x]^2)*(a + b - b*Tanh
[c + d*x]^2)^2) - (-((b*(7*a + 12*b)*Tanh[c + d*x])/(a*(a + b - b*Tanh[c +
 d*x]^2)^2)) + (3*((((a^2 + 12*a*b + 16*b^2)*ArcTanh[Tanh[c + d*x]])/a - (
Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a +
 b]])/(a*Sqrt[a + b]))/a - (4*b*(a + 2*b)*Tanh[c + d*x])/(a*(a + b - b*Tan
h[c + d*x]^2))))/a)/(2*a))/(4*a))/d
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(222) = 444$.

Time = 0.23 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.24

$$-\frac{1}{4a^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{1}{2a^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{-a-12b}{8a^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{3a+12b}{8a^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(3a^2+36ab+48b^2)\ln\left(1+\tanh\right.}{8a^5}$$

input
```
int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

output
```
1/d*(-1/4/a^3/(1+tanh(1/2*d*x+1/2*c))^4+1/2/a^3/(1+tanh(1/2*d*x+1/2*c))^3-
1/8*(-a-12*b)/a^4/(1+tanh(1/2*d*x+1/2*c))^2-1/8*(3*a+12*b)/a^4/(1+tanh(1/2
*d*x+1/2*c))+1/8/a^5*(3*a^2+36*a*b+48*b^2)*ln(1+tanh(1/2*d*x+1/2*c))+2*b/a
^5*(((-9/8*a^3-21/8*a^2*b-3/2*a*b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(27*a^2+35*
a*b-12*b^2)*a*tanh(1/2*d*x+1/2*c)^5-1/8*(27*a^2+35*a*b-12*b^2)*a*tanh(1/2*
d*x+1/2*c)^3+(-9/8*a^3-21/8*a^2*b-3/2*a*b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/
2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2+1/8*(15*a^2+60*a*b+48*b^2)*(-1/4/b^(1/2)/(a+b)^(
1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a
+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2
*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))))+1/4/a^3/(tanh(1/2*d*x+1/2*c)-1
)^4+1/2/a^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(a+12*b)/a^4/(tanh(1/2*d*x+1/2*c
)-1)^2-1/8*(3*a+12*b)/a^4/(tanh(1/2*d*x+1/2*c)-1)+1/8/a^5*(-3*a^2-36*a*b-4
8*b^2)*ln(tanh(1/2*d*x+1/2*c)-1))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6038 vs. $2(234) = 468$.

Time = 0.40 (sec) , antiderivative size = 12353, normalized size of antiderivative = 51.05

$$\int \frac{\sinh^4(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2468 vs. 2(234) = 468.

Time = 0.41 (sec) , antiderivative size = 2468, normalized size of antiderivative = 10.20

$$\int \frac{\sinh^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-3/256*(5*a^4*b + 100*a^3*b^2 + 320*a^2*b^3 + 352*a*b^4 + 128*b^5)*log((a*
e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*
b + 2*sqrt((a + b)*b)))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)*d) - 3/
64*(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a +
 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)
))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) + 3/256*(5*a^4*b + 100*a^
3*b^2 + 320*a^2*b^3 + 352*a*b^4 + 128*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2
*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b))
)/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)*d) + 3/64*(5*a^3*b + 30*a^2*b
^2 + 40*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)
*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*b +
 a^4*b^2)*sqrt((a + b)*b)*d) + 3/128*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*
e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a +
2*b + 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) +
1/64*(9*a^5*b + 110*a^4*b^2 + 216*a^3*b^3 + 112*a^2*b^4 + (9*a^5*b + 228*a
^4*b^2 + 920*a^3*b^3 + 1216*a^2*b^4 + 512*a*b^5)*e^(6*d*x + 6*c) + (27*a^5
*b + 594*a^4*b^2 + 2816*a^3*b^3 + 5696*a^2*b^4 + 5248*a*b^5 + 1792*b^6)*e^
(4*d*x + 4*c) + (27*a^5*b + 476*a^4*b^2 + 1720*a^3*b^3 + 2176*a^2*b^4 + 89
6*a*b^5)*e^(2*d*x + 2*c))/((a^9 + 2*a^8*b + a^7*b^2 + (a^9 + 2*a^8*b + a^7
*b^2)*e^(8*d*x + 8*c) + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(6*...
```

3.41.8 Giac [F]

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)^4}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^6\,\sinh(c+d\,x)^4}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*sinh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.42 $\int \frac{\sinh^3(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

3.42.1 Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx = \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d} - \frac{(a+3b)\cosh(c+dx)}{a^4d} + \frac{\cosh^3(c+dx)}{3a^3d} + \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(b+a\cosh^2(c+dx)\right)}$$

output
```
-(a+3*b)*cosh(d*x+c)/a^4/d+1/3*cosh(d*x+c)^3/a^3/d+1/4*b^2*(a+b)*cosh(d*x+
c)/a^4/d/(b+a*cosh(d*x+c)^2)^2-1/8*b*(9*a+13*b)*cosh(d*x+c)/a^4/d/(b+a*cos
h(d*x+c)^2)+5/8*(3*a+7*b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9
/2)/d
```

3.42.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.81 (sec) , antiderivative size = 1364, normalized size of antiderivative = 8.86

$$\int \frac{\sinh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-3*((3*(ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + Arc
Tan[(Sqrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]]))/Sqrt[a] + (2*Sq
rt[b]*Cosh[c + d*x]*(3*a + 10*b + 3*a*Cosh[2*(c + d*x)]))/(a + 2*b + a*Cos
h[2*(c + d*x)])^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6)/(819
2*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) - ((-((3*a - 4*b)*(ArcTan[((Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh
[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/S
qrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Si
nh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sin
h[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])) - (2*Sqrt[a]*Sqrt[b]*Cosh[c + d*x]*(3*
a^2 + 6*a*b + 8*b^2 + a*(3*a - 4*b)*Cosh[2*(c + d*x)]))/(a + 2*b + a*Cosh[
2*(c + d*x)])^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6)/(2048*
a^(3/2)*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) + ((3*(3*a^4 - 40*a^3*b + 720
*a^2*b^2 + 6720*a*b^3 + 8960*b^4)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(C
osh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a
+ b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 3*(3*a^4 - 40*
a^3*b + 720*a^2*b^2 + 6720*a*b^3 + 8960*b^4)*ArcTan[((Sqrt[a] + I*Sqrt[a +
 b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a]
+ I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + (2*
Sqrt[a]*Sqrt[b]*Cosh[c + d*x]*(9*a^5 - 90*a^4*b - 10144*a^3*b^2 - 48672...
```

3.42.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 26, 4621, 360, 25, 2345, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.42. $\int \frac{\sinh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx$

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{i\sin(ic+idx)^3}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int \frac{\sin(ic+idx)^3}{\left(b\sec(ic+idx)^2+a\right)^3}dx$$
$$\Big\downarrow \text{ 4621}$$
$$-\frac{\int \frac{\cosh^6(c+dx)\left(1-\cosh^2(c+dx)\right)}{\left(a\cosh^2(c+dx)+b\right)^3}d\cosh(c+dx)}{d}$$
$$\Big\downarrow \text{ 360}$$
$$-\frac{-\frac{\int -\frac{-4a^3\cosh^6(c+dx)+4a^2(a+b)\cosh^4(c+dx)-4ab(a+b)\cosh^2(c+dx)+b^2(a+b)}{\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{4a^4}-\frac{b^2(a+b)\cosh(c+dx)}{4a^4\left(a\cosh^2(c+dx)+b\right)^2}}{d}$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{\frac{\int \frac{-4a^3\cosh^6(c+dx)+4a^2(a+b)\cosh^4(c+dx)-4ab(a+b)\cosh^2(c+dx)+b^2(a+b)}{\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{4a^4}-\frac{b^2(a+b)\cosh(c+dx)}{4a^4\left(a\cosh^2(c+dx)+b\right)^2}}{d}$$
$$\Big\downarrow \text{ 2345}$$
$$-\frac{\frac{\frac{b(9a+13b)\cosh(c+dx)}{2\left(a\cosh^2(c+dx)+b\right)}-\frac{\int \frac{8a^2b\cosh^4(c+dx)-8ab(a+2b)\cosh^2(c+dx)+b^2(7a+11b)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{2b}}{4a^4}-\frac{b^2(a+b)\cosh(c+dx)}{4a^4\left(a\cosh^2(c+dx)+b\right)^2}}{d}$$
$$\Big\downarrow \text{ 1467}$$
$$-\frac{\frac{\frac{b(9a+13b)\cosh(c+dx)}{2\left(a\cosh^2(c+dx)+b\right)}-\frac{\int \left(8ab\cosh^2(c+dx)-8b(a+3b)+\frac{5\left(7b^3+3ab^2\right)}{a\cosh^2(c+dx)+b}\right)d\cosh(c+dx)}{2b}}{4a^4}-\frac{b^2(a+b)\cosh(c+dx)}{4a^4\left(a\cosh^2(c+dx)+b\right)^2}}{d}$$
$$\Big\downarrow \text{ 2009}$$

3.42. $\int \frac{\sinh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$

$$-\frac{\frac{\frac{b(9a+13b)\cosh(c+dx)}{2\left(a\cosh^2(c+dx)+b\right)}-\frac{\frac{5b^{3/2}(3a+7b)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}}+\frac{8}{3}ab\cosh^3(c+dx)-8b(a+3b)\cosh(c+dx)}{2b}}{4a^4}-\frac{b^2(a+b)\cosh(c+dx)}{4a^4\left(a\cosh^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-((-1/4*(b^2*(a + b)*Cosh[c + d*x])/(a^4*(b + a*Cosh[c + d*x]^2)^2) + ((b*
(9*a + 13*b)*Cosh[c + d*x])/(2*(b + a*Cosh[c + d*x]^2)) - ((5*b^(3/2)*(3*a
 + 7*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/Sqrt[a] - 8*b*(a + 3*b)*C
osh[c + d*x] + (8*a*b*Cosh[c + d*x]^3)/3)/(2*b))/(4*a^4))/d)
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
 - 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.42.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.

Time = 0.21 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.26

$$\frac{1}{3a^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2a^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a+6b}{2a^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{1}{3a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{\cdots}{2a\cdots}$$

input
```
int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

output
```
1/d*(1/3/a^3/(1+tanh(1/2*d*x+1/2*c))^3-1/2/a^3/(1+tanh(1/2*d*x+1/2*c))^2-1
/2*(a+6*b)/a^4/(1+tanh(1/2*d*x+1/2*c))-1/3/a^3/(tanh(1/2*d*x+1/2*c)-1)^3-1
/2/a^3/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/a^4*(-a-6*b)/(tanh(1/2*d*x+1/2*c)-1)+
2*b/a^4*(((-9/8*a^2+1/4*a*b+11/8*b^2)*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+15
*a^2*b+5*a*b^2+33*b^3)/(a+b)*tanh(1/2*d*x+1/2*c)^4+(-27/8*a^2-5/4*a*b+33/8
*b^2)*tanh(1/2*d*x+1/2*c)^2-9/8*a^2-5/2*a*b-11/8*b^2)/(tanh(1/2*d*x+1/2*c)
^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)^2+5/16*(3*a+7*b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+
1/2*c)^2+2*a-2*b)/(a*b)^(1/2))))
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4793 vs. 2(138) = 276.

Time = 0.35 (sec) , antiderivative size = 8667, normalized size of antiderivative = 56.28

$$\int \frac{\sinh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.42.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.42.8 Giac [F]

$$\int \frac{\sinh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx=\int \frac{\sinh(dx+c)^3}{\left(b\,\text{sech}\,(dx+c)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx=\int \frac{\cosh(c+d\,x)^6\sinh(c+d\,x)^3}{\left(a\cosh\,(c+d\,x)^2+b\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*sinh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.43 $\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3} dx$

3.43.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3} dx = -\frac{(a+6b)x}{2a^4} + \frac{\sqrt{b}(15a^2+40ab+24b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(11a+12b)\tanh(c+dx)}{8a^3(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
-1/2*(a+6*b)*x/a^4+1/8*(15*a^2+40*a*b+24*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/
(a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+
b-b*tanh(d*x+c)^2)^2+3/4*b*tanh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*b
*(11*a+12*b)*tanh(d*x+c)/a^3/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

3.43.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2544 vs. $2(187) = 374$.

Time = 22.29 (sec) , antiderivative size = 2544, normalized size of antiderivative = 13.60

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3} dx = \text{Result too large to show}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-5*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8
*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqr
t[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c
 + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(8192*b^(5/2)*d*
(a + b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d
*x]^6*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a +
b)^(5/2) + (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a
*b + 4*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*
Cosh[2*(c + d*x)])^2)))/(2048*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) + ((a +
 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*
a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c]
 - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sq
rt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b
*(Cosh[c] - Sinh[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b +
8*b^2)*d*x*Cosh[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128
*a^4*b^2*d*x*Cosh[2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 1
28*a^2*b^4*d*x*Cosh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2
048*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1
024*a*b^5*d*x*Cosh[4*c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*
a^3*b^3*d*x*Cosh[6*c + 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a...
```

3.43.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 25, 4620, 373, 402, 27, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.43. $\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3} dx$

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\sin(ic+idx)^2}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\sin(ic+idx)^2}{\left(b\sec(ic+idx)^2+a\right)^3}dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 373$$

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\int \frac{5b\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{2a}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{-\frac{\int -\frac{2(a+b)\left(9b\tanh^2(c+dx)+2a+3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{3b\tanh(c+dx)}{2a\left(a-b\tanh^2(c+dx)+b\right)^2}}{2a}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\int \frac{9b\tanh^2(c+dx)+2a+3b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{2a}-\frac{3b\tanh(c+dx)}{2a\left(a-b\tanh^2(c+dx)+b\right)^2}}{2a}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{-\frac{\int -\frac{4a^2+17ba+12b^2+b(11a+12b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(11a+12b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{2a}-\frac{3b\,\text{t}}{2a\left(a-b\,\text{t}\right.}}{2a}}{d}$$

$$\Big\downarrow \ 25$$

3.43. $\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$

$$\frac{\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\int \frac{4a^2+17ba+12b^2+b(11a+12b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(11a+12b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{2a}-\frac{3b\tanh}{2a\left(a-b\tanh\right.}}{2a}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\frac{4(a+b)(a+6b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b\left(15a^2+40ab+24b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}}{2a}}{2a}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\frac{4(a+b)(a+6b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{b\left(15a^2+40ab+24b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{b}{2a(a}}{2a}}{2a}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\frac{4(a+b)(a+6b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(11a+}{2a(a+b)\left(a\right.}}{2a}}{2a}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(2*a*(1 - Tanh[c + d*x]^2)*(a + b - b*Tanh[c + d*x]^2)^2) -
 ((-3*b*Tanh[c + d*x])/(2*a*(a + b - b*Tanh[c + d*x]^2)^2) + (((4*(a + b)*
(a + 6*b)*ArcTanh[Tanh[c + d*x]])/a - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*
ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b
)) - (b*(11*a + 12*b)*Tanh[c + d*x])/(2*a*(a + b)*(a + b - b*Tanh[c + d*x]
^2)))/(2*a))/(2*a))/d
```

3.43. $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.43. $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.43.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1319 vs. $2(169) = 338$.

Time = 0.24 (sec) , antiderivative size = 1320, normalized size of antiderivative = 7.06

Expression too large to display

input
```
int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

output
```
1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)+1/2/
d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/
a^3/(1+tanh(1/2*d*x+1/2*c))^2+1/2/d/a^3/(1+tanh(1/2*d*x+1/2*c))-1/2/d/a^3*
ln(1+tanh(1/2*d*x+1/2*c))-3/d/a^4*ln(1+tanh(1/2*d*x+1/2*c))*b+9/4/d*b/a^2/
(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+2/d*b^2/a^3/(tanh(
1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+27/4/d*b/a/(tanh(1/2*d*x
+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+23/4/d*b^2/a^2/(tanh(1/2*
d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-2/d*b^3/a^3/(tanh(1/2*
d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+27/4/d*b/a/(tanh(1/2*d
*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+23/4/d*b^2/a^2/(tanh(1/
2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3-2/d*b^3/a^3/(tanh(1/
2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+9/4/d*b/a^2/(tanh...
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4724 vs. $2(178) = 356$.

Time = 0.38 (sec) , antiderivative size = 9730, normalized size of antiderivative = 52.03

$$\int \frac{\sinh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.43.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(178) = 356$.

Time = 0.37 (sec) , antiderivative size = 1373, normalized size of antiderivative = 7.34

$$\int \frac{\sinh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
3/64*(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a
 + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*
b)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) - 3/64*(5*a^3*b + 30*a^
2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a +
 b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*
b + a^4*b^2)*sqrt((a + b)*b)*d) - 1/32*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((
a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
+ 2*b + 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d)
- 1/16*(9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a
^2*b^3 + 32*a*b^4)*e^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3
 + 832*a*b^4 + 320*b^5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^
2*b^3 + 160*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a
^7*b + a^6*b^2)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3
)*e^(6*d*x + 6*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*
b^4)*e^(4*d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x
+ 2*c))*d) + 1/16*(9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3
*b^2 + 336*a^2*b^3 + 160*a*b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2
 + 740*a^2*b^3 + 832*a*b^4 + 320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a
^3*b^2 + 64*a^2*b^3 + 32*a*b^4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^
2 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a...
```

3.43.8 Giac [F]

$$\int \frac{\sinh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx=\int \frac{\sinh(dx+c)^2}{\left(b\,\text{sech}(dx+c)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\cosh(c+d\,x)^6\,\sinh(c+d\,x)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*sinh(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.44 $\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.44.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d}+\frac{15\cosh(c+dx)}{8a^3d} -\frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2}-\frac{5\cosh^3(c+dx)}{8a^2d\left(b+a\cosh^2(c+dx)\right)}$$

output
```
15/8*cosh(d*x+c)/a^3/d-1/4*cosh(d*x+c)^5/a/d/(b+a*cosh(d*x+c)^2)^2-5/8*cos
h(d*x+c)^3/a^2/d/(b+a*cosh(d*x+c)^2)-15/8*arctan(cosh(d*x+c)*a^(1/2)/b^(1/
2))*b^(1/2)/a^(7/2)/d
```

3.44.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.49 (sec) , antiderivative size = 1272, normalized size of antiderivative = 10.97

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(5*(3*(a^2 - 4*a*b + 16*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c
] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 3*(a^2 - 4*a*b + 16
*b^2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c
]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c]
)^2]*Tanh[(d*x)/2]))/Sqrt[b]] + (8*Sqrt[a]*b^(3/2)*(a^2 + 12*a*b + 16*b^2)
*Cosh[c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)])^2 + (2*Sqrt[a]*Sqrt[b]*(3*
a^2 - 12*a*b - 80*b^2)*Cosh[c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]))*(a
+ 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6)/(4096*a^(5/2)*b^(5/2)*d*(a
 + b*Sech[c + d*x]^2)^3) + (5*((3*(ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c
 + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/
Sqrt[b]]))/Sqrt[a] + (2*Sqrt[b]*Cosh[c + d*x]*(3*a + 10*b + 3*a*Cosh[2*(c
+ d*x)]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2)*(a + 2*b + a*Cosh[2*c + 2*d*x
])^3*Sech[c + d*x]^6)/(4096*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) + (9*(-((
3*a - 4*b)*(ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*
Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - S
inh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqr
t[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqr
t[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])) - (2*Sqrt[
a]*Sqrt[b]*Cosh[c + d*x]*(3*a^2 + 6*a*b + 8*b^2 + a*(3*a - 4*b)*Cosh[2*...
```

3.44.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4621, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{\left(b\sec(ic+idx)^2+a\right)^3}dx$$

$$\Big\downarrow \text{4621}$$

$$\frac{\int \frac{\cosh^6(c+dx)}{\left(a\cosh^2(c+dx)+b\right)^3}d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 252

$$\frac{\frac{5\int \frac{\cosh^4(c+dx)}{\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{4a}-\frac{\cosh^5(c+dx)}{4a\left(a\cosh^2(c+dx)+b\right)^2}}{d}$$

$\Big\downarrow$ 252

$$\frac{\frac{5\left(\frac{3\int \frac{\cosh^2(c+dx)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{2a}-\frac{\cosh^3(c+dx)}{2a\left(a\cosh^2(c+dx)+b\right)}\right)}{4a}-\frac{\cosh^5(c+dx)}{4a\left(a\cosh^2(c+dx)+b\right)^2}}{d}$$

$\Big\downarrow$ 262

$$\frac{\frac{5\left(\frac{3\left(\frac{\cosh(c+dx)}{a}-\frac{b\int \frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a}\right)}{2a}-\frac{\cosh^3(c+dx)}{2a\left(a\cosh^2(c+dx)+b\right)}\right)}{4a}-\frac{\cosh^5(c+dx)}{4a\left(a\cosh^2(c+dx)+b\right)^2}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{5\left(\frac{3\left(\frac{\cosh(c+dx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{\cosh^3(c+dx)}{2a\left(a\cosh^2(c+dx)+b\right)}\right)}{4a}-\frac{\cosh^5(c+dx)}{4a\left(a\cosh^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/4*Cosh[c + d*x]^5/(a*(b + a*Cosh[c + d*x]^2)^2) + (5*((3*(-((Sqrt[b]*A
rcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/a^(3/2)) + Cosh[c + d*x]/a))/(2*a)
 - Cosh[c + d*x]^3/(2*a*(b + a*Cosh[c + d*x]^2))))/(4*a))/d
```

3.44. $\int \frac{\sinh(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

3.44.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.44. $\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3} dx$

3.44.4 Maple [A] (verified)

Time = 164.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{-\frac{1}{a^{3}\operatorname{sech}(dx+c)}-\frac{b\left(\frac{\frac{7\operatorname{sech}(dx+c)^{3}b}{8}+\frac{9\operatorname{sech}(dx+c)a}{8}}{\left(a+b\operatorname{sech}(dx+c)^{2}\right)^{2}}+\frac{15\arctan\left(\frac{b\operatorname{sech}(dx+c)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}}{d}$
default	$-\frac{-\frac{1}{a^{3}\operatorname{sech}(dx+c)}-\frac{b\left(\frac{\frac{7\operatorname{sech}(dx+c)^{3}b}{8}+\frac{9\operatorname{sech}(dx+c)a}{8}}{\left(a+b\operatorname{sech}(dx+c)^{2}\right)^{2}}+\frac{15\arctan\left(\frac{b\operatorname{sech}(dx+c)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}}{d}$
risch	$\frac{e^{dx+c}}{2a^{3}d}+\frac{e^{-dx-c}}{2a^{3}d}+\frac{\left(9a\,e^{6dx+6c}+27a\,e^{4dx+4c}+28b\,e^{4dx+4c}+27\,e^{2dx+2c}a+28b\,e^{2dx+2c}+9a\right)e^{dx+c}b}{4a^{3}\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^{2}d}+\frac{15\sqrt{-ab}\,\ln\Big(}{}$

input
```
int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-1/a^3/sech(d*x+c)-1/a^3*b*((7/8*sech(d*x+c)^3*b+9/8*sech(d*x+c)*a)/
(a+b*sech(d*x+c)^2)^2+15/8/(a*b)^(1/2)*arctan(b*sech(d*x+c)/(a*b)^(1/2))))
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2599 vs. 2(100) = 200.

Time = 0.32 (sec) , antiderivative size = 4829, normalized size of antiderivative = 41.63

$$\int \frac{\sinh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(8*a^2*cosh(d*x + c)^10 + 80*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + 8*a
^2*sinh(d*x + c)^10 + 20*(2*a^2 + 5*a*b)*cosh(d*x + c)^8 + 20*(18*a^2*cosh
(d*x + c)^2 + 2*a^2 + 5*a*b)*sinh(d*x + c)^8 + 160*(6*a^2*cosh(d*x + c)^3
+ (2*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(4*a^2 + 15*a*b + 12
*b^2)*cosh(d*x + c)^6 + 20*(84*a^2*cosh(d*x + c)^4 + 28*(2*a^2 + 5*a*b)*co
sh(d*x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*x + c)^6 + 8*(252*a^2*cosh
(d*x + c)^5 + 140*(2*a^2 + 5*a*b)*cosh(d*x + c)^3 + 15*(4*a^2 + 15*a*b + 1
2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*(4*a^2 + 15*a*b + 12*b^2)*cosh(
d*x + c)^4 + 20*(84*a^2*cosh(d*x + c)^6 + 70*(2*a^2 + 5*a*b)*cosh(d*x + c)
^4 + 15*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^2 + 4*a^2 + 15*a*b + 12*b^
2)*sinh(d*x + c)^4 + 80*(12*a^2*cosh(d*x + c)^7 + 14*(2*a^2 + 5*a*b)*cosh(
d*x + c)^5 + 5*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^3 + (4*a^2 + 15*a*b
 + 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 20*(2*a^2 + 5*a*b)*cosh(d*x +
c)^2 + 20*(18*a^2*cosh(d*x + c)^8 + 28*(2*a^2 + 5*a*b)*cosh(d*x + c)^6 + 1
5*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^4 + 6*(4*a^2 + 15*a*b + 12*b^2)*
cosh(d*x + c)^2 + 2*a^2 + 5*a*b)*sinh(d*x + c)^2 + 15*(a^2*cosh(d*x + c)^9
 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 + 4*(a^2 + 2*
a*b)*cosh(d*x + c)^7 + 4*(9*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x +
c)^7 + 28*(3*a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x +
 c)^6 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^5 + 2*(63*a^2*cosh(d*x ...
```

3.44.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.44.7 Maxima [F]

$$\int \frac{\sinh(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\sinh(dx+c)}{(b\,\text{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*a^2*e^(10*d*x + 10*c) + 2*a^2 + 5*(2*a^2*e^(8*c) + 5*a*b*e^(8*c))*e
^(8*d*x) + 5*(4*a^2*e^(6*c) + 15*a*b*e^(6*c) + 12*b^2*e^(6*c))*e^(6*d*x) +
 5*(4*a^2*e^(4*c) + 15*a*b*e^(4*c) + 12*b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*
e^(2*c) + 5*a*b*e^(2*c))*e^(2*d*x))/(a^5*d*e^(9*d*x + 9*c) + a^5*d*e^(d*x
+ c) + 4*(a^5*d*e^(7*c) + 2*a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(3*a^5*d*e^(5*c
) + 8*a^4*b*d*e^(5*c) + 8*a^3*b^2*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c)
+ 2*a^4*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(15/2*(b*e^(3*d*x + 3*c) -
b*e^(d*x + c))/(a^4*e^(4*d*x + 4*c) + a^4 + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*
c))*e^(2*d*x)), x)
```

3.44.8 Giac [F]

$$\int \frac{\sinh(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\sinh(dx+c)}{(b\,\text{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.44.9 Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int \frac{\sinh(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \frac{\frac{7\,b^2\cosh(c+d\,x)}{8}+\frac{9\,a\,b\cosh(c+d\,x)^3}{8}}{d\,a^5\cosh(c+d\,x)^4+2\,d\,a^4\,b\cosh(c+d\,x)^2+d\,a^3\,b^2} + \frac{\cosh(c+d\,x)}{a^3\,d} - \frac{15\,\sqrt{b}\,\text{atan}\left(\frac{\sqrt{a}\cosh(c+d\,x)}{\sqrt{b}}\right)}{8\,a^{7/2}\,d}$$

input
```
int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
((7*b^2*cosh(c + d*x))/8 + (9*a*b*cosh(c + d*x)^3)/8)/(a^5*d*cosh(c + d*x)
^4 + a^3*b^2*d + 2*a^4*b*d*cosh(c + d*x)^2) + cosh(c + d*x)/(a^3*d) - (15*
b^(1/2)*atan((a^(1/2)*cosh(c + d*x))/b^(1/2)))/(8*a^(7/2)*d)
```

3.44. $\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.45 $\int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$

3.45.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3d} - \frac{\operatorname{arctanh}(\cosh(c+dx))}{(a+b)^3d} - \frac{b\cosh^3(c+dx)}{4a(a+b)d\,(b+a\cosh^2(c+dx))^2} - \frac{b(7a+3b)\cosh(c+dx)}{8a^2(a+b)^2d\,(b+a\cosh^2(c+dx))}$$

output
```
-arctanh(cosh(d*x+c))/(a+b)^3/d-1/4*b*cosh(d*x+c)^3/a/(a+b)/d/(b+a*cosh(d*
x+c)^2)^2-1/8*b*(7*a+3*b)*cosh(d*x+c)/a^2/(a+b)^2/d/(b+a*cosh(d*x+c)^2)+1/
8*(15*a^2+10*a*b+3*b^2)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2
)/(a+b)^3/d
```

3.45.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.49 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.86

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^5(c+dx)\left(\frac{8b^2(a+b)^2}{a^2}-\frac{2b(a+b)(9a+5b)(a+2b+a\cosh(2(c+dx)))}{a^2}+\frac{\sqrt{b}(15a^2+10ab+3b^2)}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((8*b^2*(a + b)^2)/a^2 -
(2*b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a^2 + (Sqrt[b]*(
15*a^2 + 10*a*b + 3*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] -
Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt
[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c +
d*x)])^2*Sech[c + d*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTa
n[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x
)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(
d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x])/a^(5/2
) - 8*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Cosh[(c + d*x)/2]]*Sech[c + d*
x] + 8*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Sinh[(c + d*x)/2]]*Sech[c + d
*x]))/(64*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

3.45.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 26, 4621, 372, 440, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^3\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cosh^6(c+dx)}{(1-\cosh^2(c+dx))\left(a\cosh^2(c+dx)+b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$-\frac{\frac{b\cosh^3(c+dx)}{4a(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\int \frac{\cosh^2(c+dx)\left(3b-(4a+3b)\cosh^2(c+dx)\right)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{440}$$

$$-\frac{\frac{b\cosh^3(c+dx)}{4a(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\frac{\int \frac{b(7a+3b)-\left(8a^2+7ba+3b^2\right)\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}d\cosh(c+dx)}{2a(a+b)}-\frac{b(7a+3b)\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{b\cosh^3(c+dx)}{4a(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{b\left(15a^2+10ab+3b^2\right)\int \frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a+b}-\frac{8a^2\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{b\cosh^3(c+dx)}{4a(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{8a^2\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{219}$$

3.45. $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}dx$

$$-\frac{\frac{b\cosh^3(c+dx)}{4a(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{8a^2\text{arctanh}(\cosh(c+dx))}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-(((b*Cosh[c + d*x]^3)/(4*a*(a + b)*(b + a*Cosh[c + d*x]^2)^2) - (((Sqrt[b
]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(Sqrt
[a]*(a + b)) - (8*a^2*ArcTanh[Cosh[c + d*x]])/(a + b))/(2*a*(a + b)) - (b*
(7*a + 3*b)*Cosh[c + d*x])/(2*a*(a + b)*(b + a*Cosh[c + d*x]^2)))/(4*a*(a
+ b)))/d)
```

3.45.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(140) = 280$.

Time = 0.26 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.89

$$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a+b)^3}+\frac{2b\left(\frac{-\frac{\left(9a^3-a^2b-13a\,b^2-3b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8a^2}-\frac{3\left(9a^3-3a^2b+7a\,b^2+3b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8a^2}-\frac{\left(27a^3+13a^2b-23a\,b^2-9b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}}{(a+b)^3}}{d}$$

input
```
int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

output
```
1/d*(1/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))+2*b/(a+b)^3*((-1/8*(9*a^3-a^2*b-13*
a*b^2-3*b^3)/a^2*tanh(1/2*d*x+1/2*c)^6-3/8*(9*a^3-3*a^2*b+7*a*b^2+3*b^3)/a
^2*tanh(1/2*d*x+1/2*c)^4-1/8*(27*a^3+13*a^2*b-23*a*b^2-9*b^3)/a^2*tanh(1/2
*d*x+1/2*c)^2-3/8*(3*a^3+7*a^2*b+5*a*b^2+b^3)/a^2)/(tanh(1/2*d*x+1/2*c)^4*
a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^
2*b+a+b)^2+1/16*(15*a^2+10*a*b+3*b^2)/a^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*
tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))))
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4721 vs. $2(140) = 280$.

Time = 0.39 (sec) , antiderivative size = 8742, normalized size of antiderivative = 56.77

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.45.6 Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(c+dx\right)}{\left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

3.45.7 Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(b\operatorname{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((9*a^2*b*e^(7*c) + 5*a*b^2*e^(7*c))*e^(7*d*x) + (27*a^2*b*e^(5*c) +
43*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) + (27*a^2*b*e^(3*c) + 43*a*b^
2*e^(3*c) + 12*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 5*a*b^2*e^c)*e^(d*x
))/(a^6*d + 2*a^5*b*d + a^4*b^2*d + (a^6*d*e^(8*c) + 2*a^5*b*d*e^(8*c) + a
^4*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 4*a^5*b*d*e^(6*c) + 5*a^4
*b^2*d*e^(6*c) + 2*a^3*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d*e^(4*c) + 14*
a^5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) + 24*a^3*b^3*d*e^(4*c) + 8*a^2*b^4*
d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 4*a^5*b*d*e^(2*c) + 5*a^4*b^2*d*
e^(2*c) + 2*a^3*b^3*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/
(a^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a
^3*d + 3*a^2*b*d + 3*a*b^2*d + b^3*d) + 2*integrate(1/8*((15*a^2*b*e^(3*c)
 + 10*a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 10*a*b^2*
e^c + 3*b^3*e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + (a^6*e^(4
*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))*e^(4*d*x) + 2
*(a^6*e^(2*c) + 5*a^5*b*e^(2*c) + 9*a^4*b^2*e^(2*c) + 7*a^3*b^3*e^(2*c) +
2*a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

3.45.8 Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(b\operatorname{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{{\cosh\left(c+d\,x\right)}^6}{\mathrm{sinh}\left(c+d\,x\right)\,{\left(a\,{\mathrm{cosh}\left(c+d\,x\right)}^2+b\right)}^3}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(cosh(c + d*x)^6/(sinh(c + d*x)*(b + a*cosh(c + d*x)^2)^3), x)
```

3.46 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.46.1 Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{15\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} - \frac{15\coth(c+dx)}{8(a+b)^3d} + \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
-15/8*coth(d*x+c)/(a+b)^3/d+15/8*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*
b^(1/2)/(a+b)^(7/2)/d+1/4*coth(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+5/8*
coth(d*x+c)/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

3.46.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.35 (sec) , antiderivative size = 981, normalized size of antiderivative = 7.79

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$
$$= \frac{(a+2b+a\cosh(2c+2dx))^3\operatorname{sech}^6(c+dx)\left(-\frac{15ib\arctan\left(\operatorname{sech}(dx)\left(-\frac{i\cosh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4c)}}+\frac{i\sinh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(}}\right.\right.}{64\sqrt{a+b}d\sqrt{b\cosh(4c)-b\sin}}\right.}{}$$
$$+\frac{(a+2b+a\cosh(2c+2dx))\operatorname{csch}(c)\operatorname{csch}(c+dx)\operatorname{sech}(2c)\operatorname{sech}^6(c+dx)\left(-32a^4\sinh(dx)-64a^3b\sinh(dx\right.}{}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((((-15*I)/64)*b*ArcTan
[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*
c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-
(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cosh[2*c])/(Sqrt[a + b
]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + (((15*I)/64)*b*ArcTan[Sech[d*x]*(((
-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*
Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x])
- 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(Sqrt[a + b]*d*Sqrt[b*Cos
h[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b
 + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]*Sech[2*c]*Sech[c + d*x]^6*(-
32*a^4*Sinh[d*x] - 64*a^3*b*Sinh[d*x] + 22*a^2*b^2*Sinh[d*x] + 80*a*b^3*Si
nh[d*x] + 16*b^4*Sinh[d*x] + 32*a^4*Sinh[3*d*x] + 46*a^3*b*Sinh[3*d*x] - 5
4*a^2*b^2*Sinh[3*d*x] - 8*a*b^3*Sinh[3*d*x] - 48*a^4*Sinh[2*c - d*x] - 128
*a^3*b*Sinh[2*c - d*x] - 106*a^2*b^2*Sinh[2*c - d*x] + 80*a*b^3*Sinh[2*c -
 d*x] + 16*b^4*Sinh[2*c - d*x] + 48*a^4*Sinh[2*c + d*x] + 146*a^3*b*Sinh[2
*c + d*x] + 182*a^2*b^2*Sinh[2*c + d*x] + 80*a*b^3*Sinh[2*c + d*x] + 16*b^
4*Sinh[2*c + d*x] - 32*a^4*Sinh[4*c + d*x] - 82*a^3*b*Sinh[4*c + d*x] - 54
*a^2*b^2*Sinh[4*c + d*x] - 80*a*b^3*Sinh[4*c + d*x] - 16*b^4*Sinh[4*c + d*
x] - 8*a^4*Sinh[2*c + 3*d*x] + 18*a^3*b*Sinh[2*c + 3*d*x] + 54*a^2*b^2*Sin
h[2*c + 3*d*x] + 8*a*b^3*Sinh[2*c + 3*d*x] + 32*a^4*Sinh[4*c + 3*d*x] +...
```

3.46. $\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$

3.46.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 4620, 253, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^3\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{5\int \frac{\coth^2(c+dx)}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4(a+b)}+\frac{\coth(c+dx)}{4(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{5\left(\frac{3\int \frac{\coth^2(c+dx)}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{2(a+b)}+\frac{\coth(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\coth(c+dx)}{4(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{5\left(\frac{3\left(\frac{b\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a+b}-\frac{\coth(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\coth(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\coth(c+dx)}{4(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{5\left(\frac{3\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\coth(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\coth(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\coth(c+dx)}{4(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Coth[c + d*x]/(4*(a + b)*(a + b - b*Tanh[c + d*x]^2)^2) + (5*((3*((Sqrt[b
]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2) - Coth[c + d
*x]/(a + b)))/(2*(a + b)) + Coth[c + d*x]/(2*(a + b)*(a + b - b*Tanh[c + d
*x]^2))))/(4*(a + b)))/d
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(110) = 220$.

Time = 0.26 (sec) , antiderivative size = 819, normalized size of antiderivative = 6.50

Expression too large to display

input
```
int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

output
```
-1/2/d/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)-1/2/d/(a+b)^3/tanh(1/
2*d*x+1/2*c)+9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^
4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*
x+1/2*c)^7*a+9/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c
)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*
d*x+1/2*c)^7+27/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)
^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d
*x+1/2*c)^5*a-1/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*
c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2
*d*x+1/2*c)^5+27/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c
)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*
d*x+1/2*c)^3*a-1/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2
*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/
2*d*x+1/2*c)^3+9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c
)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*
d*x+1/2*c)*a+9/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c
)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*
d*x+1/2*c)+15/16/d*b^(1/2)/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^
2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-15/16/d*b^(1/2)/(a+b)^(7/2)*l
n(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b...
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3499 vs. 2(116) = 232.

Time = 0.34 (sec) , antiderivative size = 7275, normalized size of antiderivative = 57.74

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.46.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(116) = 232.

Time = 0.35 (sec) , antiderivative size = 533, normalized size of antiderivative = 4.23

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = -\frac{15\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}d} - \frac{8\,a^4-9\,a^3b-2\,a^2b^2+2\,(16\,a^4+23\,a^3b-27\,a^2b^2-4\,ab^3)}{4\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3+(3\,a^7+17\,a^6b+33\,a^5b^2+27\,a^4b^3+8\,a^3b^4)e^{(-2\,dx-2\,c)}+2\,(a^7+7\,a^6b+2}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-15/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d
*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*sqrt((a + b)*b)*d) - 1/4*(8*a^4 - 9*a^3*b - 2*a^2*b^2 + 2*(16*a^4 + 23*a^
3*b - 27*a^2*b^2 - 4*a*b^3)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 64*a^3*b + 53*a
^2*b^2 - 40*a*b^3 - 8*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 41*a^3*b + 27*a^
2*b^2 + 40*a*b^3 + 8*b^4)*e^(-6*d*x - 6*c) + (8*a^4 + 9*a^3*b + 24*a^2*b^2
 + 8*a*b^3)*e^(-8*d*x - 8*c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + (3*a
^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-2*d*x - 2*c) + 2*
(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-4*d
*x - 4*c) - 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^
2*b^5)*e^(-6*d*x - 6*c) - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*
a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-10*d
*x - 10*c))*d)
```

3.46.8 Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{(b\operatorname{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)^6}{\sinh(c+d\,x)^2\,(a\cosh(c+d\,x)^2+b)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(cosh(c + d*x)^6/(sinh(c + d*x)^2*(b + a*cosh(c + d*x)^2)^3), x)
```

3.47 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.47.1 Optimal result

Integrand size = 23, antiderivative size = 213

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx &= -\frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4d} \\ &\quad+\frac{(a-5b)\operatorname{arctanh}(\cosh(c+dx))}{2(a+b)^4d} \\ &\quad+\frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} \\ &\quad-\frac{(4a^2-9ab-b^2)\cosh(c+dx)}{8a(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} \\ &\quad-\frac{\cosh(c+dx)\coth^2(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)^2}\end{aligned}$$

output
```
1/2*(a-5*b)*arctanh(cosh(d*x+c))/(a+b)^4/d+1/4*(2*a-b)*b*cosh(d*x+c)/a/(a+
b)^2/d/(b+a*cosh(d*x+c)^2)^2-1/8*(4*a^2-9*a*b-b^2)*cosh(d*x+c)/a/(a+b)^3/d
/(b+a*cosh(d*x+c)^2)-1/2*cosh(d*x+c)*coth(d*x+c)^2/(a+b)/d/(b+a*cosh(d*x+c
)^2)^2-1/8*(15*a^2-10*a*b-b^2)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)
/a^(3/2)/(a+b)^4/d
```

3.47.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.53 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.46

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^5(c+dx)\left(-\frac{8b^2(a+b)^2}{a}+\frac{2b(a+b)(9a+b)(a+2b+a\cosh(2(c+dx)))}{a}+\frac{\sqrt{b}(-15a^2+10ab+b}{}\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((-8*b^2*(a + b)^2)/a + (
2*b*(a + b)*(9*a + b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a + (Sqrt[b]*(-15*a
^2 + 10*a*b + b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c
])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh
[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])
^2*Sech[c + d*x])/a^(3/2) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((Sqr
t[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] +
Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]
))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x])/a^(3/2) - (a
+ b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Csch[(c + d*x)/2]^2*Sech[c + d*x] +
 4*(a - 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Cosh[(c + d*x)/2]]*Sech
[c + d*x] - 4*(a - 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Sinh[(c + d*
x)/2]]*Sech[c + d*x] - (a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[(c +
 d*x)/2]^2*Sech[c + d*x]))/(64*(a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

3.47.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 26, 4621, 372, 440, 27, 402, 25, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^3\sin(ic+idx)^3}dx$$

$$\Big\downarrow \; 4621$$

$$\frac{\int \frac{\cosh^6(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2\left(a\cosh^2(c+dx)+b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\cosh^3(c+dx)}{2(a+b)\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\int \frac{\cosh^2(c+dx)\left(3b-(a-2b)\cosh^2(c+dx)\right)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^3}d\cosh(c+dx)}{2(a+b)}}{d}$$

$$\Big\downarrow \; 440$$

$$\frac{\frac{\cosh^3(c+dx)}{2(a+b)\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\frac{\int \frac{2\left((2a-b)b-\left(2a^2-8ba-b^2\right)\cosh^2(c+dx)\right)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{4a(a+b)}-\frac{b(2a-b)\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)^2}}{2(a+b)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\cosh^3(c+dx)}{2(a+b)\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\frac{\int \frac{(2a-b)b-\left(2a^2-8ba-b^2\right)\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{2a(a+b)}-\frac{b(2a-b)\cosh(c+dx)}{2a(a+b)\left(a\cosh^2(c+dx)+b\right)^2}}{2(a+b)}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\cosh^3(c+dx)}{2(a+b)\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\left(4a^2-9ab-b^2\right)\cosh(c+dx)}{2(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{\int -\frac{b\left((11a-b)b-\left(4a^2-9ba-b^2\right)\cosh^2(c+dx)\right)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}d\cosh(c+dx)}{2b(a+b)}}{2a(a+b)}-\frac{b(2a-b)}{2a(a+b)\left(a\ \text{co}\right.}}{2(a+b)}}{d}$$

$$\Big\downarrow \; 25$$

3.47. $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}dx$

$$\frac{\frac{\cosh^3(c+dx)}{2(a+b)(1-\cosh^2(c+dx))(a\cosh^2(c+dx)+b)^2}-\frac{\frac{\frac{\int\frac{b((11a-b)b-(4a^2-9ba-b^2)\cosh^2(c+dx))}{(1-\cosh^2(c+dx))(a\cosh^2(c+dx)+b)}d\cosh(c+dx)}{2b(a+b)}+\frac{(4a^2-9ab-b^2)\cosh(c+dx)}{2(a+b)(a\cosh^2(c+dx)+b)}}{2a(a+b)}-\frac{b(2a-b)\,\mathrm{co}\cdots}{2a(a+b)(a\cosh\cdots}}{2(a+b)}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\cosh^3(c+dx)}{2(a+b)(1-\cosh^2(c+dx))(a\cosh^2(c+dx)+b)^2}-\frac{\frac{\frac{\int\frac{(11a-b)b-(4a^2-9ba-b^2)\cosh^2(c+dx)}{(1-\cosh^2(c+dx))(a\cosh^2(c+dx)+b)}d\cosh(c+dx)}{2(a+b)}+\frac{(4a^2-9ab-b^2)\cosh(c+dx)}{2(a+b)(a\cosh^2(c+dx)+b)}}{2a(a+b)}-\frac{b(2a-b)\cosh(\cdots}{2a(a+b)(a\cosh^2(\cdots}}{2(a+b)}}{d}$$

$\downarrow$ 397

$$\frac{\frac{\cosh^3(c+dx)}{2(a+b)(1-\cosh^2(c+dx))(a\cosh^2(c+dx)+b)^2}-\frac{\frac{\frac{\frac{b(15a^2-10ab-b^2)\int\frac{1}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{a+b}-\frac{4a(a-5b)\int\frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{2(a+b)}+\frac{(4a^2-9\cdots}{2(a+b)(\cdots}}{2a(a+b)}\cdots}{2(a+b)}}{d}$$

$\downarrow$ 218

$$\frac{\frac{\cosh^3(c+dx)}{2(a+b)(1-\cosh^2(c+dx))(a\cosh^2(c+dx)+b)^2}-\frac{\frac{\frac{\frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{4a(a-5b)\int\frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a+b}}{2(a+b)}+\frac{(4a^2-9ab-b^{2}\cdots}{2(a+b)(a\cos\cdots}}{2a(a+b)}\cdots}{2(a+b)}}{d}$$

$\downarrow$ 219

$$\frac{\frac{\cosh^3(c+dx)}{2(a+b)(1-\cosh^2(c+dx))(a\cosh^2(c+dx)+b)^2}-\frac{\frac{\frac{\frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{4a(a-5b)\text{arctanh}(\cosh(c+dx))}{a+b}}{2(a+b)}+\frac{(4a^2-9ab-b^2)\cosh\cdots}{2(a+b)(a\cosh^2(c+d\cdots}}{2a(a+b)}\cdots}{2(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

3.47. $\int \frac{\text{csch}^3(c+dx)}{(a+b\,\text{sech}^2(c+dx))^3}\,dx$

output
```
(Cosh[c + d*x]^3/(2*(a + b)*(1 - Cosh[c + d*x]^2)*(b + a*Cosh[c + d*x]^2)^
2) - (-1/2*((2*a - b)*b*Cosh[c + d*x])/(a*(a + b)*(b + a*Cosh[c + d*x]^2)^
2) + (((Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqr
t[b]])/(Sqrt[a]*(a + b)) - (4*a*(a - 5*b)*ArcTanh[Cosh[c + d*x]])/(a + b))
/(2*(a + b)) + ((4*a^2 - 9*a*b - b^2)*Cosh[c + d*x])/(2*(a + b)*(b + a*Cos
h[c + d*x]^2)))/(2*a*(a + b)))/(2*(a + b)))/d
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

3.47. $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.47.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.64

$$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{8a^{3}+24a^{2}b+24a\,b^{2}+8b^{3}}-\frac{1}{8(a+b)^{3}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{(-2a+10b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^{4}}-\frac{2b\left(\frac{-\frac{\left(9a^{3}-5a^{2}b-13a\,b^{2}+b^{3}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{8a}-\frac{\left(27a^{3}-21a\cdots\right.}{\cdots}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\cdots\right.}\cdots}{\cdots}$$

3.47. $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

input
```
int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/(a^3+3*a^2*b+3*a*b^2+b^3)-1/8/(a+b)^3/tanh(
1/2*d*x+1/2*c)^2+1/4/(a+b)^4*(-2*a+10*b)*ln(tanh(1/2*d*x+1/2*c))-2*b/(a+b)
^4*((-1/8*(9*a^3-5*a^2*b-13*a*b^2+b^3)/a*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3
-21*a^2*b+29*a*b^2-3*b^3)/a*tanh(1/2*d*x+1/2*c)^4-1/8*(27*a^3+a^2*b-23*a*b
^2+3*b^3)/a*tanh(1/2*d*x+1/2*c)^2-1/8*(9*a^3+17*a^2*b+7*a*b^2-b^3)/a)/(tan
h(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/16*(15*a^2-10*a*b-b^2)/a/(a*b)^(1/2)*arcta
n(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))))
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10990 vs. $2(195) = 390$.

Time = 0.55 (sec) , antiderivative size = 20341, normalized size of antiderivative = 95.50

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.47.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

3.47.7 Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(b\operatorname{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/2*(a - 5*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^4*d + 4*a^3*b*d + 6*a^2*b^2
*d + 4*a*b^3*d + b^4*d) - 1/2*(a - 5*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^4
*d + 4*a^3*b*d + 6*a^2*b^2*d + 4*a*b^3*d + b^4*d) - 1/4*((4*a^3*e^(11*c) -
 9*a^2*b*e^(11*c) - a*b^2*e^(11*c))*e^(11*d*x) + (20*a^3*e^(9*c) + 23*a^2*
b*e^(9*c) - 29*a*b^2*e^(9*c) + 4*b^3*e^(9*c))*e^(9*d*x) + 2*(20*a^3*e^(7*c
) + 57*a^2*b*e^(7*c) + 47*a*b^2*e^(7*c) - 2*b^3*e^(7*c))*e^(7*d*x) + 2*(20
*a^3*e^(5*c) + 57*a^2*b*e^(5*c) + 47*a*b^2*e^(5*c) - 2*b^3*e^(5*c))*e^(5*d
*x) + (20*a^3*e^(3*c) + 23*a^2*b*e^(3*c) - 29*a*b^2*e^(3*c) + 4*b^3*e^(3*c
))*e^(3*d*x) + (4*a^3*e^c - 9*a^2*b*e^c - a*b^2*e^c)*e^(d*x))/(a^6*d + 3*a
^5*b*d + 3*a^4*b^2*d + a^3*b^3*d + (a^6*d*e^(12*c) + 3*a^5*b*d*e^(12*c) +
3*a^4*b^2*d*e^(12*c) + a^3*b^3*d*e^(12*c))*e^(12*d*x) + 2*(a^6*d*e^(10*c)
+ 7*a^5*b*d*e^(10*c) + 15*a^4*b^2*d*e^(10*c) + 13*a^3*b^3*d*e^(10*c) + 4*a
^2*b^4*d*e^(10*c))*e^(10*d*x) - (a^6*d*e^(8*c) + 3*a^5*b*d*e^(8*c) - 13*a^
4*b^2*d*e^(8*c) - 47*a^3*b^3*d*e^(8*c) - 48*a^2*b^4*d*e^(8*c) - 16*a*b^5*d
*e^(8*c))*e^(8*d*x) - 4*(a^6*d*e^(6*c) + 7*a^5*b*d*e^(6*c) + 23*a^4*b^2*d*
e^(6*c) + 37*a^3*b^3*d*e^(6*c) + 28*a^2*b^4*d*e^(6*c) + 8*a*b^5*d*e^(6*c))
*e^(6*d*x) - (a^6*d*e^(4*c) + 3*a^5*b*d*e^(4*c) - 13*a^4*b^2*d*e^(4*c) - 4
7*a^3*b^3*d*e^(4*c) - 48*a^2*b^4*d*e^(4*c) - 16*a*b^5*d*e^(4*c))*e^(4*d*x)
 + 2*(a^6*d*e^(2*c) + 7*a^5*b*d*e^(2*c) + 15*a^4*b^2*d*e^(2*c) + 13*a^3*b^
3*d*e^(2*c) + 4*a^2*b^4*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/32*((15*a...
```

3.47.8 Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(b\operatorname{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)^6}{\sinh\left(c+d\,x\right)^3\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(cosh(c + d*x)^6/(sinh(c + d*x)^3*(b + a*cosh(c + d*x)^2)^3), x)
```

3.48 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.48.1 Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = -\frac{5(3a-4b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d} + \frac{(a-2b)\coth(c+dx)}{(a+b)^4d} - \frac{\coth^3(c+dx)}{3(a+b)^3d} - \frac{ab\tanh(c+dx)}{4(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(7a-4b)b\tanh(c+dx)}{8(a+b)^4d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
(a-2*b)*coth(d*x+c)/(a+b)^4/d-1/3*coth(d*x+c)^3/(a+b)^3/d-5/8*(3*a-4*b)*ar
ctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(9/2)/d-1/4*a*b*tanh(
d*x+c)/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*(7*a-4*b)*b*tanh(d*x+c)/(a+b)
^4/d/(a+b-b*tanh(d*x+c)^2)
```

3.48.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 985 vs. $2(165) = 330$.

Time = 6.34 (sec) , antiderivative size = 985, normalized size of antiderivative = 5.97

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx =$$

$$-\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^6(c+dx)\left(\frac{480(3a-4b)b\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^{}}}\right.}{}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/6144*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((480*(3*a - 4*b)
*b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sin
h[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a
*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c
] - Sinh[c])^4]) + (Csch[c]*Csch[c + d*x]^3*Sech[2*c]*(4*(44*a^4 + 122*a^3
*b + 63*a^2*b^2 + 126*a*b^3 + 36*b^4)*Sinh[d*x] + (-96*a^4 - 71*a^3*b + 34
4*a^2*b^2 - 1208*a*b^3 + 48*b^4)*Sinh[3*d*x] + 224*a^4*Sinh[2*c - d*x] + 5
76*a^3*b*Sinh[2*c - d*x] + 124*a^2*b^2*Sinh[2*c - d*x] - 2184*a*b^3*Sinh[2
*c - d*x] + 144*b^4*Sinh[2*c - d*x] - 224*a^4*Sinh[2*c + d*x] - 657*a^3*b*
Sinh[2*c + d*x] - 538*a^2*b^2*Sinh[2*c + d*x] + 984*a*b^3*Sinh[2*c + d*x]
+ 144*b^4*Sinh[2*c + d*x] + 176*a^4*Sinh[4*c + d*x] + 569*a^3*b*Sinh[4*c +
 d*x] + 666*a^2*b^2*Sinh[4*c + d*x] + 1704*a*b^3*Sinh[4*c + d*x] - 144*b^4
*Sinh[4*c + d*x] + 48*a^4*Sinh[2*c + 3*d*x] + 111*a^3*b*Sinh[2*c + 3*d*x]
+ 360*a^2*b^2*Sinh[2*c + 3*d*x] + 312*a*b^3*Sinh[2*c + 3*d*x] - 48*b^4*Sin
h[2*c + 3*d*x] - 96*a^4*Sinh[4*c + 3*d*x] - 152*a^3*b*Sinh[4*c + 3*d*x] +
146*a^2*b^2*Sinh[4*c + 3*d*x] - 728*a*b^3*Sinh[4*c + 3*d*x] - 48*b^4*Sinh[
4*c + 3*d*x] + 48*a^4*Sinh[6*c + 3*d*x] + 192*a^3*b*Sinh[6*c + 3*d*x] + 55
8*a^2*b^2*Sinh[6*c + 3*d*x] - 168*a*b^3*Sinh[6*c + 3*d*x] + 48*b^4*Sinh[6*
c + 3*d*x] + 16*a^4*Sinh[2*c + 5*d*x] - 598*a^2*b^2*Sinh[2*c + 5*d*x] + 48
*a*b^3*Sinh[2*c + 5*d*x] + 72*a^3*b*Sinh[4*c + 5*d*x] + 150*a^2*b^2*Sin...
```

3.48.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4620, 361, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^4(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 361$$

$$\frac{\frac{1}{4}b\int \frac{\coth^4(c+dx)\left(-\frac{3a\tanh^4(c+dx)}{(a+b)^3}-\frac{4a\tanh^2(c+dx)}{b(a+b)^2}+\frac{4}{b(a+b)}\right)}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)-\frac{ab\tanh(c+dx)}{4(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \ 1582$$

$$\frac{\frac{1}{4}b\left(\frac{\int \frac{\coth^4(c+dx)\left(-\frac{(7a-4b)b^2\tanh^4(c+dx)}{a+b}-8(a-b)b\tanh^2(c+dx)+8b(a+b)\right)}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{2b^2(a+b)^3}-\frac{(7a-4b)\tanh(c+dx)}{2(a+b)^4\left(a-b\tanh^2(c+dx)+b\right)}\right)-\frac{ab\tan}{4(a+b)^3(a-b\text{t}}}{d}$$

$$\Big\downarrow \ 1584$$

$$\frac{\frac{1}{4}b\left(\frac{\int \left(8b\coth^4(c+dx)+\frac{8b(2b-a)\coth^2(c+dx)}{a+b}-\frac{5b^2(4b-3a)}{(a+b)\left(b\tanh^2(c+dx)-a-b\right)}\right)d\tanh(c+dx)}{2b^2(a+b)^3}-\frac{(7a-4b)\tanh(c+dx)}{2(a+b)^4\left(a-b\tanh^2(c+dx)+b\right)}\right)-\frac{ab\,\text{ta}}{4(a+b)^3(a-b}}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{4}b\left(\frac{-\frac{5b^{3/2}(3a-4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{8b(a-2b)\coth(c+dx)}{a+b}-\frac{8}{3}b\coth^3(c+dx)}{2b^2(a+b)^3}-\frac{(7a-4b)\tanh(c+dx)}{2(a+b)^4(a-b\tanh^2(c+dx)+b)}\right)-\frac{ab\tanh(}{4(a+b)^3(a-b\tan}}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/4*(a*b*Tanh[c + d*x])/((a + b)^3*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(
((-5*(3*a - 4*b)*b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a
+ b)^(3/2) + (8*(a - 2*b)*b*Coth[c + d*x])/(a + b) - (8*b*Coth[c + d*x]^3)
/3)/(2*b^2*(a + b)^3) - ((7*a - 4*b)*Tanh[c + d*x])/(2*(a + b)^4*(a + b -
b*Tanh[c + d*x]^2))))/4)/d
```

3.48.3.1 Defintions of rubi rules used

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

3.48. $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1442 vs. $2(149) = 298$.

Time = 0.30 (sec) , antiderivative size = 1443, normalized size of antiderivative = 8.75

Expression too large to display

input
```
int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

output
```
-1/24/d/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)*tanh(1/2*d*x+1/2*c)^3*a-1/24/d/(a^
3+3*a^2*b+3*a*b^2+b^3)/(a+b)*tanh(1/2*d*x+1/2*c)^3*b+3/8/d/(a^3+3*a^2*b+3*
a*b^2+b^3)/(a+b)*tanh(1/2*d*x+1/2*c)*a-9/8/d/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+
b)*tanh(1/2*d*x+1/2*c)*b-1/24/d/(a+b)^3/tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)^
4/tanh(1/2*d*x+1/2*c)*a-9/8/d/(a+b)^4/tanh(1/2*d*x+1/2*c)*b-9/4/d*b/(a+b)^
4/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7*a^2-5/4/d*b^2/(a
+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7*a+1/d*b^3/(a
+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-27/4/d*b/(a+
b)^4/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5*a^2+13/4/d*b^
2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+
1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-1/d*b^
3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+
1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-27/4/d*b
/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1
/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3*a^2+13/4/
d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1...
```

3.48.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7442 vs. $2(155) = 310$.

Time = 0.44 (sec) , antiderivative size = 15161, normalized size of antiderivative = 91.88

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.48.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(155) = 310$.

Time = 0.41 (sec) , antiderivative size = 782, normalized size of antiderivative = 4.74

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \frac{5\,(3\,ab-4\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)b}d}$$
$$+\frac{16}{12\,(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(a^7+12\,a^6b+38\,a^5b^2+52\,a^4b^3+33\,a^3b^4+8\,a^2b^5)e^{(-2\,dx-2\,c)}}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
5/16*(3*a*b - 4*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b)
)/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 1/12*(16*a^4 - 83*a^3*b + 6*
a^2*b^2 + 2*(8*a^4 - 299*a^2*b^2 + 24*a*b^3)*e^(-2*d*x - 2*c) - (96*a^4 +
71*a^3*b - 344*a^2*b^2 + 1208*a*b^3 - 48*b^4)*e^(-4*d*x - 4*c) - 4*(56*a^4
 + 144*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)*e^(-6*d*x - 6*c) - (176*a^
4 + 569*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4)*e^(-8*d*x - 8*c) - 6*(
8*a^4 + 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*e^(-10*d*x - 10*c) - 15*
(3*a^3*b - 4*a^2*b^2)*e^(-12*d*x - 12*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4 + (a^7 + 12*a^6*b + 38*a^5*b^2 + 52*a^4*b^3 + 33*a^3*b^4
 + 8*a^2*b^5)*e^(-2*d*x - 2*c) - (3*a^7 + 20*a^6*b + 34*a^5*b^2 - 4*a^4*b^
3 - 61*a^3*b^4 - 56*a^2*b^5 - 16*a*b^6)*e^(-4*d*x - 4*c) - (3*a^7 + 28*a^6
*b + 130*a^5*b^2 + 300*a^4*b^3 + 355*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*e^(
-6*d*x - 6*c) + (3*a^7 + 28*a^6*b + 130*a^5*b^2 + 300*a^4*b^3 + 355*a^3*b^
4 + 208*a^2*b^5 + 48*a*b^6)*e^(-8*d*x - 8*c) + (3*a^7 + 20*a^6*b + 34*a^5*
b^2 - 4*a^4*b^3 - 61*a^3*b^4 - 56*a^2*b^5 - 16*a*b^6)*e^(-10*d*x - 10*c) -
 (a^7 + 12*a^6*b + 38*a^5*b^2 + 52*a^4*b^3 + 33*a^3*b^4 + 8*a^2*b^5)*e^(-1
2*d*x - 12*c) - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*e^(-14*d
*x - 14*c))*d)
```

3.48.8 Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^4}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\cosh(c+d\,x)^6}{\sinh\left(c+d\,x\right)^4\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(cosh(c + d*x)^6/(sinh(c + d*x)^4*(b + a*cosh(c + d*x)^2)^3), x)
```

3.48. $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.49 $\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$

3.49.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

output
```
1/8*(3*a+4*b)*x+1/8*(3*a+4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)^
3*sinh(d*x+c)/d
```

3.49.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\frac{4(3a+4b)(c+dx)+8(a+b)\sinh(2(c+dx))+a\sinh(4(c+dx))}{32d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

output
```
(4*(3*a + 4*b)*(c + d*x) + 8*(a + b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x
)])/(32*d)
```

3.49.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)^2}{\csc\left(ic+idx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{ 4533}$$

$$\frac{1}{4}(3a+4b)\int \cosh^2(c+dx)dx+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{1}{4}(3a+4b)\int \sin\left(ic+idx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{1}{4}(3a+4b)\left(\frac{\int 1dx}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{4}(3a+4b)\left(\frac{\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x}{2}\right)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

output
```
(a*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + ((3*a + 4*b)*(x/2 + (Cosh[c + d*
x]*Sinh[c + d*x])/(2*d)))/4
```

3.49.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.49.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{(8a+8b)\sinh(2dx+2c)+a\sinh(4dx+4c)+12dx\left(a+\frac{4b}{3}\right)}{32d}$	44
derivativedivides	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	66
default	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	66
risch	$\frac{3ax}{8}+\frac{bx}{2}+\frac{a\,\mathrm{e}^{4dx+4c}}{64d}+\frac{\mathrm{e}^{2dx+2c}a}{8d}+\frac{\mathrm{e}^{2dx+2c}b}{8d}-\frac{\mathrm{e}^{-2dx-2c}a}{8d}-\frac{\mathrm{e}^{-2dx-2c}b}{8d}-\frac{a\,\mathrm{e}^{-4dx-4c}}{64d}$	100

input
```
int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/32*((8*a+8*b)*sinh(2*d*x+2*c)+a*sinh(4*d*x+4*c)+12*d*x*(a+4/3*b))/d
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$
$$=\frac{a\cosh(dx+c)\sinh(dx+c)^3+(3a+4b)dx+\left(a\cosh(dx+c)^3+4(a+b)\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*(a*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a + 4*b)*d*x + (a*cosh(d*x + c)^
3 + 4*(a + b)*cosh(d*x + c))*sinh(d*x + c))/d
```

3.49.6 Sympy [F]

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\int \left(a+b\,\text{sech}^2(c+dx)\right)\cosh^4(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x)**4, x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$
$$=\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{8}b\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) + 1/8*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2
*c)/d)
```

3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(55) = 110$.

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.90

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$
$$=\frac{8\,(dx+c)(3\,a+4\,b)+ae^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx+2\,c)}-\left(18\,ae^{(4\,dx+4\,c)}+24\,be^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+}\right.}{64\,d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
1/64*(8*(d*x + c)*(3*a + 4*b) + a*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) +
8*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) + 24*b*e^(4*d*x + 4*c) + 8*a*e
^(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + a)*e^(-4*d*x - 4*c))/d
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\frac{\frac{a\sinh(2\,c+2\,d\,x)}{4}+\frac{a\sinh(4\,c+4\,d\,x)}{32}+\frac{b\sinh(2\,c+2\,d\,x)}{4}}{d}+\frac{3\,a\,x}{8}+\frac{b\,x}{2}$$

input
```
int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

output
```
((a*sinh(2*c + 2*d*x))/4 + (a*sinh(4*c + 4*d*x))/32 + (b*sinh(2*c + 2*d*x)
)/4)/d + (3*a*x)/8 + (b*x)/2
```

3.50 $\int \cosh^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx$

3.50.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \cosh^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx=\frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}$$

output
```
(a+b)*sinh(d*x+c)/d+1/3*a*sinh(d*x+c)^3/d
```

3.50.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \cosh^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx=\frac{b\cosh(dx)\sinh(c)}{d}+\frac{b\cosh(c)\sinh(dx)}{d}+\frac{a\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

output
```
(b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d + (a*Sinh[c + d*x])/d +
(a*Sinh[c + d*x]^3)/(3*d)
```

3.50.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx) \left(a+b\text{sech}^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)^2}{\csc\left(ic+idx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4532}$$

$$\int \cosh(c+dx)\left(a\cosh^2(c+dx)+b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ic+idx+\frac{\pi}{2}\right)\left(b+a\sin\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3492}$$

$$\frac{i\int \left(a\sinh^2(c+dx)+a+b\right)d(-i\sinh(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-i(a+b)\sinh(c+dx)-\frac{1}{3}ia\sinh^3(c+dx)\right)}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

output
```
(I*((-I)*(a + b)*Sinh[c + d*x] - (I/3)*a*Sinh[c + d*x]^3))/d
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

3.50.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{\sinh(3dx+3c)a+9\sinh(dx+c)\left(a+\frac{4b}{3}\right)}{12d}$	31
derivativedivides	$\frac{a\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+b\sinh(dx+c)}{d}$	34
default	$\frac{a\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+b\sinh(dx+c)}{d}$	34
risch	$\frac{a\,e^{3dx+3c}}{24d}+\frac{3\,e^{dx+c}a}{8d}+\frac{e^{dx+c}b}{2d}-\frac{3\,e^{-dx-c}a}{8d}-\frac{e^{-dx-c}b}{2d}-\frac{a\,e^{-3dx-3c}}{24d}$	86

input
```
int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(sinh(3*d*x+3*c)*a+9*sinh(d*x+c)*(a+4/3*b))/d
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{a\sinh\left(dx+c\right)^3+3\left(a\cosh\left(dx+c\right)^2+3\,a+4\,b\right)\sinh\left(dx+c\right)}{12\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/12*(a*sinh(d*x + c)^3 + 3*(a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))
/d
```

3.50.6 Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)\cosh^3\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x)**3, x)
```

3.50.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(28) = 56.

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.83

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/24*a*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x
 - 3*c)/d) + 1/2*b*(e^(d*x + c)/d - e^(-d*x - c)/d)
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(28) = 56$.

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.40

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$
$$=\frac{ae^{(3\,dx+3\,c)}+9\,ae^{(dx+c)}+12\,be^{(dx+c)}-\left(9\,ae^{(2\,dx+2\,c)}+12\,be^{(2\,dx+2\,c)}+a\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
1/24*(a*e^(3*d*x + 3*c) + 9*a*e^(d*x + c) + 12*b*e^(d*x + c) - (9*a*e^(2*d
*x + 2*c) + 12*b*e^(2*d*x + 2*c) + a)*e^(-3*d*x - 3*c))/d
```

3.50.9 Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$
$$=\frac{3\,a\sinh(c+d\,x)+3\,b\sinh(c+d\,x)+a\sinh(c+d\,x)^3}{3\,d}$$

input
```
int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

output
```
(3*a*sinh(c + d*x) + 3*b*sinh(c + d*x) + a*sinh(c + d*x)^3)/(3*d)
```

3.51 $\int \cosh^2(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)\,dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \cosh^2(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)\,dx=\frac{1}{2}(a+2b)x+\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
1/2*(a+2*b)*x+1/2*a*cosh(d*x+c)*sinh(d*x+c)/d
```

3.51.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cosh^2(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)\,dx=bx+\frac{a(c+dx)}{2d}+\frac{a\sinh(2(c+dx))}{4d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

output
```
b*x + (a*(c + d*x))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

3.51.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)^2}{\csc\left(ic+idx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{2}(a+2b)\int 1dx+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}x(a+2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b)*x)/2 + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

3.51.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.51.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{a\sinh(2dx+2c)+2(a+2b)xd}{4d}$	27
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+(dx+c)b}{d}$	37
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+(dx+c)b}{d}$	37
risch	$\frac{ax}{2}+bx+\frac{\mathrm{e}^{2dx+2c}a}{8d}-\frac{\mathrm{e}^{-2dx-2c}a}{8d}$	39

input
```
int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*sinh(2*d*x+2*c)+2*(a+2*b)*x*d)/d
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \cosh^2(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx=\frac{(a+2\,b)dx+a\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*((a + 2*b)*d*x + a*cosh(d*x + c)*sinh(d*x + c))/d
```

3.51.6 Sympy [A] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \cosh^2(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx$$
$$=a\left(\begin{cases}-\frac{x\sinh^2\left(c+dx\right)}{2}+\frac{x\cosh^2\left(c+dx\right)}{2}+\frac{\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x\cosh^2\left(c\right) & \text{otherwise}\end{cases}\right)$$
$$+b\left(\begin{cases}x & \text{for } |x|<1\\ G_{2,2}^{1,1}\left(\begin{matrix}1 & 2\\ 1 & 0\end{matrix}\middle| x\right)+G_{2,2}^{0,2}\left(\begin{matrix}2,1 & \\ & 1,0\end{matrix}\middle| x\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

output
```
a*Piecewise((-x*sinh(c + d*x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*
cosh(c + d*x)/(2*d), Ne(d, 0)), (x*cosh(c)**2, True)) + b*Piecewise((x, Ab
s(x) < 1), (meijerg(((1,), (2,)), ((1,), (0,)), x) + meijerg(((2, 1), ()),
 ((), (1, 0)), x), True))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \cosh^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\frac{1}{8}\,a\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+bx$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + b*x
```

3.51.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.13

$$\int \cosh^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$=\frac{4\,(dx+c)(a+2\,b)+ae^{(2\,dx+2\,c)}-\left(2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(4*(d*x + c)*(a + 2*b) + a*e^(2*d*x + 2*c) - (2*a*e^(2*d*x + 2*c) + 4*
b*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c))/d
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\frac{a\,x}{2}+b\,x+\frac{a\sinh(2\,c+2\,d\,x)}{4\,d}$$

input
```
int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

output
```
(a*x)/2 + b*x + (a*sinh(2*c + 2*d*x))/(4*d)
```

3.52 $\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$

3.52.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\frac{b\arctan(\sinh(c+dx))}{d}+\frac{a\sinh(c+dx)}{d}$$

output
```
b*arctan(sinh(d*x+c))/d+a*sinh(d*x+c)/d
```

3.52.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\frac{b\arctan(\sinh(c+dx))}{d}+\frac{a\cosh(dx)\sinh(c)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

output
```
(b*ArcTan[Sinh[c + d*x]])/d + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*
x])/d
```

3.52.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)^2}{\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4533$$

$$b\int \text{sech}(c+dx)dx+\frac{a\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sinh(c+dx)}{d}+b\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 4257$$

$$\frac{a\sinh(c+dx)}{d}+\frac{b\arctan(\sinh(c+dx))}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

output
```
(b*ArcTan[Sinh[c + d*x]])/d + (a*Sinh[c + d*x])/d
```

3.52.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.52.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{a\sinh(dx+c)+2b\arctan(\mathrm{e}^{dx+c})}{d}$	24
default	$\frac{a\sinh(dx+c)+2b\arctan(\mathrm{e}^{dx+c})}{d}$	24
parallelrisch	$\frac{ib\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)-ib\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+a\sinh(dx+c)}{d}$	48
risch	$\frac{\mathrm{e}^{dx+c}a}{2d}-\frac{\mathrm{e}^{-dx-c}a}{2d}+\frac{ib\ln(\mathrm{e}^{dx+c}+i)}{d}-\frac{ib\ln(\mathrm{e}^{dx+c}-i)}{d}$	63

input
```
int(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*sinh(d*x+c)+2*b*arctan(exp(d*x+c)))
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.88

$$\int\cosh(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$=\frac{a\cosh(dx+c)^2+2\,a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+4\,(b\cosh(dx+c)+b\sinh(dx+c))}{2\,(d\cosh(dx+c)+d\sinh(dx+c))}$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)
^2 + 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
 + c)) - a)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

3.52.6 Sympy [F]

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\int \left(a+b\operatorname{sech}^2{(c+dx)}\right)\cosh{(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x), x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=-\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{a\sinh\left(dx+c\right)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-2*b*arctan(e^(-d*x - c))/d + a*sinh(d*x + c)/d
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\frac{4\,b\arctan\left(e^{(dx+c)}\right)+ae^{(dx+c)}-ae^{(-dx-c)}}{2\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(4*b*arctan(e^(d*x + c)) + a*e^(d*x + c) - a*e^(-d*x - c))/d
```

3.52.9 Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\frac{2\,\mathrm{atan}\left(\frac{b\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}-\frac{a\,e^{-c-d\,x}}{2\,d}+\frac{a\,e^{c+d\,x}}{2\,d}$$

input
```
int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

output
```
(2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2
)^(1/2) - (a*exp(- c - d*x))/(2*d) + (a*exp(c + d*x))/(2*d)
```

3.53 $\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

3.53.1 Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\frac{(2a+b)\arctan(\sinh(c+dx))}{2d}+\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*(2*a+b)*arctan(sinh(d*x+c))/d+1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

3.53.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\frac{a\arctan(\sinh(c+dx))}{d}+\frac{b\arctan(\sinh(c+dx))}{2d}+\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

output
```
(a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c
+ d*x]*Tanh[c + d*x])/(2*d)
```

3.53.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ic+idx+\frac{\pi}{2}\right)\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{2}(2a+b)\int \text{sech}(c+dx)dx+\frac{b\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{1}{2}(2a+b)\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{(2a+b)\arctan(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

input
```
Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

output
```
((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/
(2*d)
```

3.53.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.53.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
parts	$\frac{a\arctan(\sinh(dx+c))}{d}+\frac{b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
parallelrisch	$\frac{-i\left(\frac{b}{2}+a\right)(1+\cosh(2dx+2c))\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+i\left(\frac{b}{2}+a\right)(1+\cosh(2dx+2c))\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)+b\sinh(dx+c)}{d(1+\cosh(2dx+2c))}$
risch	$\frac{b\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(e^{2dx+2c}+1\right)^{2}}+\frac{i\ln\left(e^{dx+c}+i\right)a}{d}+\frac{ib\ln\left(e^{dx+c}+i\right)}{2d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{d}-\frac{ib\ln\left(e^{dx+c}-i\right)}{2d}$

input
```
int(sech(d*x+c)*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a*arctan(exp(d*x+c))+b*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+
c))))
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 321, normalized size of antiderivative = 8.02

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$=\frac{b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3+\left((2\,a+b)\cosh(dx+c)^4+4\,(2\,a+\right.}{}$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
 + ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3
+ (2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)
*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^3
 + (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c)
+ sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x +
c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x +
c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 +
 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

3.53.6 Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \int \left(a+b\operatorname{sech}^2(c+dx)\right)\operatorname{sech}(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*sech(c + d*x), x)
```

3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.02

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$= -b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(36) = 72.

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(2\,a+b)+\frac{4\,b\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4\,d}$$

input

```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output

```
1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(2*a + b) + 4
*b*(e^(d*x + c) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.10

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\left(2\,a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{4\,a^2+4\,a\,b+b^2}}\right)\,\sqrt{4\,a^2+4\,a\,b+b^2}}{\sqrt{d^2}} + \frac{b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{2\,b\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input

```
int((a + b/cosh(c + d*x)^2)/cosh(c + d*x),x)
```

output

```
(atan((exp(d*x)*exp(c)*(2*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(4*a*b + 4*a^
2 + b^2)^(1/2)))*(4*a*b + 4*a^2 + b^2)^(1/2))/(d^2)^(1/2) + (b*exp(c + d*x
))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x)
+ exp(4*c + 4*d*x) + 1))
```

3.54 $\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}$$

output
```
(a+b)*tanh(d*x+c)/d-1/3*b*tanh(d*x+c)^3/d
```

3.54.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\frac{a\tanh(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x])/d + (b*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

3.54.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4534, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right) \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2 \left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)^2\right) dx$$

$$\Big\downarrow \; 4534$$

$$\frac{1}{3}(3a+2b)\int \operatorname{sech}^2(c+dx)dx+\frac{b\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}+\frac{1}{3}(3a+2b)\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \; 4254$$

$$\frac{b\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}+\frac{i(3a+2b)\int 1d(-i\tanh(c+dx))}{3d}$$

$$\Big\downarrow \; 24$$

$$\frac{(3a+2b)\tanh(c+dx)}{3d}+\frac{b\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

output
```
((3*a + 2*b)*Tanh[c + d*x])/(3*d) + (b*Sech[c + d*x]^2*Tanh[c + d*x])/(3*d
)
```

3.54.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

3.54.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{a\tanh(dx+c)+b\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$	34
default	$\frac{a\tanh(dx+c)+b\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$	34
parts	$\frac{a\tanh(dx+c)}{d}+\frac{b\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$	36
parallelrisch	$\frac{(3a+2b)\sinh(3dx+3c)+3\sinh(dx+c)(a+2b)}{3d(\cosh(3dx+3c)+3\cosh(dx+c))}$	57
risch	$-\frac{2\left(3a\,\mathrm{e}^{4dx+4c}+6\,\mathrm{e}^{2dx+2c}a+6b\,\mathrm{e}^{2dx+2c}+3a+2b\right)}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^3}$	62

input
```
int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*tanh(d*x+c)+b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 5.27

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx =$$

$$-\frac{4\left((3a+b)\cosh(dx+c)^2-2b\cosh(dx+c)\sinh(dx\right.}{3\left(d\cosh(dx+c)^4+4d\cosh(dx+c)\sinh(dx+c)^3+d\sinh(dx+c)^4+4d\cosh(dx+c)^2+2\left(3d\cos\right.\right.}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-4/3*((3*a + b)*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) + (3*a +
 b)*sinh(d*x + c)^2 + 3*a + 3*b)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*si
nh(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x
+ c)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*si
nh(d*x + c) + 3*d)
```

3.54.6 Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \int \left(a+b\operatorname{sech}^2(c+dx)\right)\operatorname{sech}^2(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**2, x)
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(28) = 56$.

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.73

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$=\frac{4}{3}b\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+\frac{2a}{d(e^{(-2dx-2c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
4/3*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = -\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+3\,a+2\,b\right)}{3\,d(e^{(2\,dx+2\,c)}+1)^3}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
-2/3*(3*a*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 3*
a + 2*b)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = -\frac{2\left(3\,a+2\,b+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}+3\,a\,\mathrm{e}^{4\,c+4\,d\,x}+6\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{3\,d\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^3}$$

input
```
int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^2,x)
```

output
```
-(2*(3*a + 2*b + 6*a*exp(2*c + 2*d*x) + 3*a*exp(4*c + 4*d*x) + 6*b*exp(2*c
 + 2*d*x)))/(3*d*(exp(2*c + 2*d*x) + 1)^3)
```

3.55 $\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{(4a+3b)\arctan(\sinh(c+dx))}{8d} + \frac{(4a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output
```
1/8*(4*a+3*b)*arctan(sinh(d*x+c))/d+1/8*(4*a+3*b)*sech(d*x+c)*tanh(d*x+c)/
d+1/4*b*sech(d*x+c)^3*tanh(d*x+c)/d
```

3.55.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{a\arctan(\sinh(c+dx))}{2d} + \frac{3b\arctan(\sinh(c+dx))}{8d} + \frac{a\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{3b\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

output
```
(a*ArcTan[Sinh[c + d*x]])/(2*d) + (3*b*ArcTan[Sinh[c + d*x]])/(8*d) + (a*S
ech[c + d*x]*Tanh[c + d*x])/(2*d) + (3*b*Sech[c + d*x]*Tanh[c + d*x])/(8*d
) + (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

3.55.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4534, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ic+idx+\frac{\pi}{2}\right)^3 \left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{4}(4a+3b)\int \operatorname{sech}^3(c+dx)dx+\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}+\frac{1}{4}(4a+3b)\int \csc\left(ic+idx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}(4a+3b)\left(\frac{1}{2}\int \operatorname{sech}(c+dx)dx+\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)+\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}+\frac{1}{4}(4a+3b)\left(\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}+\frac{1}{2}\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}(4a+3b)\left(\frac{\arctan(\sinh(c+dx))}{2d}+\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)+\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

output
```
(b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d) + ((4*a + 3*b)*(ArcTan[Sinh[c + d*
x]]/(2*d) + (Sech[c + d*x]*Tanh[c + d*x])/(2*d)))/4
```

3.55.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.55.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(\left(\frac{\operatorname{sech}(dx+c)^{3}}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)}{d}$
default	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(\left(\frac{\operatorname{sech}(dx+c)^{3}}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)}{d}$
parts	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}+\frac{b\left(\left(\frac{\operatorname{sech}(dx+c)^{3}}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)}{d}$
parallelrisch	$\frac{-8i\left(\frac{3}{4}+\frac{\cosh(4dx+4c)}{4}+\cosh(2dx+2c)\right)\left(a+\frac{3b}{4}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+8i\left(\frac{3}{4}+\frac{\cosh(4dx+4c)}{4}+\cosh(2dx+2c)\right)\left(a+\frac{3b}{4}\right)\ln}{4d(\cosh(4dx+4c)+4\cosh(2dx+2c)+3)}$
risch	$\frac{e^{dx+c}\left(4a\,e^{6dx+6c}+3b\,e^{6dx+6c}+4a\,e^{4dx+4c}+11b\,e^{4dx+4c}-4\,e^{2dx+2c}a-11b\,e^{2dx+2c}-4a-3b\right)}{4d\left(e^{2dx+2c}+1\right)^{4}}+\frac{i\ln\left(e^{dx+c}+i\right)a}{2d}+\frac{3ib}{}$

input
```
int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b*((1/4*sech(d*x+c
)^3+3/8*sech(d*x+c))*tanh(d*x+c)+3/4*arctan(exp(d*x+c))))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 1112, normalized size of antiderivative = 15.89

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*((4*a + 3*b)*cosh(d*x + c)^7 + 7*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x +
c)^6 + (4*a + 3*b)*sinh(d*x + c)^7 + (4*a + 11*b)*cosh(d*x + c)^5 + (21*(4
*a + 3*b)*cosh(d*x + c)^2 + 4*a + 11*b)*sinh(d*x + c)^5 + 5*(7*(4*a + 3*b)
*cosh(d*x + c)^3 + (4*a + 11*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a + 11
*b)*cosh(d*x + c)^3 + (35*(4*a + 3*b)*cosh(d*x + c)^4 + 10*(4*a + 11*b)*co
sh(d*x + c)^2 - 4*a - 11*b)*sinh(d*x + c)^3 + (21*(4*a + 3*b)*cosh(d*x + c
)^5 + 10*(4*a + 11*b)*cosh(d*x + c)^3 - 3*(4*a + 11*b)*cosh(d*x + c))*sinh
(d*x + c)^2 + ((4*a + 3*b)*cosh(d*x + c)^8 + 8*(4*a + 3*b)*cosh(d*x + c)*s
inh(d*x + c)^7 + (4*a + 3*b)*sinh(d*x + c)^8 + 4*(4*a + 3*b)*cosh(d*x + c)
^6 + 4*(7*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*sinh(d*x + c)^6 + 8*(7*
(4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^5
 + 6*(4*a + 3*b)*cosh(d*x + c)^4 + 2*(35*(4*a + 3*b)*cosh(d*x + c)^4 + 30*
(4*a + 3*b)*cosh(d*x + c)^2 + 12*a + 9*b)*sinh(d*x + c)^4 + 8*(7*(4*a + 3*
b)*cosh(d*x + c)^5 + 10*(4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(4*a + 3*b)*cosh(d*x + c)^2 + 4*(7*(4*a + 3*b
)*cosh(d*x + c)^6 + 15*(4*a + 3*b)*cosh(d*x + c)^4 + 9*(4*a + 3*b)*cosh(d*
x + c)^2 + 4*a + 3*b)*sinh(d*x + c)^2 + 8*((4*a + 3*b)*cosh(d*x + c)^7 + 3
*(4*a + 3*b)*cosh(d*x + c)^5 + 3*(4*a + 3*b)*cosh(d*x + c)^3 + (4*a + 3*b)
*cosh(d*x + c))*sinh(d*x + c) + 4*a + 3*b)*arctan(cosh(d*x + c) + sinh(d*x
 + c)) - (4*a + 3*b)*cosh(d*x + c) + (7*(4*a + 3*b)*cosh(d*x + c)^6 + 5...
```

3.55.6 Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\int\left(a+b\operatorname{sech}^2\left(c+dx\right)\right)\operatorname{sech}^3\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**3, x)
```

3.55.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(64) = 128$.

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx =$$

$$-\frac{1}{4}b\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3e^{(-dx-c)}+11e^{(-3dx-3c)}-11e^{(-5dx-5c)}-3e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)$$
$$-a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) -
 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^
(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1)))
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.23

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$

$$=\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)(4a+3b)+\frac{4\left(4a\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+16a\left(e^{(dx+c)}\right.\right.}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16d}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a + 3*b)
+ 4*(4*a*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b*(e^(d*x + c) - e^(-d*x - c))
^3 + 16*a*(e^(d*x + c) - e^(-d*x - c)) + 20*b*(e^(d*x + c) - e^(-d*x - c))
)/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

3.55.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 283, normalized size of antiderivative = 4.04

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(4\,a\sqrt{d^2}+3\,b\sqrt{d^2}\right)}{d\sqrt{16\,a^2+24\,a\,b+9\,b^2}}\right)\sqrt{16\,a^2+24\,a\,b+9\,b^2}}{4\sqrt{d^2}}$$
$$-\frac{\frac{a\,e^{5\,c+5\,d\,x}}{d}+\frac{2\,e^{3\,c+3\,d\,x}\,(a+2\,b)}{d}+\frac{a\,e^{c+d\,x}}{d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}-\frac{e^{c+d\,x}\,(2\,a-b)}{2\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$
$$-\frac{2\,b\,e^{c+d\,x}}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,(4\,a+3\,b)}{4\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(4*a*(d^2)^(1/2) + 3*b*(d^2)^(1/2)))/(d*(24*a*b + 1
6*a^2 + 9*b^2)^(1/2)))*(24*a*b + 16*a^2 + 9*b^2)^(1/2))/(4*(d^2)^(1/2)) -
((a*exp(5*c + 5*d*x))/d + (2*exp(3*c + 3*d*x)*(a + 2*b))/d + (a*exp(c + d*
x))/d)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp
(8*c + 8*d*x) + 1) - (exp(c + d*x)*(2*a - b))/(2*d*(2*exp(2*c + 2*d*x) + e
xp(4*c + 4*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(
4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(4*a + 3*b))/(4*d*(e
xp(2*c + 2*d*x) + 1))
```

3.56 $\int \mathrm{sech}^4(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx$

3.56.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \mathrm{sech}^4(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx=\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+2b)\tanh^3(c+dx)}{3d}+\frac{b\tanh^5(c+dx)}{5d}$$

output
```
(a+b)*tanh(d*x+c)/d-1/3*(a+2*b)*tanh(d*x+c)^3/d+1/5*b*tanh(d*x+c)^5/d
```

3.56.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \mathrm{sech}^4(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx=\frac{a\tanh(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}-\frac{a\tanh^3(c+dx)}{3d}-\frac{2b\tanh^3(c+dx)}{3d}+\frac{b\tanh^5(c+dx)}{5d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x])/d + (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (2
*b*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)/(5*d)
```

3.56.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4534, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ic+idx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{5}(5a+4b)\int \operatorname{sech}^4(c+dx)dx+\frac{b\tanh(c+dx)\operatorname{sech}^4(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\tanh(c+dx)\operatorname{sech}^4(c+dx)}{5d}+\frac{1}{5}(5a+4b)\int \csc\left(ic+idx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\tanh(c+dx)\operatorname{sech}^4(c+dx)}{5d}+\frac{i(5a+4b)\int\left(1-\tanh^2(c+dx)\right)d(-i\tanh(c+dx))}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\tanh(c+dx)\operatorname{sech}^4(c+dx)}{5d}+\frac{i(5a+4b)\left(\frac{1}{3}i\tanh^3(c+dx)-i\tanh(c+dx)\right)}{5d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

output
```
(b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) + ((I/5)*(5*a + 4*b)*((-I)*Tanh[c
+ d*x] + (I/3)*Tanh[c + d*x]^3))/d
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.56.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+b\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{d}$	56
default	$\frac{a\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+b\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{d}$	56
parts	$\frac{a\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}+\frac{b\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{d}$	58
parallelrisch	$\frac{(50a+40b)\sinh(3dx+3c)+(10a+8b)\sinh(5dx+5c)+40\sinh(dx+c)(a+2b)}{15d(\cosh(5dx+5c)+5\cosh(3dx+3c)+10\cosh(dx+c))}$	85
risch	$-\frac{4(15a\,e^{6dx+6c}+35a\,e^{4dx+4c}+40b\,e^{4dx+4c}+25\,e^{2dx+2c}a+20b\,e^{2dx+2c}+5a+4b)}{15d(e^{2dx+2c}+1)^5}$	86

input
```
int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b*(8/15+1/5*sech(d*x+c)^4+4/15*
sech(d*x+c)^2)*tanh(d*x+c))
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 343, normalized size of antiderivative = 6.86

$$\int \operatorname{sech}^4(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right) \, dx =$$

$$-\frac{}{15 \left(d \cosh(dx+c)^7 + 7\, d \cosh(dx+c) \sinh(dx+c)^6 + d \sinh(dx+c)^7 + 5\, d \cosh(dx+c)^5 + \left(21\, d \cosh\right.\right.}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-8/15*(2*(5*a + b)*cosh(d*x + c)^3 + 6*(5*a + b)*cosh(d*x + c)*sinh(d*x +
c)^2 + (5*a - 2*b)*sinh(d*x + c)^3 + 30*(a + b)*cosh(d*x + c) + (3*(5*a -
2*b)*cosh(d*x + c)^2 + 5*a + 10*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d
*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 +
 (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5
*d*cosh(d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)
^5 + 50*d*cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cos
h(d*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x +
 c)^2 + 5*d)*sinh(d*x + c))
```

3.56.6 Sympy [F]

$$\int \operatorname{sech}^4(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right) \, dx = \int \left(a+b\operatorname{sech}^2(c+dx)\right) \operatorname{sech}^4(c+dx) \, dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**4, x)
```

3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(46) = 92$.

Time = 0.19 (sec) , antiderivative size = 300, normalized size of antiderivative = 6.00

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$= \frac{16}{15}b\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}+}\right.$$
$$\left.+\frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)\right.$$

input
```
integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
16/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x
- 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x -
 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.70

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx =$$
$$-\frac{4\left(15\,ae^{(6\,dx+6\,c)}+35\,ae^{(4\,dx+4\,c)}+40\,be^{(4\,dx+4\,c)}+25\,ae^{(2\,dx+2\,c)}+20\,be^{(2\,dx+2\,c)}+5\,a+4\,b\right)}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^5}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
-4/15*(15*a*e^(6*d*x + 6*c) + 35*a*e^(4*d*x + 4*c) + 40*b*e^(4*d*x + 4*c)
+ 25*a*e^(2*d*x + 2*c) + 20*b*e^(2*d*x + 2*c) + 5*a + 4*b)/(d*(e^(2*d*x +
2*c) + 1)^5)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 292, normalized size of antiderivative = 5.84

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$= -\frac{\frac{8(a+2b)}{15d}+\frac{4a\,e^{2c+2dx}}{5d}}{3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1} - \frac{\frac{8a\,e^{2c+2dx}}{5d}+\frac{8a\,e^{6c+6dx}}{5d}+\frac{16\,e^{4c+4dx}(a+2b)}{5d}}{5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1} - \frac{\frac{2a}{5d}+\frac{6a\,e^{4c+4dx}}{5d}+\frac{8\,e^{2c+2dx}(a+2b)}{5d}}{4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1} - \frac{2a}{5d\left(2\,e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

output
```
- ((8*(a + 2*b))/(15*d) + (4*a*exp(2*c + 2*d*x))/(5*d))/(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*a*exp(2*c + 2*d*x))/(
5*d) + (8*a*exp(6*c + 6*d*x))/(5*d) + (16*exp(4*c + 4*d*x)*(a + 2*b))/(5*d
))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp
(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*a)/(5*d) + (6*a*exp(4*c + 4*
d*x))/(5*d) + (8*exp(2*c + 2*d*x)*(a + 2*b))/(5*d))/(4*exp(2*c + 2*d*x) +
6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*a)/(5
*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.57 $\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

3.57.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{1}{8}\left(3a^2+8ab+8b^2\right)x+\frac{3a(a+2b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)}{4d}$$

output
```
1/8*(3*a^2+8*a*b+8*b^2)*x+3/8*a*(a+2*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*co
sh(d*x+c)^3*sinh(d*x+c)*(a+b-b*tanh(d*x+c)^2)/d
```

3.57.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.71

$$\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{4(3a^2+8ab+8b^2)(c+dx)+8a(a+2b)\sinh(2(c+dx))+a^2\sinh(4(c+dx))}{32d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(4*(3*a^2 + 8*a*b + 8*b^2)*(c + d*x) + 8*a*(a + 2*b)*Sinh[2*(c + d*x)] + a
^2*Sinh[4*(c + d*x)])/(32*d)
```

3.57.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4634, 315, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^2}{\sec(ic+idx)^4}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\left(-b\tanh^2(c+dx)+a+b\right)^2}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{a\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{(a+b)(3a+4b)-b(a+4b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\int \frac{(a+b)(3a+4b)-b(a+4b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{a\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2+8ab+8b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{3a(a+2b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{a\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2+8ab+8b^2\right)\text{arctanh}(\tanh(c+dx))+\frac{3a(a+2b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{a\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2))/(4*(1 - Tanh[c + d*x]^2)^2)
 + (((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[Tanh[c + d*x]])/2 + (3*a*(a + 2*b)*Ta
nh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/4)/d
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.57.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{8a(a+2b)\sinh(2dx+2c)+a^2\sinh(4dx+4c)+12\left(a^2+\frac{8}{3}ab+\frac{8}{3}b^2\right)dx}{32d}$
derivativedivides	$\frac{a^2\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+(dx+c)b^2}{d}$
default	$\frac{a^2\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+(dx+c)b^2}{d}$
risch	$\frac{3a^2x}{8}+abx+b^2x+\frac{a^2e^{4dx+4c}}{64d}+\frac{a^2e^{2dx+2c}}{8d}+\frac{a\,e^{2dx+2c}b}{4d}-\frac{a^2e^{-2dx-2c}}{8d}-\frac{a\,e^{-2dx-2c}b}{4d}-\frac{a^2e^{-4dx-4c}}{64d}$

input
```
int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(8*a*(a+2*b)*sinh(2*d*x+2*c)+a^2*sinh(4*d*x+4*c)+12*(a^2+8/3*a*b+8/3*
b^2)*d*x)/d
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \cosh^4(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{a^2\cosh(dx+c)\sinh(dx+c)^3+(3a^2+8ab+8b^2)dx+\left(a^2\cosh(dx+c)^3+4(a^2+2ab)\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*(a^2*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*d*x + (a^
2*cosh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx &= \frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)\\&\quad+\frac{1}{4}ab\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+b^2x\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/4*a*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x
 - 2*c)/d) + b^2*x
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{a^2e^{(4dx+4c)}+8a^2e^{(2dx+2c)}+16abe^{(2dx+2c)}+8(3a^2+8ab+8b^2)(dx+c)-\left(18a^2e^{(4dx+4c)}+48abe^{(4dx}\right.}{64d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/64*(a^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*e^(2*d*x + 2*c)
 + 8*(3*a^2 + 8*a*b + 8*b^2)*(d*x + c) - (18*a^2*e^(4*d*x + 4*c) + 48*a*b*
e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*
e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c))/d
```

3.57.9 Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=\frac{3\,a^2\,x}{8}+b^2\,x+a\,b\,x+\frac{a^2\sinh(2\,c+2\,d\,x)}{4\,d}+\frac{a^2\sinh(4\,c+4\,d\,x)}{32\,d}+\frac{a\,b\sinh(2\,c+2\,d\,x)}{2\,d}$$

input
```
int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(3*a^2*x)/8 + b^2*x + a*b*x + (a^2*sinh(2*c + 2*d*x))/(4*d) + (a^2*sinh(4*
c + 4*d*x))/(32*d) + (a*b*sinh(2*c + 2*d*x))/(2*d)
```

3.58 $\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx$

3.58.1 Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx = \frac{b^2\arctan(\sinh(c+dx))}{d} + \frac{a(a+2b)\sinh(c+dx)}{d} + \frac{a^2\sinh^3(c+dx)}{3d}$$

output
```
b^2*arctan(sinh(d*x+c))/d+a*(a+2*b)*sinh(d*x+c)/d+1/3*a^2*sinh(d*x+c)^3/d
```

3.58.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx = \frac{b^2\arctan(\sinh(c+dx))}{d} + \frac{2ab\cosh(dx)\sinh(c)}{d} + \frac{2ab\cosh(c)\sinh(dx)}{d} + \frac{a^2\sinh(c+dx)}{d} + \frac{a^2\sinh^3(c+dx)}{3d}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(b^2*ArcTan[Sinh[c + d*x]])/d + (2*a*b*Cosh[d*x]*Sinh[c])/d + (2*a*b*Cosh[
c]*Sinh[d*x])/d + (a^2*Sinh[c + d*x])/d + (a^2*Sinh[c + d*x]^3)/(3*d)
```

3.58.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^2}{\sec(ic+idx)^3}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(a\sinh^2(c+dx)+a+b\right)^2}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{b^2}{\sinh^2(c+dx)+1}+a^2\sinh^2(c+dx)+a(a+2b)\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}a^2\sinh^3(c+dx)+a(a+2b)\sinh(c+dx)+b^2\arctan(\sinh(c+dx))}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(b^2*ArcTan[Sinh[c + d*x]] + a*(a + 2*b)*Sinh[c + d*x] + (a^2*Sinh[c + d*x
]^3)/3)/d
```

3.58.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.58.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+2ab\sinh(dx+c)+2b^2\arctan\left(\mathrm{e}^{dx+c}\right)}{d}$
default	$\frac{a^2\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+2ab\sinh(dx+c)+2b^2\arctan\left(\mathrm{e}^{dx+c}\right)}{d}$
parallelrisch	$\frac{-12ib^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+12ib^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)+a^2\sinh(3dx+3c)+9\sinh(dx+c)\left(a+\frac{8b}{3}\right)a}{12d}$
risch	$\frac{a^2\mathrm{e}^{3dx+3c}}{24d}+\frac{3a^2\mathrm{e}^{dx+c}}{8d}+\frac{\mathrm{e}^{dx+c}ab}{d}-\frac{3a^2\mathrm{e}^{-dx-c}}{8d}-\frac{a\,\mathrm{e}^{-dx-c}b}{d}-\frac{a^2\mathrm{e}^{-3dx-3c}}{24d}+\frac{ib^2\ln(\mathrm{e}^{dx+c}+i)}{d}-\frac{ib^2\ln(\mathrm{e}^{d}}{d}$

input
```
int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+2*a*b*sinh(d*x+c)+2*b^2*arcta
n(exp(d*x+c)))
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 414, normalized size of antiderivative = 8.45

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{a^2\cosh{(dx+c)}^6+6\,a^2\cosh{(dx+c)}\sinh{(dx+c)}^5+a^2\sinh{(dx+c)}^6+3\,(3\,a^2+8\,ab)\cosh{(dx+c)}^4+}{}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh
(d*x + c)^6 + 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2
 + 3*a^2 + 8*a*b)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + 3*(3*a^2 +
8*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^2
+ 3*(5*a^2*cosh(d*x + c)^4 + 6*(3*a^2 + 8*a*b)*cosh(d*x + c)^2 - 3*a^2 - 8
*a*b)*sinh(d*x + c)^2 - a^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c
)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c
)^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(a^2*cosh(d*x + c)^5 + 2*(3
*a^2 + 8*a*b)*cosh(d*x + c)^3 - (3*a^2 + 8*a*b)*cosh(d*x + c))*sinh(d*x +
c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x
+ c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

3.58.6 Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \int \left(a+b\,\text{sech}^2\,(c+dx)\right)^2\cosh^3\,(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x)**3, x)
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.14

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{24}\,a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$
$$+ab\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)-\frac{2\,b^2\arctan\left(e^{(-dx-c)}\right)}{d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d
*x - 3*c)/d) + a*b*(e^(d*x + c)/d - e^(-d*x - c)/d) - 2*b^2*arctan(e^(-d*x
 - c))/d
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.92

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{48\,b^2\arctan\left(e^{(dx+c)}\right)+a^2e^{(3\,dx+3\,c)}+9\,a^2e^{(dx+c)}+24\,abe^{(dx+c)}-\left(9\,a^2e^{(2\,dx+2\,c)}+24\,abe^{(2\,dx+2\,c)}+a^2\right)e^{(-3}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(48*b^2*arctan(e^(d*x + c)) + a^2*e^(3*d*x + 3*c) + 9*a^2*e^(d*x + c)
 + 24*a*b*e^(d*x + c) - (9*a^2*e^(2*d*x + 2*c) + 24*a*b*e^(2*d*x + 2*c) +
a^2)*e^(-3*d*x - 3*c))/d
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{2\,\text{atan}\left(\frac{b^2\,e^{d\,x}\,e^c\,\sqrt{d^2}}{d\,\sqrt{b^4}}\right)\,\sqrt{b^4}}{\sqrt{d^2}} - \frac{e^{-c-d\,x}\,(3\,a^2+8\,b\,a)}{8\,d} - \frac{a^2\,e^{-3\,c-3\,d\,x}}{24\,d} + \frac{a^2\,e^{3\,c+3\,d\,x}}{24\,d} + \frac{a\,e^{c+d\,x}\,(3\,a+8\,b)}{8\,d}$$

input
```
int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(2*atan((b^2*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^4)^(1/2)))*(b^4)^(1/2))/(d
^2)^(1/2) - (exp(- c - d*x)*(8*a*b + 3*a^2))/(8*d) - (a^2*exp(- 3*c - 3*d*
x))/(24*d) + (a^2*exp(3*c + 3*d*x))/(24*d) + (a*exp(c + d*x)*(3*a + 8*b))/
(8*d)
```

3.59 $\int \cosh^2(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

3.59.1 Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=\frac{1}{2}a(a+4b)x+\frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)}{d}$$

output
```
1/2*a*(a+4*b)*x+1/2*a^2*cosh(d*x+c)*sinh(d*x+c)/d+b^2*tanh(d*x+c)/d
```

3.59.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=2abx+\frac{a^2(c+dx)}{2d}+\frac{a^2\sinh(2(c+dx))}{4d}+\frac{b^2\tanh(c+dx)}{d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
2*a*b*x + (a^2*(c + d*x))/(2*d) + (a^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*Tan
h[c + d*x])/d
```

3.59.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^2}{\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\left(-b\tanh^2(c+dx)+a+b\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^2+\frac{a(a+2b)-2ab\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}+\frac{1}{2}a(a+4b)\text{arctanh}(\tanh(c+dx))+b^2\tanh(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a*(a + 4*b)*ArcTanh[Tanh[c + d*x]])/2 + b^2*Tanh[c + d*x] + (a^2*Tanh[c
+ d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

3.59.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.59.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(dx+c)+b^2\tanh(dx+c)}{d}$	51
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(dx+c)+b^2\tanh(dx+c)}{d}$	51
parallelrisch	$\frac{a^2\sinh(3dx+3c)+4adx(a+4b)\cosh(dx+c)+\sinh(dx+c)(a^2+8b^2)}{8d\cosh(dx+c)}$	60
risch	$\frac{a^2x}{2}+2abx+\frac{a^2e^{2dx+2c}}{8d}-\frac{a^2e^{-2dx-2c}}{8d}-\frac{2b^2}{d(e^{2dx+2c}+1)}$	68

input
```
int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(d*x+c)+b^2*tan
h(d*x+c))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{a^2\sinh(dx+c)^3+4\left((a^2+4ab)dx-2b^2\right)\cosh(dx+c)+\left(3a^2\cosh(dx+c)^2+a^2+8b^2\right)\sinh(dx+c)}{8d\cosh(dx+c)}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*(a^2*sinh(d*x + c)^3 + 4*((a^2 + 4*a*b)*d*x - 2*b^2)*cosh(d*x + c) + (
3*a^2*cosh(d*x + c)^2 + a^2 + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

3.59.6 Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=\int\left(a+b\,\text{sech}^2(c+dx)\right)^2\cosh^2(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x)**2, x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=\frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+2abx+\frac{2b^2}{d(e^{(-2dx-2c)}+1)}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*b^2/(
d*(e^(-2*d*x - 2*c) + 1))
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.72

$$\int \cosh^2(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \, dx = \frac{a^2 e^{(2\,dx+2\,c)} + 4\left(a^2+4\,ab\right)(dx+c) - \frac{a^2 e^{(4\,dx+4\,c)}+4\,abe^{(4\,dx+4\,c)}+2\,a^2 e^{(2\,dx+2\,c)}+4\,abe^{(2\,dx+2\,c)}+16\,b^2 e^{(2\,dx+2\,c)}+a^2}{e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/8*(a^2*e^(2*d*x + 2*c) + 4*(a^2 + 4*a*b)*(d*x + c) - (a^2*e^(4*d*x + 4*c
) + 4*a*b*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 4*a*b*e^(2*d*x + 2*c)
+ 16*b^2*e^(2*d*x + 2*c) + a^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \cosh^2(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \, dx = \frac{a^2\,e^{2\,c+2\,d\,x}}{8\,d} - \frac{a^2\,e^{-2\,c-2\,d\,x}}{8\,d} - \frac{2\,b^2}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} + \frac{a\,x\,(a+4\,b)}{2}$$

input
```
int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(a^2*exp(2*c + 2*d*x))/(8*d) - (a^2*exp(- 2*c - 2*d*x))/(8*d) - (2*b^2)/(d
*(exp(2*c + 2*d*x) + 1)) + (a*x*(a + 4*b))/2
```

3.60 $\int \cosh(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx$

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \cosh(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx=\frac{b(4a+b)\arctan(\sinh(c+dx))}{2d}+\frac{a^2\sinh(c+dx)}{d}+\frac{b^2\mathrm{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*b*(4*a+b)*arctan(sinh(d*x+c))/d+a^2*sinh(d*x+c)/d+1/2*b^2*sech(d*x+c)*
tanh(d*x+c)/d
```

3.60.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

$$\int \cosh(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx=\frac{2ab\arctan(\sinh(c+dx))}{d}+\frac{b^2\arctan(\sinh(c+dx))}{2d}+\frac{a^2\cosh(dx)\sinh(c)}{d}+\frac{a^2\cosh(c)\sinh(dx)}{d}+\frac{b^2\mathrm{sech}(c+dx)\tanh(c+dx)}{2d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2
*Cosh[d*x]*Sinh[c])/d + (a^2*Cosh[c]*Sinh[d*x])/d + (b^2*Sech[c + d*x]*Tan
h[c + d*x])/(2*d)
```

3.60.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^2}{\sec(ic+idx)}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(a\sinh^2(c+dx)+a+b\right)^2}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(a^2+\frac{2ab\sinh^2(c+dx)+b(2a+b)}{\left(\sinh^2(c+dx)+1\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sinh(c+dx)+\frac{1}{2}b(4a+b)\arctan(\sinh(c+dx))+\frac{b^2\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((b*(4*a + b)*ArcTan[Sinh[c + d*x]])/2 + a^2*Sinh[c + d*x] + (b^2*Sinh[c +
 d*x])/(2*(1 + Sinh[c + d*x]^2)))/d
```

3.60.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.60.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^{2} \sinh \left(d x +c \right)+4 a b \arctan \left({\mathrm e}^{d x +c}\right)+b^{2} \left(\frac{\operatorname{sech}\left(d x +c \right) \tanh \left(d x +c \right)}{2}+\arctan \left({\mathrm e}^{d x +c}\right)\right)}{d}$
default	$\frac{a^{2} \sinh \left(d x +c \right)+4 a b \arctan \left({\mathrm e}^{d x +c}\right)+b^{2} \left(\frac{\operatorname{sech}\left(d x +c \right) \tanh \left(d x +c \right)}{2}+\arctan \left({\mathrm e}^{d x +c}\right)\right)}{d}$
parallelrisch	$\frac{-4 i \left(a +\frac{b}{4}\right) \left(1+\cosh \left(2 d x +2 c \right)\right) b \ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)+4 i \left(a +\frac{b}{4}\right) \left(1+\cosh \left(2 d x +2 c \right)\right) b \ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+i\right)+a^{2} \sinh \left(3 d x \right.}{2 d \left(1+\cosh \left(2 d x +2 c \right)\right)}$
risch	$\frac{a^{2} {\mathrm e}^{d x +c}}{2 d}-\frac{a^{2} {\mathrm e}^{-d x -c}}{2 d}+\frac{b^{2} {\mathrm e}^{d x +c} \left({\mathrm e}^{2 d x +2 c}-1\right)}{d \left({\mathrm e}^{2 d x +2 c}+1\right)^{2}}+\frac{2 i b a \ln \left({\mathrm e}^{d x +c}+i\right)}{d}+\frac{i b^{2} \ln \left({\mathrm e}^{d x +c}+i\right)}{2 d}-\frac{2 i b a \ln \left({\mathrm e}^{d x +c}-i\right)}{d}-$

input
```
int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*sinh(d*x+c)+4*a*b*arctan(exp(d*x+c))+b^2*(1/2*sech(d*x+c)*tanh(d*
x+c)+arctan(exp(d*x+c))))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 653, normalized size of antiderivative = 11.66

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx$$
$$= \frac{a^2\cosh(dx+c)^6 + 6a^2\cosh(dx+c)\sinh(dx+c)^5 + a^2\sinh(dx+c)^6 + (a^2+2b^2)\cosh(dx+c)^4 + (15$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(
d*x + c)^6 + (a^2 + 2*b^2)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2
 + 2*b^2)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 2*b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c)^2 + (15*a^2*cosh(d
*x + c)^4 + 6*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2
 - a^2 + 2*((4*a*b + b^2)*cosh(d*x + c)^5 + 5*(4*a*b + b^2)*cosh(d*x + c)*
sinh(d*x + c)^4 + (4*a*b + b^2)*sinh(d*x + c)^5 + 2*(4*a*b + b^2)*cosh(d*x
 + c)^3 + 2*(5*(4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c)^
3 + 2*(5*(4*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + b^2)*cosh(d*x + c))*si
nh(d*x + c)^2 + (4*a*b + b^2)*cosh(d*x + c) + (5*(4*a*b + b^2)*cosh(d*x +
c)^4 + 6*(4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c))*arcta
n(cosh(d*x + c) + sinh(d*x + c)) + 2*(3*a^2*cosh(d*x + c)^5 + 2*(a^2 + 2*b
^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(
d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*c
osh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh
(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*
cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

3.60.6 Sympy [F]

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx = \int \left(a+b\,\text{sech}^2(c+dx)\right)^2\cosh(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x), x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.80

$$\int \cosh(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$= -b^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) - \frac{4\,ab\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{a^2\sinh\left(dx+c\right)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-b^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(
-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a*b*arctan(e^(-d*x - c))/d + a
^2*sinh(d*x + c)/d
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(52) = 104.

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \cosh(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{2\,a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) + \left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)(4\,ab+b^2) + \frac{4\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/4*(2*a^2*(e^(d*x + c) - e^(-d*x - c)) + (pi + 2*arctan(1/2*(e^(2*d*x + 2
*c) - 1)*e^(-d*x - c)))*(4*a*b + b^2) + 4*b^2*(e^(d*x + c) - e^(-d*x - c))
/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

3.60.9 Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.07

$$\int \cosh(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx$$

$$= \frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(b^2\,\sqrt{d^2}+4\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{16\,a^2\,b^2+8\,a\,b^3+b^4}}\right)\,\sqrt{16\,a^2\,b^2+8\,a\,b^3+b^4}}{\sqrt{d^2}} + \frac{a^2\,e^{c+d\,x}}{2\,d} - \frac{a^2\,e^{-c-d\,x}}{2\,d} + \frac{b^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{2\,b^2\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(atan((exp(d*x)*exp(c)*(b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(1/2)))/(d*(8*a*b^3
+ b^4 + 16*a^2*b^2)^(1/2)))*(8*a*b^3 + b^4 + 16*a^2*b^2)^(1/2))/(d^2)^(1/2
) + (a^2*exp(c + d*x))/(2*d) - (a^2*exp(- c - d*x))/(2*d) + (b^2*exp(c + d
*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^2*exp(c + d*x))/(d*(2*exp(2*c + 2*d
*x) + exp(4*c + 4*d*x) + 1))
```

3.61 $\int \mathrm{sech}(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 90

$$\begin{aligned}\int \mathrm{sech}(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx &= \frac{(8a^2+8ab+3b^2)\arctan(\sinh(c+dx))}{8d} + \frac{3b(2a+b)\mathrm{sech}(c+dx)\tanh(c+dx)}{8d} \\ &\quad + \frac{b\mathrm{sech}^3(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{4d}\end{aligned}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c))/d+3/8*b*(2*a+b)*sech(d*x+c)*ta
nh(d*x+c)/d+1/4*b*sech(d*x+c)^3*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

3.61.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \mathrm{sech}(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx &= \frac{a^2\arctan(\sinh(c+dx))}{d} + \frac{ab\arctan(\sinh(c+dx))}{d} \\ &\quad + \frac{3b^2\arctan(\sinh(c+dx))}{8d} \\ &\quad + \frac{ab\mathrm{sech}(c+dx)\tanh(c+dx)}{d} \\ &\quad + \frac{3b^2\mathrm{sech}(c+dx)\tanh(c+dx)}{8d} \\ &\quad + \frac{b^2\mathrm{sech}^3(c+dx)\tanh(c+dx)}{4d}\end{aligned}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Sinh[c + d*x]])/d + (a*b*ArcTan[Sinh[c + d*x]])/d + (3*b^2*Arc
Tan[Sinh[c + d*x]])/(8*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d + (3*b^2*S
ech[c + d*x]*Tanh[c + d*x])/(8*d) + (b^2*Sech[c + d*x]^3*Tanh[c + d*x])/(4
*d)
```

3.61.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4635, 315, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(ic+idx)\left(a+b\sec(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \frac{\left(a\sinh^2(c+dx)+a+b\right)^2}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\frac{1}{4}\int \frac{a(4a+b)\sinh^2(c+dx)+(a+b)(4a+3b)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)+\frac{b\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^2+8ab+3b^2\right)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{3b(2a+b)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{b\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^2+8ab+3b^2\right)\arctan(\sinh(c+dx))+\frac{3b(2a+b)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{b\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((b*Sinh[c + d*x]*(a + b + a*Sinh[c + d*x]^2))/(4*(1 + Sinh[c + d*x]^2)^2)
 + (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/2 + (3*b*(2*a + b)*Sin
h[c + d*x])/(2*(1 + Sinh[c + d*x]^2)))/4)/d
```

3.61.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.61.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2a^2\arctan(e^{dx+c})+2ab\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+b^2\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\operatorname{arct}}{}\right.}{d}$
default	$\frac{2a^2\arctan(e^{dx+c})+2ab\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+b^2\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\operatorname{arct}}{}\right.}{d}$
parts	$\frac{a^2\arctan(\sinh(dx+c))}{d}+\frac{b^2\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)}{d}+\frac{2ab\left(\frac{\operatorname{sech}(dx+c)\tanh(d}{2}\right.}{}$
parallelrisch	$\frac{-4i\left(\frac{3}{8}b^2+ab+a^2\right)\left(\frac{3}{4}+\frac{\cosh(4dx+4c)}{4}+\cosh(2dx+2c)\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+4i\left(\frac{3}{8}b^2+ab+a^2\right)\left(\frac{3}{4}+\frac{\cosh(4dx+4c)}{4}+\cosh(2\right.}{d(\cosh(4dx+4c)+4\cosh(2dx+2c)+3}$
risch	$\frac{b\,e^{dx+c}\left(8a\,e^{6dx+6c}+3b\,e^{6dx+6c}+8a\,e^{4dx+4c}+11b\,e^{4dx+4c}-8\,e^{2dx+2c}a-11b\,e^{2dx+2c}-8a-3b\right)}{4d\left(e^{2dx+2c}+1\right)^4}+\frac{i\ln(e^{dx+c}+i)a^2}{d}+$

input
```
int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^2*arctan(exp(d*x+c))+2*a*b*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(ex
p(d*x+c)))+b^2*((1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+3/4*arctan
(exp(d*x+c))))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1372 vs. 2(84) = 168.

Time = 0.27 (sec) , antiderivative size = 1372, normalized size of antiderivative = 15.24

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/4*((8*a*b + 3*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 3*b^2)*cosh(d*x + c)*sin
h(d*x + c)^6 + (8*a*b + 3*b^2)*sinh(d*x + c)^7 + (8*a*b + 11*b^2)*cosh(d*x
 + c)^5 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a*b + 11*b^2)*sinh(d*x +
 c)^5 + 5*(7*(8*a*b + 3*b^2)*cosh(d*x + c)^3 + (8*a*b + 11*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^4 - (8*a*b + 11*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 3*b
^2)*cosh(d*x + c)^4 + 10*(8*a*b + 11*b^2)*cosh(d*x + c)^2 - 8*a*b - 11*b^2
)*sinh(d*x + c)^3 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(8*a*b + 11*b
^2)*cosh(d*x + c)^3 - 3*(8*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 +
((8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(
d*x + c)*sinh(d*x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*
a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d
*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b +
 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + 6*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*
b + 3*b^2)*cosh(d*x + c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 +
24*a^2 + 24*a*b + 9*b^2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*co
sh(d*x + c)^5 + 10*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*
a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*co
sh(d*x + c)^2 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 +
 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + ...
```

3.61.6 Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\int\left(a+b\operatorname{sech}^2(c+dx)\right)^2\operatorname{sech}(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x), x)
```

3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(84) = 168$.

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.23

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{1}{4}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3e^{(-dx-c)}+11e^{(-3dx-3c)}-11e^{(-5dx-5c)}-3e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right) - 2ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)+\frac{a^2\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c)
 - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^
(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arc
tan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x -
2*c) + e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(84) = 168$.

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)(8a^2+8ab+3b^2)+\frac{4\left(8ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3\cdots\right.}{\left(\left(e^{(dx+c)}-e^{(-\cdots\right.\right.}}{16d}$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 8*a
*b + 3*b^2) + 4*(8*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b^2*(e^(d*x + c)
 - e^(-d*x - c))^3 + 32*a*b*(e^(d*x + c) - e^(-d*x - c)) + 20*b^2*(e^(d*x
+ c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

3.61.9 Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.37

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(8\,a^2\,\sqrt{d^2}+3\,b^2\,\sqrt{d^2}+8\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4+128\,a^3\,b+112\,a^2\,b^2+48\,a\,b^3+9\,b^4}}\right)\sqrt{64\,a^4+128\,a^3\,b+112\,a^2\,b^2+48\,a\,b^3+9\,b^4}}{4\,\sqrt{d^2}}$$
$$-\frac{6\,b^2\,e^{c+d\,x}}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$+\frac{4\,b^2\,e^{c+d\,x}}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$+\frac{e^{c+d\,x}\left(3\,b^2+8\,a\,b\right)}{4\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{e^{c+d\,x}\left(8\,a\,b-b^2\right)}{2\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x),x)
```

output
```
(atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 8*a*b*(d^2
)^(1/2)))/(d*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2)))
*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2))/(4*(d^2)^(1/
2)) - (6*b^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) + 1)) + (4*b^2*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*ex
p(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (exp(c + d*
x)*(8*a*b + 3*b^2))/(4*d*(exp(2*c + 2*d*x) + 1)) - (exp(c + d*x)*(8*a*b -
b^2))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.62 $\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

3.62.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{2b(a+b)\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
(a+b)^2*tanh(d*x+c)/d-2/3*b*(a+b)*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d
```

3.62.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.75

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{d}-\frac{2ab\tanh^3(c+dx)}{3d}-\frac{2b^2\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d -
(2*a*b*Tanh[c + d*x]^3)/(3*d) - (2*b^2*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[
c + d*x]^5)/(5*d)
```

3.62.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \left(-b\tanh^2(c+dx)+a+b\right)^2d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(b^2\tanh^4(c+dx)-2ab\left(\frac{b}{a}+1\right)\tanh^2(c+dx)+a^2\left(\frac{b(2a+b)}{a^2}+1\right)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2}{3}b(a+b)\tanh^3(c+dx)+(a+b)^2\tanh(c+dx)+\frac{1}{5}b^2\tanh^5(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + b)^2*Tanh[c + d*x] - (2*b*(a + b)*Tanh[c + d*x]^3)/3 + (b^2*Tanh[c +
 d*x]^5)/5)/d
```

3.62.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.62.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a^2\tanh(dx+c)+2ab\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+b^2\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{d}$
default	$\frac{a^2\tanh(dx+c)+2ab\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+b^2\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{d}$
parts	$\frac{a^2\tanh(dx+c)}{d}+\frac{b^2\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{d}+\frac{2ab\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
parallelrisch	$\frac{(45a^2+100ab+40b^2)\sinh(3dx+3c)+(15a^2+20ab+8b^2)\sinh(5dx+5c)+30\left(a^2+\frac{8}{3}ab+\frac{8}{3}b^2\right)\sinh(dx+c)}{15d(\cosh(5dx+5c)+5\cosh(3dx+3c)+10\cosh(dx+c))}$
risch	$-\frac{2(15a^2e^{8dx+8c}+60a^2e^{6dx+6c}+60ab\,e^{6dx+6c}+90a^2e^{4dx+4c}+140ab\,e^{4dx+4c}+80\,e^{4dx+4c}b^2+60a^2e^{2dx+2c}+100ab\,e^{2dx}}{15d\left(e^{2dx+2c}+1\right)^5}$

input
```
int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*tanh(d*x+c)+2*a*b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b^2*(8/15+1
/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(49) = 98.

Time = 0.24 (sec) , antiderivative size = 404, normalized size of antiderivative = 7.62

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx =$$
$$-\frac{4\left((15a^2+10ab+4b^2)\cosh(dx+c)^4-8(5ab+2b^2)\cosh(dx+c)\sinh(dx+c)^3+(15\right.}{15\left(d\cosh(dx+c)^6+6d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6+6d\cosh(dx+c)^4+3(5d\cosh\right.}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-4/15*((15*a^2 + 10*a*b + 4*b^2)*cosh(d*x + c)^4 - 8*(5*a*b + 2*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (15*a^2 + 10*a*b + 4*b^2)*sinh(d*x + c)^4 + 20*
(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 10*a*b + 4*b^2)*cos
h(d*x + c)^2 + 30*a^2 + 40*a*b + 10*b^2)*sinh(d*x + c)^2 + 45*a^2 + 70*a*b
 + 40*b^2 - 8*((5*a*b + 2*b^2)*cosh(d*x + c)^3 + 5*(a*b + b^2)*cosh(d*x +
c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5
+ d*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*
sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c
)^3 + 15*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2
 + 5*d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5
*d*cosh(d*x + c))*sinh(d*x + c) + 10*d)
```

3.62.6 Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\operatorname{sech}^2(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(49) = 98$.

Time = 0.21 (sec) , antiderivative size = 324, normalized size of antiderivative = 6.11

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{16}{15}b^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}+}\right.$$
$$+\frac{8}{3}ab\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+\frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
16/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1
0*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6
*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-
2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8
*c) + e^(-10*d*x - 10*c) + 1))) + 8/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*
d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d
*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a^2/(d*(e^(-2
*d*x - 2*c) + 1))
```

3.62.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(49) = 98$.

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.94

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx =$$
$$-\frac{2\left(15\,a^2e^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}+60\,abe^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}+140\,abe^{(4\,dx+4\,c)}+80\,b^2e^{(4\,dx+4\,c)}+\right.}{15\,d(e^{(2\,dx+2\,c)}+1)^5}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-2/15*(15*a^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x +
 6*c) + 90*a^2*e^(4*d*x + 4*c) + 140*a*b*e^(4*d*x + 4*c) + 80*b^2*e^(4*d*x
 + 4*c) + 60*a^2*e^(2*d*x + 2*c) + 100*a*b*e^(2*d*x + 2*c) + 40*b^2*e^(2*d
*x + 2*c) + 15*a^2 + 20*a*b + 8*b^2)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 452, normalized size of antiderivative = 8.53

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{2a(a+2b)}{5d}+\frac{2a^2e^{2c+2dx}}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1} - \frac{\frac{2a^2}{5d}+\frac{2a^2e^{8c+8dx}}{5d}+\frac{4e^{4c+4dx}(3a^2+8ab+8b^2)}{5d}+\frac{8ae^{2c+2dx}(a+2b)}{5d}+\frac{8ae^{6c+6dx}(a+2b)}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1} - \frac{\frac{2a(a+2b)}{5d}+\frac{2a^2e^{6c+6dx}}{5d}+\frac{2e^{2c+2dx}(3a^2+8ab+8b^2)}{5d}+\frac{6ae^{4c+4dx}(a+2b)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1} - \frac{\frac{2(3a^2+8ab+8b^2)}{15d}+\frac{2a^2e^{4c+4dx}}{5d}+\frac{4ae^{2c+2dx}(a+2b)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1} - \frac{2a^2}{5d\left(e^{2c+2dx}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

output
```
- ((2*a*(a + 2*b))/(5*d) + (2*a^2*exp(2*c + 2*d*x))/(5*d))/(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1) - ((2*a^2)/(5*d) + (2*a^2*exp(8*c + 8*d*x))/(
5*d) + (4*exp(4*c + 4*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(5*d) + (8*a*exp(2*c +
 2*d*x)*(a + 2*b))/(5*d) + (8*a*exp(6*c + 6*d*x)*(a + 2*b))/(5*d))/(5*exp(
2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d
*x) + exp(10*c + 10*d*x) + 1) - ((2*a*(a + 2*b))/(5*d) + (2*a^2*exp(6*c +
6*d*x))/(5*d) + (2*exp(2*c + 2*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(5*d) + (6*a*
exp(4*c + 4*d*x)*(a + 2*b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x
) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(8*a*b + 3*a^2 + 8*b^
2))/(15*d) + (2*a^2*exp(4*c + 4*d*x))/(5*d) + (4*a*exp(2*c + 2*d*x)*(a + 2
*b))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
1) - (2*a^2)/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.63 $\int \mathbf{sech}^3(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

3.63.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{(8a^2+12ab+5b^2)\arctan(\sinh(c+dx))}{16d} + \frac{(8a^2+12ab+5b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{b(8a+5b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{b\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{6d}$$

output
```
1/16*(8*a^2+12*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/16*(8*a^2+12*a*b+5*b^2)*
sech(d*x+c)*tanh(d*x+c)/d+1/24*b*(8*a+5*b)*sech(d*x+c)^3*tanh(d*x+c)/d+1/6
*b*sech(d*x+c)^5*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

3.63.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.46

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{a^2\arctan(\sinh(c+dx))}{2d}+\frac{3ab\arctan(\sinh(c+dx))}{4d}+\frac{5b^2\arctan(\sinh(c+dx))}{16d}$$
$$+\frac{a^2\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{3ab\operatorname{sech}(c+dx)\tanh(c+dx)}{4d}$$
$$+\frac{5b^2\operatorname{sech}(c+dx)\tanh(c+dx)}{16d}+\frac{ab\operatorname{sech}^3(c+dx)\tanh(c+dx)}{2d}$$
$$+\frac{5b^2\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d}+\frac{b^2\operatorname{sech}^5(c+dx)\tanh(c+dx)}{6d}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Sinh[c + d*x]])/(2*d) + (3*a*b*ArcTan[Sinh[c + d*x]])/(4*d) +
(5*b^2*ArcTan[Sinh[c + d*x]])/(16*d) + (a^2*Sech[c + d*x]*Tanh[c + d*x])/(
2*d) + (3*a*b*Sech[c + d*x]*Tanh[c + d*x])/(4*d) + (5*b^2*Sech[c + d*x]*Ta
nh[c + d*x])/(16*d) + (a*b*Sech[c + d*x]^3*Tanh[c + d*x])/(2*d) + (5*b^2*S
ech[c + d*x]^3*Tanh[c + d*x])/(24*d) + (b^2*Sech[c + d*x]^5*Tanh[c + d*x])
/(6*d)
```

3.63.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4635, 315, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^2\,dx$$
$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(a\sinh^2(c+dx)+a+b\right)^2}{\left(\sinh^2(c+dx)+1\right)^4}d\sinh(c+dx)}{d}$$

$\Big\downarrow$ 315

$$\frac{\frac{1}{6}\int \frac{3a(2a+b)\sinh^2(c+dx)+(a+b)(6a+5b)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)+\frac{b\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$\Big\downarrow$ 298

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+12ab+5b^2\right)\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)+\frac{b(8a+5b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{b\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$\Big\downarrow$ 215

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+12ab+5b^2\right)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{b(8a+5b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{b\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+12ab+5b^2\right)\left(\frac{1}{2}\arctan(\sinh(c+dx))+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{b(8a+5b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{b\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((b*Sinh[c + d*x]*(a + b + a*Sinh[c + d*x]^2))/(6*(1 + Sinh[c + d*x]^2)^3)
 + ((b*(8*a + 5*b)*Sinh[c + d*x])/(4*(1 + Sinh[c + d*x]^2)^2) + (3*(8*a^2
+ 12*a*b + 5*b^2)*(ArcTan[Sinh[c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Sinh[c
+ d*x]^2))))/4)/6)/d
```

3.63.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.63.4 Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^2\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+2ab\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)+b^2\left(\left(\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+2ab\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)+b^2\left(\left(\right.\right.}{d}$
parts	$\frac{a^2\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)}{d}+\frac{b^2\left(\left(\frac{\operatorname{sech}(dx+c)^5}{6}+\frac{5\operatorname{sech}(dx+c)^3}{24}+\frac{5\operatorname{sech}(dx+c)}{16}\right)\tanh(dx+c)+\frac{5\arctan(}{8}\right.}{d}$
parallelrisch	$-360i\left(\frac{2}{3}+\frac{\cosh(6dx+6c)}{15}+\frac{2\cosh(4dx+4c)}{5}+\cosh(2dx+2c)\right)\left(a^2+\frac{5}{8}b^2+\frac{3}{2}ab\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+360i\left(\frac{2}{3}+\frac{\cosh(6dx+6c)}{15}\right.$
risch	$e^{dx+c}(24a^2e^{10dx+10c}+36ab\,e^{10dx+10c}+15b^2e^{10dx+10c}+72a^2e^{8dx+8c}+204ab\,e^{8dx+8c}+85b^2e^{8dx+8c}+48a^2e^{6dx+6c}+168$

input
```
int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+2*a*b*((1/4*sech
(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+3/4*arctan(exp(d*x+c)))+b^2*((1/6*s
ech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+c))*tanh(d*x+c)+5/8*arctan(e
xp(d*x+c))))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2946 vs. $2(120) = 240$.

Time = 0.28 (sec) , antiderivative size = 2946, normalized size of antiderivative = 23.02

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 12*a*b + 5
*b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 12*a*b + 5*b^2)*sinh(d*x
 + c)^11 + (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 12*
a*b + 5*b^2)*cosh(d*x + c)^2 + 72*a^2 + 204*a*b + 85*b^2)*sinh(d*x + c)^9
+ 9*(55*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + (72*a^2 + 204*a*b + 85*
b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x
 + c)^7 + 6*(165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 6*(72*a^2 + 20
4*a*b + 85*b^2)*cosh(d*x + c)^2 + 8*a^2 + 28*a*b + 33*b^2)*sinh(d*x + c)^7
 + 42*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 2*(72*a^2 + 204*a*b +
 85*b^2)*cosh(d*x + c)^3 + (8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c))*sinh(d
*x + c)^6 - 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 +
12*a*b + 5*b^2)*cosh(d*x + c)^6 + 21*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x
+ c)^4 + 21*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^2 - 8*a^2 - 28*a*b - 3
3*b^2)*sinh(d*x + c)^5 + 6*(165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 +
 21*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^5 + 35*(8*a^2 + 28*a*b + 33*
b^2)*cosh(d*x + c)^3 - 5*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^4 - (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^3 + (495*(8*a^2 + 12*a
*b + 5*b^2)*cosh(d*x + c)^8 + 84*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)
^6 + 210*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^4 - 60*(8*a^2 + 28*a*b +
33*b^2)*cosh(d*x + c)^2 - 72*a^2 - 204*a*b - 85*b^2)*sinh(d*x + c)^3 + ...
```

3.63.6 Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2\operatorname{sech}^3\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(120) = 240.

Time = 0.30 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.72

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{1}{24}b^2\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\ldots}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,d\ldots}}\right) - \frac{1}{2}ab\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right) - a^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*b^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3
*c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) -
15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e
^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x
- 12*c) + 1))) - 1/2*a*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*
e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d
*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1
))) - a^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(
2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(120) = 240.

Time = 0.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.29

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(8\,a^2+12\,ab+5\,b^2)+\frac{4\left(24\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+36\,ab\left(e^{(dx+c)}-e^{(-d\ldots}\right.\right.}{\ldots}}{\ldots}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 1
2*a*b + 5*b^2) + 4*(24*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 36*a*b*(e^(d*x
 + c) - e^(-d*x - c))^5 + 15*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 192*a^2*
(e^(d*x + c) - e^(-d*x - c))^3 + 384*a*b*(e^(d*x + c) - e^(-d*x - c))^3 +
160*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a^2*(e^(d*x + c) - e^(-d*x -
c)) + 960*a*b*(e^(d*x + c) - e^(-d*x - c)) + 528*b^2*(e^(d*x + c) - e^(-d*
x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 569, normalized size of antiderivative = 4.45

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(8\,a^2\,\sqrt{d^2}+5\,b^2\,\sqrt{d^2}+12\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4+192\,a^3\,b+224\,a^2\,b^2+120\,a\,b^3+25\,b^4}}\right)\,\sqrt{64\,a^4+192\,a^3\,b+224\,a^2\,b^2+120\,a\,b^3+25\,b^4}}{8\,\sqrt{d^2}}$$
$$- \frac{\frac{2\,a^2\,e^{c+d\,x}}{3\,d}+\frac{2\,a^2\,e^{9\,c+9\,d\,x}}{3\,d}+\frac{4\,e^{5\,c+5\,d\,x}\,\left(3\,a^2+8\,a\,b+8\,b^2\right)}{3\,d}+\frac{8\,a\,e^{3\,c+3\,d\,x}\,\left(a+2\,b\right)}{3\,d}+\frac{8\,a\,e^{7\,c+7\,d\,x}\,\left(a+2\,b\right)}{3\,d}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}$$
$$+ \frac{2\,e^{c+d\,x}\,\left(4\,a\,b-11\,b^2\right)}{3\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$+ \frac{16\,b^2\,e^{c+d\,x}}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$+ \frac{e^{c+d\,x}\,\left(8\,a^2+12\,a\,b+5\,b^2\right)}{8\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(-16\,a^2+12\,a\,b+5\,b^2\right)}{12\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$
$$- \frac{e^{c+d\,x}\,\left(20\,a\,b-b^2\right)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 5*b^2*(d^2)^(1/2) + 12*a*b*(d^
2)^(1/2)))/(d*(120*a*b^3 + 192*a^3*b + 64*a^4 + 25*b^4 + 224*a^2*b^2)^(1/2
)))*(120*a*b^3 + 192*a^3*b + 64*a^4 + 25*b^4 + 224*a^2*b^2)^(1/2))/(8*(d^2
)^(1/2)) - ((2*a^2*exp(c + d*x))/(3*d) + (2*a^2*exp(9*c + 9*d*x))/(3*d) +
(4*exp(5*c + 5*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(3*d) + (8*a*exp(3*c + 3*d*x)
*(a + 2*b))/(3*d) + (8*a*exp(7*c + 7*d*x)*(a + 2*b))/(3*d))/(6*exp(2*c + 2
*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) +
6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) + (2*exp(c + d*x)*(4*a*b -
11*b^2))/(3*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x
) + exp(8*c + 8*d*x) + 1)) + (16*b^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x
) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(1
0*c + 10*d*x) + 1)) + (exp(c + d*x)*(12*a*b + 8*a^2 + 5*b^2))/(8*d*(exp(2*
c + 2*d*x) + 1)) + (exp(c + d*x)*(12*a*b - 16*a^2 + 5*b^2))/(12*d*(2*exp(2
*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (exp(c + d*x)*(20*a*b - b^2))/(3*d*
(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

3.63. $\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

3.64 $\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

3.64.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d}+\frac{b(2a+3b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

output
```
(a+b)^2*tanh(d*x+c)/d-1/3*(a+b)*(a+3*b)*tanh(d*x+c)^3/d+1/5*b*(2*a+3*b)*ta
nh(d*x+c)^5/d-1/7*b^2*tanh(d*x+c)^7/d
```

3.64.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{4ab\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^3(c+dx)}{d}+\frac{2ab\tanh^5(c+dx)}{5d}+\frac{3b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d -
(a^2*Tanh[c + d*x]^3)/(3*d) - (4*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c
+ d*x]^3)/d + (2*a*b*Tanh[c + d*x]^5)/(5*d) + (3*b^2*Tanh[c + d*x]^5)/(5*d
) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

3.64.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \ 4634$$

$$\frac{\int \left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 290$$

$$\frac{\int \left(-b^2\tanh^6(c+dx)+b(2a+3b)\tanh^4(c+dx)+(-a-3b)(a+b)\tanh^2(c+dx)+(a+b)^2\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{5}b(2a+3b)\tanh^5(c+dx)-\frac{1}{3}(a+b)(a+3b)\tanh^3(c+dx)+(a+b)^2\tanh(c+dx)-\frac{1}{7}b^2\tanh^7(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + b)^2*Tanh[c + d*x] - ((a + b)*(a + 3*b)*Tanh[c + d*x]^3)/3 + (b*(2*a
 + 3*b)*Tanh[c + d*x]^5)/5 - (b^2*Tanh[c + d*x]^7)/7)/d
```

3.64.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.64.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)+b^2\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}(dx-}{35}\right.}{d}$
default	$\frac{a^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)+b^2\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}(dx-}{35}\right.}{d}$
parts	$\frac{a^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}+\frac{b^2\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}(dx+c)^4}{35}+\frac{8\operatorname{sech}(dx+c)^2}{35}\right)\tanh(dx+c)}{d}+\frac{2ab\left(\frac{8}{15}+\frac{\mathrm{se}}{}\right.}{}$
parallelrisch	$\frac{(1050a^2+2352ab+1008b^2)\sinh(3dx+3c)+(490a^2+784ab+336b^2)\sinh(5dx+5c)+(70a^2+112ab+48b^2)\sinh(7dx+7c)+}{105d(\cosh(7dx+7c)+7\cosh(5dx+5c)+21\cosh(3dx+3c)+35\cosh(dx+c))}$
risch	$-\frac{4(105a^2e^{10dx+10c}+455a^2e^{8dx+8c}+560ab\,e^{8dx+8c}+770a^2e^{6dx+6c}+1400ab\,e^{6dx+6c}+840b^2e^{6dx+6c}+630a^2e^{4dx+4c}+1}{105d(e^{2dx+2c}+1)^7}$

input
```
int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(8/15+1/5*sech(d*x+c)^4
+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b^2*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*
x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(74) = 148$.

Time = 0.25 (sec) , antiderivative size = 677, normalized size of antiderivative = 8.46

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx =$$

$$-\frac{8\,(2\,(3\cdots}{105\,\left(d\cosh\left(dx+c\right)^9+9\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8+d\sinh\left(dx+c\right)^9+7\,d\cosh\left(dx+c\right)^7+\left(36\,d\,c\cdots\right.\right.}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-8/105*(2*(35*a^2 + 14*a*b + 6*b^2)*cosh(d*x + c)^5 + 10*(35*a^2 + 14*a*b
+ 6*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (35*a^2 - 28*a*b - 12*b^2)*sinh(d
*x + c)^5 + 14*(25*a^2 + 34*a*b + 6*b^2)*cosh(d*x + c)^3 + (10*(35*a^2 - 2
8*a*b - 12*b^2)*cosh(d*x + c)^2 + 105*a^2 + 84*a*b - 84*b^2)*sinh(d*x + c)
^3 + 2*(10*(35*a^2 + 14*a*b + 6*b^2)*cosh(d*x + c)^3 + 21*(25*a^2 + 34*a*b
 + 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 28*(25*a^2 + 46*a*b + 24*b^2)*c
osh(d*x + c) + (5*(35*a^2 - 28*a*b - 12*b^2)*cosh(d*x + c)^4 + 63*(5*a^2 +
 4*a*b - 4*b^2)*cosh(d*x + c)^2 + 70*a^2 + 112*a*b + 168*b^2)*sinh(d*x + c
))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c
)^9 + 7*d*cosh(d*x + c)^7 + (36*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 +
 7*(12*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 22*d*cosh(
d*x + c)^5 + (126*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 + 20*d)*sinh(d
*x + c)^5 + (126*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 110*d*cosh(d*
x + c))*sinh(d*x + c)^4 + 42*d*cosh(d*x + c)^3 + (84*d*cosh(d*x + c)^6 + 2
45*d*cosh(d*x + c)^4 + 200*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^3 + (36
*d*cosh(d*x + c)^7 + 147*d*cosh(d*x + c)^5 + 220*d*cosh(d*x + c)^3 + 126*d
*cosh(d*x + c))*sinh(d*x + c)^2 + 56*d*cosh(d*x + c) + (9*d*cosh(d*x + c)^
8 + 49*d*cosh(d*x + c)^6 + 100*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^2 +
14*d)*sinh(d*x + c))
```

3.64.6 Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\operatorname{sech}^4(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(74) = 148$.

Time = 0.21 (sec) , antiderivative size = 671, normalized size of antiderivative = 8.39

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{32}{35}b^2\left(\frac{7e^{(-2dx-2c)}}{d(7e^{(-2dx-2c)}+21e^{(-4dx-4c)}+35e^{(-6dx-6c)}+35e^{(-8dx-8c)}+21e^{(-10dx-10c)}+7e^{(-12dx-12c)}+e\right.$$
$$+\frac{32}{15}ab\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{}{d(5e^{(-2dx-2c}}\right.$$
$$+\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
32/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e
^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
 - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c
) + 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-1
0*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 32/15*a
*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4
*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2
*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.46

$$\int \operatorname{sech}^4(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx =$$

$$-\frac{4 \left(105 a^2 e^{(10 dx+10 c)} + 455 a^2 e^{(8 dx+8 c)} + 560 abe^{(8 dx+8 c)} + 770 a^2 e^{(6 dx+6 c)} + 1400 abe^{(6 dx+6 c)} + 840 b^2 e^{(6 \cdots}\right.}{\cdots}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-4/105*(105*a^2*e^(10*d*x + 10*c) + 455*a^2*e^(8*d*x + 8*c) + 560*a*b*e^(8
*d*x + 8*c) + 770*a^2*e^(6*d*x + 6*c) + 1400*a*b*e^(6*d*x + 6*c) + 840*b^2
*e^(6*d*x + 6*c) + 630*a^2*e^(4*d*x + 4*c) + 1176*a*b*e^(4*d*x + 4*c) + 50
4*b^2*e^(4*d*x + 4*c) + 245*a^2*e^(2*d*x + 2*c) + 392*a*b*e^(2*d*x + 2*c)
+ 168*b^2*e^(2*d*x + 2*c) + 35*a^2 + 56*a*b + 24*b^2)/(d*(e^(2*d*x + 2*c)
+ 1)^7)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 692, normalized size of antiderivative = 8.65

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{\frac{32\,a\,(a+2\,b)}{105\,d}+\frac{8\,a^2\,e^{2\,c+2\,d\,x}}{21\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1} - \frac{\frac{8\,a^2\,e^{2\,c+2\,d\,x}}{7\,d}+\frac{8\,a^2\,e^{10\,c+10\,d\,x}}{7\,d}+\frac{16\,e^{6\,c+6\,d\,x}\,(3\,a^2+8\,a\,b+8\,b^2)}{7\,d}+\frac{32\,a\,e^{4\,c+4\,d\,x}\,(a+2\,b)}{7\,d}+\frac{32\,a\,e^{8\,c+8\,d\,x}\,(a+2\,b)}{7\,d}}{7\,e^{2\,c+2\,d\,x}+21\,e^{4\,c+4\,d\,x}+35\,e^{6\,c+6\,d\,x}+35\,e^{8\,c+8\,d\,x}+21\,e^{10\,c+10\,d\,x}+7\,e^{12\,c+12\,d\,x}+e^{14\,c+14\,d\,x}+1} - \frac{\frac{4\,a^2}{21\,d}+\frac{20\,a^2\,e^{8\,c+8\,d\,x}}{21\,d}+\frac{8\,e^{4\,c+4\,d\,x}\,(3\,a^2+8\,a\,b+8\,b^2)}{7\,d}+\frac{32\,a\,e^{2\,c+2\,d\,x}\,(a+2\,b)}{21\,d}+\frac{64\,a\,e^{6\,c+6\,d\,x}\,(a+2\,b)}{21\,d}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1} - \frac{\frac{32\,a\,(a+2\,b)}{105\,d}+\frac{16\,a^2\,e^{6\,c+6\,d\,x}}{21\,d}+\frac{16\,e^{2\,c+2\,d\,x}\,(3\,a^2+8\,a\,b+8\,b^2)}{35\,d}+\frac{64\,a\,e^{4\,c+4\,d\,x}\,(a+2\,b)}{35\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1} - \frac{\frac{4\,(3\,a^2+8\,a\,b+8\,b^2)}{35\,d}+\frac{4\,a^2\,e^{4\,c+4\,d\,x}}{7\,d}+\frac{32\,a\,e^{2\,c+2\,d\,x}\,(a+2\,b)}{35\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1} - \frac{4\,a^2}{21\,d\,(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1)}$$

input
```
int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

output
```
- ((32*a*(a + 2*b))/(105*d) + (8*a^2*exp(2*c + 2*d*x))/(21*d))/(3*exp(2*c
+ 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*a^2*exp(2*c +
2*d*x))/(7*d) + (8*a^2*exp(10*c + 10*d*x))/(7*d) + (16*exp(6*c + 6*d*x)*(8
*a*b + 3*a^2 + 8*b^2))/(7*d) + (32*a*exp(4*c + 4*d*x)*(a + 2*b))/(7*d) + (
32*a*exp(8*c + 8*d*x)*(a + 2*b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c +
 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x
) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((4*a^2)/(21*d) + (20
*a^2*exp(8*c + 8*d*x))/(21*d) + (8*exp(4*c + 4*d*x)*(8*a*b + 3*a^2 + 8*b^2
))/(7*d) + (32*a*exp(2*c + 2*d*x)*(a + 2*b))/(21*d) + (64*a*exp(6*c + 6*d*
x)*(a + 2*b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6
*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d
*x) + 1) - ((32*a*(a + 2*b))/(105*d) + (16*a^2*exp(6*c + 6*d*x))/(21*d) +
(16*exp(2*c + 2*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(35*d) + (64*a*exp(4*c + 4*d
*x)*(a + 2*b))/(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(
6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((4*(8*a*b +
 3*a^2 + 8*b^2))/(35*d) + (4*a^2*exp(4*c + 4*d*x))/(7*d) + (32*a*exp(2*c +
 2*d*x)*(a + 2*b))/(35*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (4*a^2)/(21*d*(2*exp(2*c + 2*d*x)
 + exp(4*c + 4*d*x) + 1))
```

3.65 $\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

3.65.1 Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{3}{8}a\left(a^2+4ab+8b^2\right)x + \frac{3a^2(a+4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a^3\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{b^3\tanh(c+dx)}{d}$$

output
```
3/8*a*(a^2+4*a*b+8*b^2)*x+3/8*a^2*(a+4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a^
3*cosh(d*x+c)^3*sinh(d*x+c)/d+b^3*tanh(d*x+c)/d
```

3.65.2 Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{12a(a^2+4ab+8b^2)(c+dx)+8a^2(a+3b)\sinh(2(c+dx))+a^3\sinh(4(c+dx))+32b^3\tanh(c+dx)}{32d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(12*a*(a^2 + 4*a*b + 8*b^2)*(c + d*x) + 8*a^2*(a + 3*b)*Sinh[2*(c + d*x)]
+ a^3*Sinh[4*(c + d*x)] + 32*b^3*Tanh[c + d*x])/(32*d)
```

3.65.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^3}{\sec(ic+idx)^4}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{\left(-b\tanh^2(c+dx)+a+b\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(b^3+\frac{3ab^2\tanh^4(c+dx)-3ab(a+2b)\tanh^2(c+dx)+a\left(a^2+3ba+3b^2\right)}{\left(1-\tanh^2(c+dx)\right)^3}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^3\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}+\frac{3}{8}a\left(a^2+4ab+8b^2\right)\text{arctanh}(\tanh(c+dx))+\frac{3a^2(a+4b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)}+b^3\tanh(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((3*a*(a^2 + 4*a*b + 8*b^2)*ArcTanh[Tanh[c + d*x]])/8 + b^3*Tanh[c + d*x]
+ (a^3*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + (3*a^2*(a + 4*b)*Tanh[
c + d*x])/(8*(1 - Tanh[c + d*x]^2)))/d
```

3.65.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.65.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{a^3\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a\,b^2(dx+c)+b^3\tanh}{d}$
default	$\frac{a^3\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a\,b^2(dx+c)+b^3\tanh}{d}$
parallelrisch	$\frac{(9a^3+24a^2b)\sinh(3dx+3c)+a^3\sinh(5dx+5c)+24adx(a^2+4ab+8b^2)\cosh(dx+c)+8\sinh(dx+c)(a^3+3a^2b+8b^3)}{64d\cosh(dx+c)}$
risch	$\frac{3a^3x}{8}+\frac{3b\,a^2x}{2}+3a\,b^2x+\frac{a^3e^{4dx+4c}}{64d}+\frac{a^3e^{2dx+2c}}{8d}+\frac{3a^2e^{2dx+2c}b}{8d}-\frac{a^3e^{-2dx-2c}}{8d}-\frac{3a^2e^{-2dx-2c}b}{8d}-$

input
```
int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+3
*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a*b^2*(d*x+c)+b^3*tan
h(d*x+c))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.82

$$\int \cosh^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^3 \, dx$$
$$= \frac{a^3 \sinh(dx+c)^5 + \left(10\, a^3 \cosh(dx+c)^2 + 9\, a^3 + 24\, a^2 b\right) \sinh(dx+c)^3 - 8\left(8\, b^3 - 3\left(a^3 + 4\, a^2 b + 8\, ab^2\right)d}{64\, d \cos}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/64*(a^3*sinh(d*x + c)^5 + (10*a^3*cosh(d*x + c)^2 + 9*a^3 + 24*a^2*b)*si
nh(d*x + c)^3 - 8*(8*b^3 - 3*(a^3 + 4*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)
+ (5*a^3*cosh(d*x + c)^4 + 8*a^3 + 24*a^2*b + 64*b^3 + 9*(3*a^3 + 8*a^2*b)
*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \cosh^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.55

$$\int \cosh^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^3 \, dx$$
$$= \frac{1}{64}\, a^3 \left(24\, x + \frac{e^{(4\, dx+4\, c)}}{d} + \frac{8\, e^{(2\, dx+2\, c)}}{d} - \frac{8\, e^{(-2\, dx-2\, c)}}{d} - \frac{e^{(-4\, dx-4\, c)}}{d}\right) + \frac{3}{8}\, a^2 b \left(4\, x + \frac{e^{(2\, dx+2\, c)}}{d} - \frac{e^{(-2\, dx-2\, c)}}{d}\right) + 3\, ab^2 x + \frac{2\, b^3}{d\left(e^{(-2\, dx-2\, c)} + 1\right)}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 3/8*a^2*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d
*x - 2*c)/d) + 3*a*b^2*x + 2*b^3/(d*(e^(-2*d*x - 2*c) + 1))
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(78) = 156.

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.11

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{a^3e^{(4\,dx+4\,c)} + 8\,a^3e^{(2\,dx+2\,c)} + 24\,a^2be^{(2\,dx+2\,c)} + 24\,(a^3+4\,a^2b+8\,ab^2)(dx+c) - \frac{128\,b^3}{e^{(2\,dx+2\,c)}+1} - \left(18\,a^3e^{(4\,dx+\right.}{64\,d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/64*(a^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*
c) + 24*(a^3 + 4*a^2*b + 8*a*b^2)*(d*x + c) - 128*b^3/(e^(2*d*x + 2*c) + 1
) - (18*a^3*e^(4*d*x + 4*c) + 72*a^2*b*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*
x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-4*d
*x - 4*c))/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{3\,a\,x\,(a^2+4\,a\,b+8\,b^2)}{8} - \frac{2\,b^3}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{a^3\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d} + \frac{a^3\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d} - \frac{a^2\,\mathrm{e}^{-2\,c-2\,d\,x}\,(a+3\,b)}{8\,d} + \frac{a^2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+3\,b)}{8\,d}$$

input
```
int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(3*a*x*(4*a*b + a^2 + 8*b^2))/8 - (2*b^3)/(d*(exp(2*c + 2*d*x) + 1)) - (a^
3*exp(- 4*c - 4*d*x))/(64*d) + (a^3*exp(4*c + 4*d*x))/(64*d) - (a^2*exp(-
2*c - 2*d*x)*(a + 3*b))/(8*d) + (a^2*exp(2*c + 2*d*x)*(a + 3*b))/(8*d)
```

3.65. $\int \cosh^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^3 \, dx$

3.66 $\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

3.66.1 Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\frac{b^2(6a+b)\arctan(\sinh(c+dx))}{2d}+\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{b^3\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*b^2*(6*a+b)*arctan(sinh(d*x+c))/d+a^2*(a+3*b)*sinh(d*x+c)/d+1/3*a^3*si
nh(d*x+c)^3/d+1/2*b^3*sech(d*x+c)*tanh(d*x+c)/d
```

3.66.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.02 (sec) , antiderivative size = 483, normalized size of antiderivative = 5.96

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$=\frac{\coth^3(c+dx)\text{csch}^2(c+dx)(a\cosh(c+dx)+b\text{sech}(c+dx))^3\left(-256\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};-\sinh^2(c\right.\right.}{}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Coth[c + d*x]^3*Csch[c + d*x]^2*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(-2
56*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]
*Sinh[c + d*x]^8*(a + b + a*Sinh[c + d*x]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c
 + d*x]^2]]*(b^3*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 - 47*S
inh[c + d*x]^6) + 3*a^2*b*Cosh[c + d*x]^4*(2401 + 1875*Sinh[c + d*x]^2 + 2
43*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cosh[c + d*x]^6*(2401 + 1875*S
inh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*(2401 +
4276*Sinh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 148*Sinh[c + d*x]^6 + Sinh[c
 + d*x]^8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*(36015 + 16120*Sinh[c + d*x]
^2 + 1473*Sinh[c + d*x]^4) + 3*a*b^2*(36015 + 52135*Sinh[c + d*x]^2 + 1759
3*Sinh[c + d*x]^4 + 753*Sinh[c + d*x]^6) + 3*a^2*b*(36015 + 88150*Sinh[c +
 d*x]^2 + 69728*Sinh[c + d*x]^4 + 19786*Sinh[c + d*x]^6 + 753*Sinh[c + d*x
]^8) + a^3*(36015 + 124165*Sinh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 8951
4*Sinh[c + d*x]^6 + 19579*Sinh[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(3780
*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

3.66.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^3}{\sec(ic+idx)^3}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(a\sinh^2(c+dx)+a+b\right)^3}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\sinh^2(c+dx)a^3+(a+3b)a^2+\frac{3a\sinh^2(c+dx)b^2+(3a+b)b^2}{\left(\sinh^2(c+dx)+1\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}a^3\sinh^3(c+dx)+a^2(a+3b)\sinh(c+dx)+\frac{1}{2}b^2(6a+b)\arctan(\sinh(c+dx))+\frac{b^3\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((b^2*(6*a + b)*ArcTan[Sinh[c + d*x]])/2 + a^2*(a + 3*b)*Sinh[c + d*x] + (
a^3*Sinh[c + d*x]^3)/3 + (b^3*Sinh[c + d*x])/(2*(1 + Sinh[c + d*x]^2)))/d
```

3.66.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.66.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+3a^2b\sinh(dx+c)+6a\,b^2\arctan\left(e^{dx+c}\right)+b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+3a^2b\sinh(dx+c)+6a\,b^2\arctan\left(e^{dx+c}\right)+b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
parallelrisch	$\frac{-72i(1+\cosh(2dx+2c))\left(a+\frac{b}{6}\right)b^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+72i(1+\cosh(2dx+2c))\left(a+\frac{b}{6}\right)b^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)+(11a^3}{24d(1+\cosh(2dx+2c))}$
risch	$\frac{a^3e^{3dx+3c}}{24d}+\frac{3a^3e^{dx+c}}{8d}+\frac{3a^2e^{dx+c}b}{2d}-\frac{3a^3e^{-dx-c}}{8d}-\frac{3a^2e^{-dx-c}b}{2d}-\frac{a^3e^{-3dx-3c}}{24d}+\frac{b^3e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(e^{2dx+2c}+1\right)^2}+$

input
```
int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+3*a^2*b*sinh(d*x+c)+6*a*b^2*a
rctan(exp(d*x+c))+b^3*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c))))
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. 2(75) = 150.

Time = 0.27 (sec) , antiderivative size = 1409, normalized size of antiderivative = 17.40

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*si
nh(d*x + c)^10 + (11*a^3 + 36*a^2*b)*cosh(d*x + c)^8 + (45*a^3*cosh(d*x +
c)^2 + 11*a^3 + 36*a^2*b)*sinh(d*x + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 + (1
1*a^3 + 36*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^3 + 18*a^2*b + 1
2*b^3)*cosh(d*x + c)^6 + 2*(105*a^3*cosh(d*x + c)^4 + 5*a^3 + 18*a^2*b + 1
2*b^3 + 14*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^
3*cosh(d*x + c)^5 + 14*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^3 + 3*(5*a^3 + 18
*a^2*b + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^3 + 18*a^2*b + 12
*b^3)*cosh(d*x + c)^4 + 2*(105*a^3*cosh(d*x + c)^6 + 35*(11*a^3 + 36*a^2*b
)*cosh(d*x + c)^4 - 5*a^3 - 18*a^2*b - 12*b^3 + 15*(5*a^3 + 18*a^2*b + 12*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*a^3*cosh(d*x + c)^7 + 7*(11*
a^3 + 36*a^2*b)*cosh(d*x + c)^5 + 5*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x +
 c)^3 - (5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 -
 (11*a^3 + 36*a^2*b)*cosh(d*x + c)^2 + (45*a^3*cosh(d*x + c)^8 + 28*(11*a^
3 + 36*a^2*b)*cosh(d*x + c)^6 + 30*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x +
c)^4 - 11*a^3 - 36*a^2*b - 12*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^2 + 24*((6*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(6*a*b^2 + b^3)
*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a*b^2 + b^3)*sinh(d*x + c)^7 + 2*(6*a*
b^2 + b^3)*cosh(d*x + c)^5 + (12*a*b^2 + 2*b^3 + 21*(6*a*b^2 + b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^5 + 5*(7*(6*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(...
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \cosh^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.66.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(75) = 150$.

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.21

$$\int \cosh^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$= -b^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right)$$
$$+ \frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$
$$+ \frac{3}{2}\,a^2b\left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) - \frac{6\,ab^2\arctan\left(e^{(-dx-c)}\right)}{d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(
-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*
e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 3/2*a^2*b*(e^(d*x
 + c)/d - e^(-d*x - c)/d) - 6*a*b^2*arctan(e^(-d*x - c))/d
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(75) = 150$.

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.01

$$\int \cosh^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 12\,a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 36\,a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + \frac{24\,b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4} + 6\,(}{24\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 12*a^3*(e^(d*x + c) - e^(-d*x -
 c)) + 36*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 24*b^3*(e^(d*x + c) - e^(-d
*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4) + 6*(pi + 2*arctan(1/2*(e^(2
*d*x + 2*c) - 1)*e^(-d*x - c)))*(6*a*b^2 + b^3))/d
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.69

$$\int \cosh^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx$$
$$=\frac{\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\left(b^3\sqrt{d^2}+6\,a\,b^2\sqrt{d^2}\right)}{d\sqrt{36\,a^2\,b^4+12\,a\,b^5+b^6}}\right)\sqrt{36\,a^2\,b^4+12\,a\,b^5+b^6}}{\sqrt{d^2}}-\frac{a^3\,e^{-3\,c-3\,d\,x}}{24\,d}+\frac{a^3\,e^{3\,c+3\,d\,x}}{24\,d}-\frac{3\,a^2\,e^{-c-d\,x}\,(a+4\,b)}{8\,d}+\frac{3\,a^2\,e^{c+d\,x}\,(a+4\,b)}{8\,d}+\frac{b^3\,e^{c+d\,x}}{d\,(e^{2\,c+2\,d\,x}+1)}-\frac{2\,b^3\,e^{c+d\,x}}{d\,(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1)}$$

input
```
int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(b^3*(d^2)^(1/2) + 6*a*b^2*(d^2)^(1/2)))/(d*(12*a*b
^5 + b^6 + 36*a^2*b^4)^(1/2)))*(12*a*b^5 + b^6 + 36*a^2*b^4)^(1/2))/(d^2)^
(1/2) - (a^3*exp(- 3*c - 3*d*x))/(24*d) + (a^3*exp(3*c + 3*d*x))/(24*d) -
(3*a^2*exp(- c - d*x)*(a + 4*b))/(8*d) + (3*a^2*exp(c + d*x)*(a + 4*b))/(8
*d) + (b^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^3*exp(c + d*x))
/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.67 $\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

3.67.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\frac{1}{2}a^2(a+6b)x+\frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}$$

output
```
1/2*a^2*(a+6*b)*x+1/2*a^3*cosh(d*x+c)*sinh(d*x+c)/d+b^2*(3*a+b)*tanh(d*x+c
)/d-1/3*b^3*tanh(d*x+c)^3/d
```

3.67.2 Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\frac{6a^2(a+6b)(c+dx)+3a^3\sinh(2(c+dx))+4b^2\left(9a+2b+b\text{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(6*a^2*(a + 6*b)*(c + d*x) + 3*a^3*Sinh[2*(c + d*x)] + 4*b^2*(9*a + 2*b +
b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

3.67.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^3}{\sec(ic+idx)^2}dx$$

$$\Big\downarrow \; 4634$$

$$\frac{\int \frac{\left(-b\tanh^2(c+dx)+a+b\right)^3}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 300$$

$$\frac{\int \left(-\tanh^2(c+dx)b^3+(3a+b)b^2+\frac{a^2(a+3b)-3a^2b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{a^3\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}+\frac{1}{2}a^2(a+6b)\text{arctanh}(\tanh(c+dx))+b^2(3a+b)\tanh(c+dx)-\frac{1}{3}b^3\tanh^3(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a^2*(a + 6*b)*ArcTanh[Tanh[c + d*x]])/2 + b^2*(3*a + b)*Tanh[c + d*x] -
(b^3*Tanh[c + d*x]^3)/3 + (a^3*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

3.67.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.67.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b(dx+c)+3a\,b^2\tanh(dx+c)+b^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b(dx+c)+3a\,b^2\tanh(dx+c)+b^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
risch	$\frac{a^3x}{2}+3b\,a^2x+\frac{a^3\mathrm{e}^{2dx+2c}}{8d}-\frac{a^3\mathrm{e}^{-2dx-2c}}{8d}-\frac{2b^2\left(9a\,\mathrm{e}^{4dx+4c}+18\,\mathrm{e}^{2dx+2c}a+6b\,\mathrm{e}^{2dx+2c}+9a+2b\right)}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^3}$
parallelrisch	$\frac{12a^2dx(a+6b)\cosh(3dx+3c)+\left(9a^3+72a\,b^2+16b^3\right)\sinh(3dx+3c)+3a^3\sinh(5dx+5c)+36a^2dx(a+6b)\cosh(dx+c)+6\sin}{24d(\cosh(3dx+3c)+3\cosh(dx+c))}$

input
```
int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(d*x+c)+3*a*b
^2*tanh(d*x+c)+b^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(66) = 132$.

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.75

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{3\,a^3\sinh\left(dx+c\right)^5 - 4\left(18\,ab^2+4\,b^3-3\left(a^3+6\,a^2b\right)dx\right)\cosh\left(dx+c\right)^3 - 12\left(18\,ab^2+4\,b^3-3\left(a^3+6\,a^2b\right)\right.}{}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*(3*a^3*sinh(d*x + c)^5 - 4*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)
*cosh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (30*a^3*cosh(d*x + c)^2 + 9*a^3 + 72*a*b^2 + 16*b^3
)*sinh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x
 + c) + 3*(5*a^3*cosh(d*x + c)^4 + 2*a^3 + 24*a*b^2 + 16*b^3 + (9*a^3 + 72
*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(66) = 132$.

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.22

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{1}{8}a^3\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+3a^2bx$$
$$+\frac{4}{3}b^3\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+\frac{6ab^2}{d(e^{(-2dx-2c)}+1)}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + 3*a^2*b*x + 4/3*b
^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) + 1))) + 6*a*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(66) = 132$.

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.11

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$=\frac{3a^3e^{(2dx+2c)}+12(a^3+6a^2b)(dx+c)-3\left(2a^3e^{(2dx+2c)}+12a^2be^{(2dx+2c)}+a^3\right)e^{(-2dx-2c)}-\frac{16(9ab^2e^{(4dx+}}{}}{24d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(3*a^3*e^(2*d*x + 2*c) + 12*(a^3 + 6*a^2*b)*(d*x + c) - 3*(2*a^3*e^(2
*d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-2*d*x - 2*c) - 16*(9*a*b
^2*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*c) + 6*b^3*e^(2*d*x + 2*c) + 9*
a*b^2 + 2*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.07

$$\int \cosh^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{a^2\,x\,(a+6\,b)}{2} - \frac{\frac{2\,a\,b^2}{d}+\frac{4\,e^{2\,c+2\,d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,a\,b^2\,e^{4\,c+4\,d\,x}}{d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1} - \frac{\frac{2\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,a\,b^2\,e^{2\,c+2\,d\,x}}{d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1} - \frac{a^3\,e^{-2\,c-2\,d\,x}}{8\,d} + \frac{a^3\,e^{2\,c+2\,d\,x}}{8\,d} - \frac{2\,a\,b^2}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(a^2*x*(a + 6*b))/2 - ((2*a*b^2)/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + 2*b^3)
)/(3*d) + (2*a*b^2*exp(4*c + 4*d*x))/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a*b^2 + 2*b^3))/(3*d) + (2*a*b^2*e
xp(2*c + 2*d*x))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (a^3*exp
(- 2*c - 2*d*x))/(8*d) + (a^3*exp(2*c + 2*d*x))/(8*d) - (2*a*b^2)/(d*(exp(
2*c + 2*d*x) + 1))
```

3.68 $\int \cosh(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \cosh(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx = \frac{3b(8a^2+4ab+b^2)\arctan(\sinh(c+dx))}{8d} + \frac{a^3\sinh(c+dx)}{d} + \frac{3b^2(4a+b)\mathrm{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b^3\mathrm{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output
```
3/8*b*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c))/d+a^3*sinh(d*x+c)/d+3/8*b^2*(4
*a+b)*sech(d*x+c)*tanh(d*x+c)/d+1/4*b^3*sech(d*x+c)^3*tanh(d*x+c)/d
```

3.68.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.23 (sec) , antiderivative size = 575, normalized size of antiderivative = 6.18

$$\int \cosh(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{\cosh(c+dx)\coth^5(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\left(256\,{}_6F_5\left(\frac{3}{2},2,2,2,2,2;1,1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)\right.}{}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/7560*(Cosh[c + d*x]*Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3*(256*Hype
rgeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*
Sinh[c + d*x]^8*(a + b + a*Sinh[c + d*x]^2)^3 + 384*HypergeometricPFQ[{3/2
, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b +
 a*Sinh[c + d*x]^2)^2*(7*b + a*(7 + 5*Sinh[c + d*x]^2)) + (315*ArcTanh[Sqr
t[-Sinh[c + d*x]^2]]*(b^3*(16807 + 15000*Sinh[c + d*x]^2 + 2187*Sinh[c + d
*x]^4 - 62*Sinh[c + d*x]^6) + a^3*Cosh[c + d*x]^4*(16807 + 24604*Sinh[c +
d*x]^2 + 11562*Sinh[c + d*x]^4 + 1468*Sinh[c + d*x]^6 + 7*Sinh[c + d*x]^8)
 + 3*a*b^2*(16807 + 29406*Sinh[c + d*x]^2 + 15312*Sinh[c + d*x]^4 + 1858*S
inh[c + d*x]^6 + 9*Sinh[c + d*x]^8) + 3*a^2*b*(16807 + 43812*Sinh[c + d*x]
^2 + 40442*Sinh[c + d*x]^4 + 14956*Sinh[c + d*x]^6 + 1719*Sinh[c + d*x]^8
+ 8*Sinh[c + d*x]^10)))/Sqrt[-Sinh[c + d*x]^2] - 21*(b^3*(252105 + 140965*
Sinh[c + d*x]^2 + 8226*Sinh[c + d*x]^4) + 3*a*b^2*(252105 + 357055*Sinh[c
+ d*x]^2 + 133071*Sinh[c + d*x]^4 + 6393*Sinh[c + d*x]^6) + 3*a^2*b*(25210
5 + 573145*Sinh[c + d*x]^2 + 437991*Sinh[c + d*x]^4 + 120431*Sinh[c + d*x]
^6 + 5640*Sinh[c + d*x]^8) + a^3*(252105 + 789235*Sinh[c + d*x]^2 + 922986
*Sinh[c + d*x]^4 + 491574*Sinh[c + d*x]^6 + 107725*Sinh[c + d*x]^8 + 4887*
Sinh[c + d*x]^10))))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

3.68.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^3}{\sec(ic+idx)}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(a\sinh^2(c+dx)+a+b\right)^3}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(a^3+\frac{3a^2b\sinh^4(c+dx)+3ab(2a+b)\sinh^2(c+dx)+b(3a^2+3ba+b^2)}{(\sinh^2(c+dx)+1)^3}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sinh(c+dx)+\frac{3}{8}b(8a^2+4ab+b^2)\arctan(\sinh(c+dx))+\frac{3b^2(4a+b)\sinh(c+dx)}{8(\sinh^2(c+dx)+1)}+\frac{b^3\sinh(c+dx)}{4(\sinh^2(c+dx)+1)^2}}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]])/8 + a^3*Sinh[c + d*x] +
 (b^3*Sinh[c + d*x])/(4*(1 + Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + b)*Sinh[c
 + d*x])/(8*(1 + Sinh[c + d*x]^2)))/d
```

3.68.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.68.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a^3 \sinh(dx+c)+6a^2 b \arctan(e^{dx+c})+3a b^2 \left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+b^3\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\right.}{d}$
default	$\frac{a^3 \sinh(dx+c)+6a^2 b \arctan(e^{dx+c})+3a b^2 \left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+b^3\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\right.}{d}$
parallelrisch	$\frac{-48i\left(\frac{3}{4}+\frac{\cosh(4dx+4c)}{4}+\cosh(2dx+2c)\right)\left(a^2+\frac{1}{2}ab+\frac{1}{8}b^2\right)b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+48i\left(\frac{3}{4}+\frac{\cosh(4dx+4c)}{4}+\cosh(2dx+2c)\right)}{4d(\cosh(4dx+4c}$
risch	$\frac{a^3 e^{dx+c}}{2d}-\frac{a^3 e^{-dx-c}}{2d}+\frac{b^2 e^{dx+c}\left(12a\, e^{6dx+6c}+3b\, e^{6dx+6c}+12a\, e^{4dx+4c}+11b\, e^{4dx+4c}-12\, e^{2dx+2c}a-11b\, e^{2dx+2c}-1\right.}{4d\left(e^{2dx+2c}+1\right)^4}$

input
```
int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*sinh(d*x+c)+6*a^2*b*arctan(exp(d*x+c))+3*a*b^2*(1/2*sech(d*x+c)*t
anh(d*x+c)+arctan(exp(d*x+c)))+b^3*((1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*ta
nh(d*x+c)+3/4*arctan(exp(d*x+c))))
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1992 vs. 2(87) = 174.

Time = 0.29 (sec) , antiderivative size = 1992, normalized size of antiderivative = 21.42

$$\int \cosh(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*a^3*cosh(d*x + c)^10 + 20*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + 2*a^3
*sinh(d*x + c)^10 + 3*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^8 + 3*(30*a^3*
cosh(d*x + c)^2 + 2*a^3 + 4*a*b^2 + b^3)*sinh(d*x + c)^8 + 24*(10*a^3*cosh
(d*x + c)^3 + (2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (4*
a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)^6 + (420*a^3*cosh(d*x + c)^4 + 4*a^
3 + 12*a*b^2 + 11*b^3 + 84*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 6*(84*a^3*cosh(d*x + c)^5 + 28*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*
x + c)^3 + (4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4
*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + (420*a^3*cosh(d*x + c)^6 + 210
*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^4 - 4*a^3 - 12*a*b^2 - 11*b^3 + 15*
(4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*a^3*c
osh(d*x + c)^7 + 42*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^3 + 1
2*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - (4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^3 - 2*a^3 - 3*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^2 +
 3*(30*a^3*cosh(d*x + c)^8 + 28*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^6 +
5*(4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)^4 - 2*a^3 - 4*a*b^2 - b^3 - 2*
(4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((8*a^2*b
 + 4*a*b^2 + b^3)*cosh(d*x + c)^9 + 9*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x +
 c)*sinh(d*x + c)^8 + (8*a^2*b + 4*a*b^2 + b^3)*sinh(d*x + c)^9 + 4*(8*a^2
*b + 4*a*b^2 + b^3)*cosh(d*x + c)^7 + 4*(8*a^2*b + 4*a*b^2 + b^3 + 9*(8...
```

3.68.6 Sympy [F]

$$\int \cosh(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx=\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^3\cosh\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*cosh(c + d*x), x)
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(87) = 174$.

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.38

$$\int \cosh(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx = -\frac{1}{4}b^3\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right) - 3\,ab^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) - \frac{6\,a^2b\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{a^3\sinh(dx+c)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*b^3*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c)
 - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^
(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3*a*b^2*(a
rctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x
 - 2*c) + e^(-4*d*x - 4*c) + 1))) - 6*a^2*b*arctan(e^(-d*x - c))/d + a^3*si
nh(d*x + c)/d
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(87) = 174$.

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.14

$$\int \cosh(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx = \frac{8\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(8\,a^2b+4\,ab^2+b^3)+\frac{4\left(12\,ab^2\left(e^{(dx+c)}\right.\right.}{}}{16\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/16*(8*a^3*(e^(d*x + c) - e^(-d*x - c)) + 3*(pi + 2*arctan(1/2*(e^(2*d*x
+ 2*c) - 1)*e^(-d*x - c)))*(8*a^2*b + 4*a*b^2 + b^3) + 4*(12*a*b^2*(e^(d*x
 + c) - e^(-d*x - c))^3 + 3*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 48*a*b^2*
(e^(d*x + c) - e^(-d*x - c)) + 20*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d
*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.70

$$\begin{aligned}
&\int \cosh(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx \\
&= \frac{a^3\,\mathrm{e}^{c+d\,x}}{2\,d}-\frac{a^3\,\mathrm{e}^{-c-d\,x}}{2\,d} \\
&\quad+\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^3\,\sqrt{d^2}+4\,a\,b^2\,\sqrt{d^2}+8\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4\,b^2+64\,a^3\,b^3+32\,a^2\,b^4+8\,a\,b^5+b^6}}\right)\,\sqrt{64\,a^4\,b^2+64\,a^3\,b^3+32\,a^2\,b^4+8\,a\,b^5+b^6}}{4\,\sqrt{d^2}} \\
&\quad-\frac{6\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{\mathrm{e}^{c+d\,x}\,\left(12\,a\,b^2-b^3\right)}{2\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} \\
&\quad+\frac{4\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{3\,\mathrm{e}^{c+d\,x}\,\left(b^3+4\,a\,b^2\right)}{4\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}
\end{aligned}$$

input
```
int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(a^3*exp(c + d*x))/(2*d) - (a^3*exp(- c - d*x))/(2*d) + (3*atan((exp(d*x)*
exp(c)*(b^3*(d^2)^(1/2) + 4*a*b^2*(d^2)^(1/2) + 8*a^2*b*(d^2)^(1/2)))/(d*(
8*a*b^5 + b^6 + 32*a^2*b^4 + 64*a^3*b^3 + 64*a^4*b^2)^(1/2)))*(8*a*b^5 + b
^6 + 32*a^2*b^4 + 64*a^3*b^3 + 64*a^4*b^2)^(1/2))/(4*(d^2)^(1/2)) - (6*b^3
*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d
*x) + 1)) - (exp(c + d*x)*(12*a*b^2 - b^3))/(2*d*(2*exp(2*c + 2*d*x) + exp
(4*c + 4*d*x) + 1)) + (4*b^3*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (3*exp(c + d*
x)*(4*a*b^2 + b^3))/(4*d*(exp(2*c + 2*d*x) + 1))
```

3.69 $\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 147

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx &= \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\arctan(\sinh(c+dx))}{16d} \\ &+ \frac{b(44a^2+44ab+15b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} \\ &+ \frac{5b(2a+b)\operatorname{sech}^3(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{24d} \\ &+ \frac{b\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d}\end{aligned}$$

output
```
1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/48*b*(44*a^2+44*a
*b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d+5/24*b*(2*a+b)*sech(d*x+c)^3*(a+b+a*s
inh(d*x+c)^2)*tanh(d*x+c)/d+1/6*b*sech(d*x+c)^5*(a+b+a*sinh(d*x+c)^2)^2*ta
nh(d*x+c)/d
```

3.69.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.49 (sec) , antiderivative size = 1430, normalized size of antiderivative = 9.73

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Coth[c + d*x]^6*Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(117228825*(a + b
)^3*Sinh[c + d*x]^2 + 274542345*a*(a + b)^2*Sinh[c + d*x]^4 + 70189350*(a
+ b)^3*Sinh[c + d*x]^4 + 215549775*a^2*(a + b)*Sinh[c + d*x]^6 + 168951510
*a*(a + b)^2*Sinh[c + d*x]^6 + 4093425*(a + b)^3*Sinh[c + d*x]^6 + 5800945
5*a^3*Sinh[c + d*x]^8 + 135323370*a^2*(a + b)*Sinh[c + d*x]^8 + 9514449*a*
(a + b)^2*Sinh[c + d*x]^8 + 36772890*a^3*Sinh[c + d*x]^10 + 7808535*a^2*(a
 + b)*Sinh[c + d*x]^10 - 75520*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2,
2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 - 13824*(a + b)^3*
HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]
^2]*Sinh[c + d*x]^10 - 1024*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2,
2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 + 2160711
*a^3*Sinh[c + d*x]^12 - 189696*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2
, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 38400*a*(a + b
)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c +
d*x]^2]*Sinh[c + d*x]^12 - 3072*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2,
2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 1
58976*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -S
inh[c + d*x]^2]*Sinh[c + d*x]^14 - 35328*a^2*(a + b)*HypergeometricPFQ[{3/
2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^14
- 3072*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1,...
```

3.69.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4635, 315, 401, 25, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(ic+idx)\left(a+b\sec(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \; 4635$$

$$\frac{\int \frac{\left(a\sinh^2(c+dx)+a+b\right)^3}{\left(\sinh^2(c+dx)+1\right)^4}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 315$$

$$\frac{\frac{1}{6}\int \frac{\left(a\sinh^2(c+dx)+a+b\right)\left(a(6a+b)\sinh^2(c+dx)+(a+b)(6a+5b)\right)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)+\frac{b\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)^2}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\Big\downarrow \; 401$$

$$\frac{\frac{1}{6}\left(\frac{5b(2a+b)\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{4\left(\sinh^2(c+dx)+1\right)^2}-\frac{1}{4}\int -\frac{a\left(24a^2+14ba+5b^2\right)\sinh^2(c+dx)+(a+b)\left(24a^2+34ba+15b^2\right)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)\right)+\frac{b\,s\ldots}{}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\int \frac{a\left(24a^2+14ba+5b^2\right)\sinh^2(c+dx)+(a+b)\left(24a^2+34ba+15b^2\right)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)+\frac{5b(2a+b)\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{b\sin\ldots}{}}{d}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{b\left(44a^2+44ab+15b^2\right)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{5b(2a+b)\sinh(c+dx)(\ldots}{4(\sinh^2(c+\ldots}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}(2a+b)\left(8a^2+8ab+5b^2\right)\arctan(\sinh(c+dx))+\frac{b\left(44a^2+44ab+15b^2\right)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{5b(2a+b)\sinh(c+dx)\left(a\sinh^2(c+\ldots\right.}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

3.69. $\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3dx$

output
```
((b*Sinh[c + d*x]*(a + b + a*Sinh[c + d*x]^2)^2)/(6*(1 + Sinh[c + d*x]^2)^
3) + ((5*b*(2*a + b)*Sinh[c + d*x]*(a + b + a*Sinh[c + d*x]^2))/(4*(1 + Si
nh[c + d*x]^2)^2) + ((3*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c +
d*x]])/2 + (b*(44*a^2 + 44*a*b + 15*b^2)*Sinh[c + d*x])/(2*(1 + Sinh[c + d
*x]^2)))/4)/6)/d
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.69. $\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.69.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2a^3\arctan\left(e^{dx+c}\right)+3a^2b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a\,b^2\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3}{}\right.}{d}$
default	$\frac{2a^3\arctan\left(e^{dx+c}\right)+3a^2b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a\,b^2\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3}{}\right.}{d}$
parts	$\frac{a^3\arctan(\sinh(dx+c))}{d}+\frac{b^3\left(\left(\frac{\operatorname{sech}(dx+c)^5}{6}+\frac{5\operatorname{sech}(dx+c)^3}{24}+\frac{5\operatorname{sech}(dx+c)}{16}\right)\tanh(dx+c)+\frac{5\arctan\left(e^{dx+c}\right)}{8}\right)}{d}+\frac{3a^2b(}{}$
parallelrisch	$\frac{-15i\left(\frac{b}{2}+a\right)\left(\frac{2}{3}+\frac{\cosh(6dx+6c)}{15}+\frac{2\cosh(4dx+4c)}{5}+\cosh(2dx+2c)\right)\left(a^2+ab+\frac{5}{8}b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+15i\left(\frac{b}{2}+a\right)\left(\frac{2}{3}+\right.}{}$
risch	$\frac{b\,e^{dx+c}\left(72a^2e^{10dx+10c}+54ab\,e^{10dx+10c}+15b^2e^{10dx+10c}+216a^2e^{8dx+8c}+306ab\,e^{8dx+8c}+85b^2e^{8dx+8c}+144a^2e^{6dx+6c}+\right.}{}$

input
```
int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^3*arctan(exp(d*x+c))+3*a^2*b*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(
exp(d*x+c)))+3*a*b^2*((1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+3/4*
arctan(exp(d*x+c)))+b^3*((1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d
*x+c))*tanh(d*x+c)+5/8*arctan(exp(d*x+c))))
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3465 vs. $2(139) = 278$.

Time = 0.29 (sec) , antiderivative size = 3465, normalized size of antiderivative = 23.57

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*(3*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^11 + 33*(24*a^2*b + 18
*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(24*a^2*b + 18*a*b^2 +
5*b^3)*sinh(d*x + c)^11 + (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^9
 + (216*a^2*b + 306*a*b^2 + 85*b^3 + 165*(24*a^2*b + 18*a*b^2 + 5*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d
*x + c)^3 + (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c))*sinh(d*x + c)^
8 + 18*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^7 + 18*(55*(24*a^2*b +
18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 14*a*b^2 + 11*b^3 + 2*(216*a
^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(24*a
^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 2*(216*a^2*b + 306*a*b^2 + 85*b
^3)*cosh(d*x + c)^3 + 3*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(
d*x + c)^6 - 18*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 18*(77*(24
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 7*(216*a^2*b + 306*a*b^2 + 85
*b^3)*cosh(d*x + c)^4 - 8*a^2*b - 14*a*b^2 - 11*b^3 + 21*(8*a^2*b + 14*a*b
^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 18*a*b^
2 + 5*b^3)*cosh(d*x + c)^7 + 7*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x +
 c)^5 + 35*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - 5*(8*a^2*b + 14
*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - (216*a^2*b + 306*a*b^2 +
 85*b^3)*cosh(d*x + c)^3 + (495*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c
)^8 + 84*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^6 + 630*(8*a^2*...
```

3.69.6 Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3\operatorname{sech}\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x), x)
```

3.69.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(139) = 278$.

Time = 0.29 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.48

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = -\frac{1}{24}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx}}\right. - \frac{3}{4}ab^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right) - 3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) + \frac{a^3\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3
*c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) -
15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e
^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x
- 12*c) + 1))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 1
1*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2
*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) +
 1))) - 3*a^2*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c)
)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*arctan(sinh(d*x +
 c))/d
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(139) = 278$.

Time = 0.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.11

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)+\frac{4\left(72\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+54\,ab}{}}{}$$

input
```
integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(16*a^3 +
24*a^2*b + 18*a*b^2 + 5*b^3) + 4*(72*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5
+ 54*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 15*b^3*(e^(d*x + c) - e^(-d*x
- c))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 + 576*a*b^2*(e^(d*x + c
) - e^(-d*x - c))^3 + 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 1152*a^2*b*
(e^(d*x + c) - e^(-d*x - c)) + 1440*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 5
28*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^
3)/d
```

3.69.9 Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.64

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^c\,\left(16\,a^3\,\sqrt{d^2}+5\,b^3\,\sqrt{d^2}+18\,a\,b^2\,\sqrt{d^2}+24\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{256\,a^6+768\,a^5\,b+1152\,a^4\,b^2+1024\,a^3\,b^3+564\,a^2\,b^4+180\,a\,b^5+25\,b^6}}\right)\,\sqrt{256\,a^6+768\,a^5\,b+1152\,a^4\,b^2+1024\,a^3\,b^3+}}{8\,\sqrt{d^2}}$$
$$-\frac{e^{c+d\,x}\,\left(54\,a\,b^2-b^3\right)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$+\frac{80\,b^3\,e^{c+d\,x}}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$+\frac{6\,e^{c+d\,x}\,\left(2\,a\,b^2-3\,b^3\right)}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$-\frac{32\,b^3\,e^{c+d\,x}}{3\,d\,\left(6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}$$
$$+\frac{e^{c+d\,x}\,\left(24\,a^2\,b+18\,a\,b^2+5\,b^3\right)}{8\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(-72\,a^2\,b+18\,a\,b^2+5\,b^3\right)}{12\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x),x)
```

output
```
(atan((exp(d*x)*exp(c)*(16*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 18*a*b^2*
(d^2)^(1/2) + 24*a^2*b*(d^2)^(1/2)))/(d*(180*a*b^5 + 768*a^5*b + 256*a^6 +
 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2)))*(180*a*b^5 +
768*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^
(1/2))/(8*(d^2)^(1/2)) - (exp(c + d*x)*(54*a*b^2 - b^3))/(3*d*(3*exp(2*c +
 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (80*b^3*exp(c + d*
x))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) +
 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*exp(c + d*x)*(2*a*b^2
- 3*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x)
 + exp(8*c + 8*d*x) + 1)) - (32*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d*x)
 + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp
(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (exp(c + d*x)*(18*a*b^2 + 24*
a^2*b + 5*b^3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(18*a*b^2 - 7
2*a^2*b + 5*b^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.69. $\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.70 $\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.70.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b(a+b)^2\tanh^3(c+dx)}{d}+\frac{3b^2(a+b)\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^7(c+dx)}{7d}$$

output
```
(a+b)^3*tanh(d*x+c)/d-b*(a+b)^2*tanh(d*x+c)^3/d+3/5*b^2*(a+b)*tanh(d*x+c)^
5/d-1/7*b^3*tanh(d*x+c)^7/d
```

3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(74) = 148.

Time = 3.62 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.19

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\frac{a^3\tanh(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{3ab^2\tanh(c+dx)}{d}+\frac{b^3\tanh(c+dx)}{d}-\frac{a^2b\tanh^3(c+dx)}{d}-\frac{2ab^2\tanh^3(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{3b^3\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^7(c+dx)}{7d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(a^3*Tanh[c + d*x])/d + (3*a^2*b*Tanh[c + d*x])/d + (3*a*b^2*Tanh[c + d*x]
)/d + (b^3*Tanh[c + d*x])/d - (a^2*b*Tanh[c + d*x]^3)/d - (2*a*b^2*Tanh[c
+ d*x]^3)/d - (b^3*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]^5)/(5*d) +
(3*b^3*Tanh[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*x]^7)/(7*d)
```

3.70.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^2(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(ic+idx)^2 \left(a+b\sec(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \left(-b\tanh^2(c+dx)+a+b\right)^3 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\int \left(-b^3\tanh^6(c+dx)+3b^2(a+b)\tanh^4(c+dx)-3b(a+b)^2\tanh^2(c+dx)+a^3\left(\frac{b(3a^2+3ba+b^2)}{a^3}+1\right)\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3}{5}b^2(a+b)\tanh^5(c+dx)-b(a+b)^2\tanh^3(c+dx)+(a+b)^3\tanh(c+dx)-\frac{1}{7}b^3\tanh^7(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + b)^3*Tanh[c + d*x] - b*(a + b)^2*Tanh[c + d*x]^3 + (3*b^2*(a + b)*Ta
nh[c + d*x]^5)/5 - (b^3*Tanh[c + d*x]^7)/7)/d
```

3.70.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.70.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{a^3\tanh(dx+c)+3a^2b\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a\,b^2\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)+b^3\left(\frac{16}{35}+\frac{\mathrm{sec}}{}\right.}{d}$
default	$\frac{a^3\tanh(dx+c)+3a^2b\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a\,b^2\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)+b^3\left(\frac{16}{35}+\frac{\mathrm{sec}}{}\right.}{d}$
parts	$\frac{a^3\tanh(dx+c)}{d}+\frac{b^3\left(\frac{16}{35}+\frac{\mathrm{sech}(dx+c)^6}{7}+\frac{6\,\mathrm{sech}(dx+c)^4}{35}+\frac{8\,\mathrm{sech}(dx+c)^2}{35}\right)\tanh(dx+c)}{d}+\frac{3a\,b^2\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}}{}\right.}{d}$
parallelrisch	$\frac{(315a^3+1050a^2b+1176a\,b^2+336b^3)\sinh(3dx+3c)+(175a^3+490a^2b+392a\,b^2+112b^3)\sinh(5dx+5c)+(35a^3+70a^2b+56}{35d(\cosh(7dx+7c)+7\cosh(5dx+5c)+21\cosh(3dx+3c)+35\cos}$
risch	$-\frac{2(35a^3e^{12dx+12c}+210a^3e^{10dx+10c}+210a^2b\,e^{10dx+10c}+525a^3e^{8dx+8c}+910a^2b\,e^{8dx+8c}+560a\,b^2e^{8dx+8c}+700a^3e^{6dx}}{}$

input
```
int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tanh(d*x+c)+3*a^2*b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a*b^2*(
8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(16/35+1/7*sech
(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(70) = 140$.

Time = 0.27 (sec) , antiderivative size = 816, normalized size of antiderivative = 11.03

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$
$$-\frac{4\left((35a^3+35a^2b+28ab^2+8b^3)\cosh(dx+c)^6-6(35a^2b+28ab^2+8b^3)\cosh(dx+c)\sinh(dx+c)\right.}{}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-4/35*((35*a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^6 - 6*(35*a^2*
b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 35*a^2*b +
 28*a*b^2 + 8*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*
b^3)*cosh(d*x + c)^4 + (210*a^3 + 350*a^2*b + 196*a*b^2 + 56*b^3 + 15*(35*
a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5
*(35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 7*a*b^2 + 2
*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 350*a^3 + 770*a^2*b + 700*a*b^2 + 2
80*b^3 + 7*(75*a^3 + 155*a^2*b + 124*a*b^2 + 24*b^3)*cosh(d*x + c)^2 + (15
*(35*a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 525*a^3 + 1085*a
^2*b + 868*a*b^2 + 168*b^3 + 84*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^2 - 2*(3*(35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*
x + c)^5 + 56*(5*a^2*b + 7*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 7*(25*a^2*b +
44*a*b^2 + 24*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*
cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 8*d*cosh(d*x + c)^6 +
4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(14*d*cosh(d*x + c)^3 +
9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d
*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x
 + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^3 + 56*d
*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 42*d*co
sh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 9*d*co...
```

3.70.6 Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\operatorname{sech}^2(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**2, x)
```

3.70.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(70) = 140$.

Time = 0.21 (sec) , antiderivative size = 695, normalized size of antiderivative = 9.39

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$=\frac{32}{35}b^3\left(\frac{7e^{(-2dx-2c)}}{d(7e^{(-2dx-2c)}+21e^{(-4dx-4c)}+35e^{(-6dx-6c)}+35e^{(-8dx-8c)}+21e^{(-10dx-10c)}+7e^{(-12dx-12c)}+e\right.$$
$$+\frac{16}{5}ab^2\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{}{d(5e^{(-2dx-2}}\right.$$
$$+4a^2b\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right.$$
$$+\frac{2a^3}{d(e^{(-2dx-2c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
32/35*b^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e
^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
 - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c
) + 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-1
0*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 16/5*a*
b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-
4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) + 1))) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2
*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a^3/(d*(e^(-2*d*x -
 2*c) + 1))
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(70) = 140$.

Time = 0.31 (sec) , antiderivative size = 302, normalized size of antiderivative = 4.08

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{2\left(35\,a^3e^{(12\,dx+12\,c)}+210\,a^3e^{(10\,dx+10\,c)}+210\,a^2be^{(10\,dx+10\,c)}+525\,a^3e^{(8\,dx+8\,c)}+910\,a^2be^{(8\,dx+8\,c)}+560\,a\right.}{}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-2/35*(35*a^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) + 210*a^2*b*e^
(10*d*x + 10*c) + 525*a^3*e^(8*d*x + 8*c) + 910*a^2*b*e^(8*d*x + 8*c) + 56
0*a*b^2*e^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 1540*a^2*b*e^(6*d*x +
6*c) + 1400*a*b^2*e^(6*d*x + 6*c) + 560*b^3*e^(6*d*x + 6*c) + 525*a^3*e^(4
*d*x + 4*c) + 1260*a^2*b*e^(4*d*x + 4*c) + 1176*a*b^2*e^(4*d*x + 4*c) + 33
6*b^3*e^(4*d*x + 4*c) + 210*a^3*e^(2*d*x + 2*c) + 490*a^2*b*e^(2*d*x + 2*c
) + 392*a*b^2*e^(2*d*x + 2*c) + 112*b^3*e^(2*d*x + 2*c) + 35*a^3 + 70*a^2*
b + 56*a*b^2 + 16*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 978, normalized size of antiderivative = 13.22

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$

$$= -\frac{\frac{2\left(5a^3+18a^2b+24ab^2+16b^3\right)}{35d}+\frac{2a^3e^{6c+6dx}}{7d}+\frac{6ae^{2c+2dx}\left(5a^2+16ab+16b^2\right)}{35d}+\frac{6a^2e^{4c+4dx}(a+2b)}{7d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$

$$-\frac{\frac{2a^2(a+2b)}{7d}+\frac{2a^3e^{2c+2dx}}{7d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$

$$-\frac{\frac{2a\left(5a^2+16ab+16b^2\right)}{35d}+\frac{8e^{2c+2dx}\left(5a^3+18a^2b+24ab^2+16b^3\right)}{35d}+\frac{2a^3e^{8c+8dx}}{7d}+\frac{12ae^{4c+4dx}\left(5a^2+16ab+16b^2\right)}{35d}+\frac{8a^2e^{6c+6a}}{7c}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$

$$-\frac{\frac{2a^3}{7d}+\frac{8e^{6c+6dx}\left(5a^3+18a^2b+24ab^2+16b^3\right)}{7d}+\frac{2a^3e^{12c+12dx}}{7d}+\frac{6ae^{4c+4dx}\left(5a^2+16ab+16b^2\right)}{7d}+\frac{6ae^{8c+8dx}\left(5a^2+16ab+16l\right.}{7d}}{7e^{2c+2dx}+21e^{4c+4dx}+35e^{6c+6dx}+35e^{8c+8dx}+21e^{10c+10dx}+7e^{12c+12d}}$$

$$-\frac{\frac{2a\left(5a^2+16ab+16b^2\right)}{35d}+\frac{2a^3e^{4c+4dx}}{7d}+\frac{4a^2e^{2c+2dx}(a+2b)}{7d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$

$$-\frac{\frac{2a^2(a+2b)}{7d}+\frac{4e^{4c+4dx}\left(5a^3+18a^2b+24ab^2+16b^3\right)}{7d}+\frac{2a^3e^{10c+10dx}}{7d}+\frac{2ae^{2c+2dx}\left(5a^2+16ab+16b^2\right)}{7d}+\frac{4ae^{6c+6dx}\left(5a^2+16\right.}{7d}}{6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+}$$

$$-\frac{2a^3}{7d\left(e^{2c+2dx}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

output
```
- ((2*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(35*d) + (2*a^3*exp(6*c + 6*
d*x))/(7*d) + (6*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (6
*a^2*exp(4*c + 4*d*x)*(a + 2*b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c +
4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*a^2*(a + 2*b))/(
7*d) + (2*a^3*exp(2*c + 2*d*x))/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d
*x) + 1) - ((2*a*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (8*exp(2*c + 2*d*x)*(
24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(35*d) + (2*a^3*exp(8*c + 8*d*x))/(
7*d) + (12*a*exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (8*a^2*e
xp(6*c + 6*d*x)*(a + 2*b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x
) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (
(2*a^3)/(7*d) + (8*exp(6*c + 6*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3)
)/(7*d) + (2*a^3*exp(12*c + 12*d*x))/(7*d) + (6*a*exp(4*c + 4*d*x)*(16*a*b
 + 5*a^2 + 16*b^2))/(7*d) + (6*a*exp(8*c + 8*d*x)*(16*a*b + 5*a^2 + 16*b^2
))/(7*d) + (12*a^2*exp(2*c + 2*d*x)*(a + 2*b))/(7*d) + (12*a^2*exp(10*c +
10*d*x)*(a + 2*b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*e
xp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c
 + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((2*a*(16*a*b + 5*a^2 + 16*b^2))/(3
5*d) + (2*a^3*exp(4*c + 4*d*x))/(7*d) + (4*a^2*exp(2*c + 2*d*x)*(a + 2*b))
/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) -
 ((2*a^2*(a + 2*b))/(7*d) + (4*exp(4*c + 4*d*x)*(24*a*b^2 + 18*a^2*b + ...
```

3.70. $\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.71 $\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.71.1 Optimal result

Integrand size = 23, antiderivative size = 196

$$\begin{aligned}
&\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx\\
&=\frac{(64a^3+144a^2b+120ab^2+35b^3)\arctan(\sinh(c+dx))}{128d}\\
&\quad+\frac{(64a^3+144a^2b+120ab^2+35b^3)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d}\\
&\quad+\frac{b(72a^2+92ab+35b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d}\\
&\quad+\frac{b(12a+7b)\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{48d}\\
&\quad+\frac{b\operatorname{sech}^7(c+dx)\left(a+b+a\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d}
\end{aligned}$$

output
```
1/128*(64*a^3+144*a^2*b+120*a*b^2+35*b^3)*arctan(sinh(d*x+c))/d+1/128*(64*
a^3+144*a^2*b+120*a*b^2+35*b^3)*sech(d*x+c)*tanh(d*x+c)/d+1/192*b*(72*a^2+
92*a*b+35*b^2)*sech(d*x+c)^3*tanh(d*x+c)/d+1/48*b*(12*a+7*b)*sech(d*x+c)^5
*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d+1/8*b*sech(d*x+c)^7*(a+b+a*sinh(d*x+c
)^2)^2*tanh(d*x+c)/d
```

3.71.2 Mathematica [A] (verified)

Time = 10.73 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.52

$$\int \text{sech}^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$=\frac{\left(b+a\cosh^2(c+dx)\right)^3\text{sech}^8(c+dx)\left(6(64a^3+144a^2b+120ab^2+35b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\cosh^8(\right.}{}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^8*(6*(64*a^3 + 144*a^2*b + 120*a*
b^2 + 35*b^3)*ArcTan[Tanh[(c + d*x)/2]]*Cosh[c + d*x]^8 + 48*b^3*Sech[c]*S
inh[d*x] + 8*b^2*(24*a + 7*b)*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 2*b*(144
*a^2 + 120*a*b + 35*b^2)*Cosh[c + d*x]^4*Sech[c]*Sinh[d*x] + 3*(64*a^3 + 1
44*a^2*b + 120*a*b^2 + 35*b^3)*Cosh[c + d*x]^6*Sech[c]*Sinh[d*x] + 48*b^3*
Cosh[c + d*x]*Tanh[c] + 8*b^2*(24*a + 7*b)*Cosh[c + d*x]^3*Tanh[c] + 2*b*(
144*a^2 + 120*a*b + 35*b^2)*Cosh[c + d*x]^5*Tanh[c] + 3*(64*a^3 + 144*a^2*
b + 120*a*b^2 + 35*b^3)*Cosh[c + d*x]^7*Tanh[c]))/(48*d*(a + 2*b + a*Cosh[
2*(c + d*x)])^3)
```

3.71.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4635, 315, 401, 25, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \sec(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^3dx$$
$$\Big\downarrow \ 4635$$
$$\frac{\int \frac{\left(a\sinh^2(c+dx)+a+b\right)^3}{\left(\sinh^2(c+dx)+1\right)^5}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{1}{8}\int \frac{\left(a\sinh^2(c+dx)+a+b\right)\left(a(8a+3b)\sinh^2(c+dx)+(a+b)(8a+7b)\right)}{\left(\sinh^2(c+dx)+1\right)^4}d\sinh(c+dx)+\frac{b\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)^2}{8\left(\sinh^2(c+dx)+1\right)^4}}{d}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{1}{8}\left(\frac{b(12a+7b)\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{6\left(\sinh^2(c+dx)+1\right)^3}-\frac{1}{6}\int -\frac{3a\left(16a^2+18ba+7b^2\right)\sinh^2(c+dx)+(a+b)\left(48a^2+78ba+35b^2\right)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)\right)+}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{3a\left(16a^2+18ba+7b^2\right)\sinh^2(c+dx)+(a+b)\left(48a^2+78ba+35b^2\right)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)+\frac{b(12a+7b)\sinh(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{6\left(\sinh^2(c+dx)+1\right)^3}\right)+\frac{b\,s}{}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3+144a^2b+120ab^2+35b^3\right)\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)+\frac{b\left(72a^2+92ab+35b^2\right)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{b(12a+7b}{}\right.}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3+144a^2b+120ab^2+35b^3\right)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{b\left(72a^2+92ab+35b^2\right)\text{si}}{4\left(\sinh^2(c+dx)+\right.}\right.\right.}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{b\left(72a^2+92ab+35b^2\right)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}+\frac{3}{4}\left(64a^3+144a^2b+120ab^2+35b^3\right)\left(\frac{1}{2}\arctan(\sinh(c+dx))+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)\right.\right.}{d}$$

input

```
Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

output

```
((b*Sinh[c + d*x]*(a + b + a*Sinh[c + d*x]^2)^2)/(8*(1 + Sinh[c + d*x]^2)^
4) + ((b*(12*a + 7*b)*Sinh[c + d*x]*(a + b + a*Sinh[c + d*x]^2))/(6*(1 + S
inh[c + d*x]^2)^3) + ((b*(72*a^2 + 92*a*b + 35*b^2)*Sinh[c + d*x])/(4*(1 +
 Sinh[c + d*x]^2)^2) + (3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*(ArcTa
n[Sinh[c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Sinh[c + d*x]^2))))/4)/6)/8)/d
```

3.71. $\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.71.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(\mathrm{e}^{dx+c})\right)+3a^2b\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(\mathrm{e}^{dx+c}\right)}{4}\right)+3a\,b^2}{}$
default	$\frac{a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(\mathrm{e}^{dx+c})\right)+3a^2b\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(\mathrm{e}^{dx+c}\right)}{4}\right)+3a\,b^2}{}$
parts	$\frac{a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(\mathrm{e}^{dx+c})\right)}{d}+\frac{b^3\left(\left(\frac{\operatorname{sech}(dx+c)^7}{8}+\frac{7\operatorname{sech}(dx+c)^5}{48}+\frac{35\operatorname{sech}(dx+c)^3}{192}+\frac{35\operatorname{sech}(dx+c)}{128}\right)\tanh(\right.}{d}$
parallelrisch	$\frac{-10752i\left(a^3+\frac{9}{4}a^2b+\frac{15}{8}a\,b^2+\frac{35}{64}b^3\right)\left(\frac{5}{8}+\frac{\cosh(8dx+8c)}{56}+\frac{\cosh(6dx+6c)}{7}+\frac{\cosh(4dx+4c)}{2}+\cosh(2dx+2c)\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{}$
risch	$\frac{\mathrm{e}^{dx+c}\left(-360a\,b^2+960a^3\mathrm{e}^{12dx+12c}+2681b^3\mathrm{e}^{10dx+10c}+2760a\,b^2\mathrm{e}^{12dx+12c}-192a^3-2681\,\mathrm{e}^{4dx+4c}b^3+960a^3\mathrm{e}^{8dx+8c}-432\right.}{}$

input
```
int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a^2*b*((1/4*se
ch(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+3/4*arctan(exp(d*x+c)))+3*a*b^2*(
(1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+c))*tanh(d*x+c)+5/8*ar
ctan(exp(d*x+c)))+b^3*((1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35/192*sech(d
*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)+35/64*arctan(exp(d*x+c))))
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6114 vs. $2(186) = 372$.

Time = 0.31 (sec) , antiderivative size = 6114, normalized size of antiderivative = 31.19

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.71.6 Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\operatorname{sech}^3(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**3, x)
```

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(186) = 372$.

Time = 0.28 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.84

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{1}{192}b^3\left(\frac{105\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{105\,e^{(-dx-c)}+805\,e^{(-3\,dx-3\,c)}+2681\,e^{(-5\,dx-5\,c)}+5053\,e^{(-7\,dx-7\,c)}-50\ldots}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\ldots}\right.$$

$$-\frac{1}{8}ab^2\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\ldots}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx\ldots}}\right.$$

$$-\frac{3}{4}a^2b\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$

$$-a^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/192*b^3*(105*arctan(e^(-d*x - c))/d - (105*e^(-d*x - c) + 805*e^(-3*d*x
 - 3*c) + 2681*e^(-5*d*x - 5*c) + 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x -
 9*c) - 2681*e^(-11*d*x - 11*c) - 805*e^(-13*d*x - 13*c) - 105*e^(-15*d*x
- 15*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c
) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) +
8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(15*arctan(e^
(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5
*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15*e^(-11*d*x - 11*c))/
(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^
(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^
2*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11
*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d
*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^3*(arctan(e^(
-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) +
e^(-4*d*x - 4*c) + 1)))
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(186) = 372$.

Time = 0.31 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.47

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(64\,a^3+144\,a^2b+120\,ab^2+35\,b^3\right)+\frac{4\left(192\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/768*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(64*a^3 +
 144*a^2*b + 120*a*b^2 + 35*b^3) + 4*(192*a^3*(e^(d*x + c) - e^(-d*x - c))
^7 + 432*a^2*b*(e^(d*x + c) - e^(-d*x - c))^7 + 360*a*b^2*(e^(d*x + c) - e
^(-d*x - c))^7 + 105*b^3*(e^(d*x + c) - e^(-d*x - c))^7 + 2304*a^3*(e^(d*x
 + c) - e^(-d*x - c))^5 + 6336*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 + 5280
*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 1540*b^3*(e^(d*x + c) - e^(-d*x -
c))^5 + 9216*a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 29952*a^2*b*(e^(d*x + c)
 - e^(-d*x - c))^3 + 28032*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 8176*b^3
*(e^(d*x + c) - e^(-d*x - c))^3 + 12288*a^3*(e^(d*x + c) - e^(-d*x - c)) +
 46080*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 50688*a*b^2*(e^(d*x + c) - e^(
-d*x - c)) + 17856*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d
*x - c))^2 + 4)^4)/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 931, normalized size of antiderivative = 4.75

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(64\,a^3\,\sqrt{d^2}+35\,b^3\,\sqrt{d^2}+120\,a\,b^2\,\sqrt{d^2}+144\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{4096\,a^6+18432\,a^5\,b+36096\,a^4\,b^2+39040\,a^3\,b^3+24480\,a^2\,b^4+8400\,a\,b^5+1225\,b^6}}\right)\sqrt{4096\,a^6+18432\,a^5\,b+36096\,a^4\,b^2}}{64\,\sqrt{d^2}}$$
$$-\frac{\frac{a^3\,e^{c+dx}}{2\,d}+\frac{2\,e^{7\,c+7\,dx}\left(5\,a^3+18\,a^2\,b+24\,a\,b^2+16\,b^3\right)}{d}+\frac{a^3\,e^{13\,c+13\,dx}}{2\,d}+\frac{3\,a\,e^{5\,c+5\,dx}\left(5\,a^2+16\,a\,b+16\,b^2\right)}{2\,d}+\frac{3\,a\,e^{9\,c+9\,dx}\left(5\,a^2+16\,a\,b+\right.}{2\,d}}{8\,e^{2\,c+2\,dx}+28\,e^{4\,c+4\,dx}+56\,e^{6\,c+6\,dx}+70\,e^{8\,c+8\,dx}+56\,e^{10\,c+10\,dx}+28\,e^{12\,c+12\,dx}+8\,e}$$
$$+\frac{2\,e^{c+dx}\left(48\,a\,b^2-37\,b^3\right)}{3\,d\,\left(5\,e^{2\,c+2\,dx}+10\,e^{4\,c+4\,dx}+10\,e^{6\,c+6\,dx}+5\,e^{8\,c+8\,dx}+e^{10\,c+10\,dx}+1\right)}$$
$$+\frac{e^{c+dx}\left(24\,a^2\,b-120\,a\,b^2+b^3\right)}{4\,d\,\left(4\,e^{2\,c+2\,dx}+6\,e^{4\,c+4\,dx}+4\,e^{6\,c+6\,dx}+e^{8\,c+8\,dx}+1\right)}$$
$$-\frac{16\,b^3\,e^{c+dx}}{d\,\left(7\,e^{2\,c+2\,dx}+21\,e^{4\,c+4\,dx}+35\,e^{6\,c+6\,dx}+35\,e^{8\,c+8\,dx}+21\,e^{10\,c+10\,dx}+7\,e^{12\,c+12\,dx}+e^{14\,c+14\,dx}+1\right)}$$
$$+\frac{e^{c+dx}\left(64\,a^3+144\,a^2\,b+120\,a\,b^2+35\,b^3\right)}{64\,d\,\left(e^{2\,c+2\,dx}+1\right)}$$
$$-\frac{4\,e^{c+dx}\left(6\,a\,b^2-29\,b^3\right)}{3\,d\,\left(6\,e^{2\,c+2\,dx}+15\,e^{4\,c+4\,dx}+20\,e^{6\,c+6\,dx}+15\,e^{8\,c+8\,dx}+6\,e^{10\,c+10\,dx}+e^{12\,c+12\,dx}+1\right)}$$
$$+\frac{e^{c+dx}\left(-144\,a^3+144\,a^2\,b+120\,a\,b^2+35\,b^3\right)}{96\,d\,\left(2\,e^{2\,c+2\,dx}+e^{4\,c+4\,dx}+1\right)}$$
$$+\frac{e^{c+dx}\left(-288\,a^2\,b+24\,a\,b^2+7\,b^3\right)}{24\,d\,\left(3\,e^{2\,c+2\,dx}+3\,e^{4\,c+4\,dx}+e^{6\,c+6\,dx}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(64*a^3*(d^2)^(1/2) + 35*b^3*(d^2)^(1/2) + 120*a*b^
2*(d^2)^(1/2) + 144*a^2*b*(d^2)^(1/2)))/(d*(8400*a*b^5 + 18432*a^5*b + 409
6*a^6 + 1225*b^6 + 24480*a^2*b^4 + 39040*a^3*b^3 + 36096*a^4*b^2)^(1/2)))*
(8400*a*b^5 + 18432*a^5*b + 4096*a^6 + 1225*b^6 + 24480*a^2*b^4 + 39040*a^
3*b^3 + 36096*a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) - ((a^3*exp(c + d*x))/(2*d)
 + (2*exp(7*c + 7*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/d + (a^3*ex
p(13*c + 13*d*x))/(2*d) + (3*a*exp(5*c + 5*d*x)*(16*a*b + 5*a^2 + 16*b^2))
/(2*d) + (3*a*exp(9*c + 9*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(2*d) + (3*a^2*e
xp(3*c + 3*d*x)*(a + 2*b))/d + (3*a^2*exp(11*c + 11*d*x)*(a + 2*b))/d)/(8*
exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c
+ 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14
*d*x) + exp(16*c + 16*d*x) + 1) + (2*exp(c + d*x)*(48*a*b^2 - 37*b^3))/(3*
d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(
8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (exp(c + d*x)*(24*a^2*b - 120*a*
b^2 + b^3))/(4*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*
d*x) + exp(8*c + 8*d*x) + 1)) - (16*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*
x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*
exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (ex
p(c + d*x)*(120*a*b^2 + 144*a^2*b + 64*a^3 + 35*b^3))/(64*d*(exp(2*c + 2*d
*x) + 1)) - (4*exp(c + d*x)*(6*a*b^2 - 29*b^3))/(3*d*(6*exp(2*c + 2*d*x...
```

3.71. $\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.72 $\int \text{sech}^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^3 dx$

3.72.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \text{sech}^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^3 dx = \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d} + \frac{3b(a+b)(a+2b)\tanh^5(c+dx)}{5d} - \frac{b^2(3a+4b)\tanh^7(c+dx)}{7d} + \frac{b^3\tanh^9(c+dx)}{9d}$$

output
```
(a+b)^3*tanh(d*x+c)/d-1/3*(a+b)^2*(a+4*b)*tanh(d*x+c)^3/d+3/5*b*(a+b)*(a+2
*b)*tanh(d*x+c)^5/d-1/7*b^2*(3*a+4*b)*tanh(d*x+c)^7/d+1/9*b^3*tanh(d*x+c)^
9/d
```

3.72.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(108) = 216.

Time = 5.16 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.20

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{a^3\tanh(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{3ab^2\tanh(c+dx)}{d}+\frac{b^3\tanh(c+dx)}{d}-\frac{a^3\tanh^3(c+dx)}{3d}-\frac{2a^2b\tanh^3(c+dx)}{d}-\frac{3ab^2\tanh^3(c+dx)}{d}-\frac{4b^3\tanh^3(c+dx)}{3d}+\frac{3a^2b\tanh^5(c+dx)}{5d}+\frac{9ab^2\tanh^5(c+dx)}{5d}+\frac{6b^3\tanh^5(c+dx)}{5d}-\frac{3ab^2\tanh^7(c+dx)}{7d}-\frac{4b^3\tanh^7(c+dx)}{7d}+\frac{b^3\tanh^9(c+dx)}{9d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(a^3*Tanh[c + d*x])/d + (3*a^2*b*Tanh[c + d*x])/d + (3*a*b^2*Tanh[c + d*x]
)/d + (b^3*Tanh[c + d*x])/d - (a^3*Tanh[c + d*x]^3)/(3*d) - (2*a^2*b*Tanh[
c + d*x]^3)/d - (3*a*b^2*Tanh[c + d*x]^3)/d - (4*b^3*Tanh[c + d*x]^3)/(3*d
) + (3*a^2*b*Tanh[c + d*x]^5)/(5*d) + (9*a*b^2*Tanh[c + d*x]^5)/(5*d) + (6
*b^3*Tanh[c + d*x]^5)/(5*d) - (3*a*b^2*Tanh[c + d*x]^7)/(7*d) - (4*b^3*Tan
h[c + d*x]^7)/(7*d) + (b^3*Tanh[c + d*x]^9)/(9*d)
```

3.72.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4634, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \; 4634$$

$$\frac{\int \left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^3 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 290$$

$$\frac{\int \left(b^3\tanh^8(c+dx)-b^2(3a+4b)\tanh^6(c+dx)+3b(a+b)(a+2b)\tanh^4(c+dx)-(a+b)^2(a+4b)\tanh^2(c+d\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{7}b^2(3a+4b)\tanh^7(c+dx)+\frac{3}{5}b(a+b)(a+2b)\tanh^5(c+dx)-\frac{1}{3}(a+b)^2(a+4b)\tanh^3(c+dx)+(a+b)^3\tan}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + b)^3*Tanh[c + d*x] - ((a + b)^2*(a + 4*b)*Tanh[c + d*x]^3)/3 + (3*b*
(a + b)*(a + 2*b)*Tanh[c + d*x]^5)/5 - (b^2*(3*a + 4*b)*Tanh[c + d*x]^7)/7
 + (b^3*Tanh[c + d*x]^9)/9)/d
```

3.72.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.72.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)+3a\,b^2\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}}{}\right.}{}$
default	$\frac{a^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)+3a\,b^2\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}}{}\right.}{}$
parts	$\frac{a^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}+\frac{b^3\left(\frac{128}{315}+\frac{\operatorname{sech}(dx+c)^8}{9}+\frac{8\operatorname{sech}(dx+c)^6}{63}+\frac{16\operatorname{sech}(dx+c)^4}{105}+\frac{64\operatorname{sech}(dx+c)^2}{315}\right)\tanh(dx+}{d}$
parallelrisch	$\frac{(9660a^3+32256a^2b+36288a\,b^2+10752b^3)\sinh(3dx+3c)+(6300a^3+18144a^2b+15552a\,b^2+4608b^3)\sinh(5dx+5c)+(189}{315d(\cosh(9dx+9c)+9\cosh(7dx+7c}$
risch	$-\frac{4(216a\,b^2+1995a^3e^{12dx+12c}+105a^3+2304\,e^{4dx+4c}b^3+7875a^3e^{8dx+8c}+252a^2b+1944\,e^{2dx+2c}a\,b^2+2268a^2b\,e^{2dx+2c}}{}$

input
```
int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(8/15+1/5*sech(d*x+c)
^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+3*a*b^2*(16/35+1/7*sech(d*x+c)^6+6/35*s
ech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(128/315+1/9*sech(d*x+c)^
8+8/63*sech(d*x+c)^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c
))
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. 2(100) = 200.

Time = 0.27 (sec) , antiderivative size = 1190, normalized size of antiderivative = 11.02

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-8/315*(2*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)^7 + 14*(1
05*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (10
5*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*sinh(d*x + c)^7 + 6*(245*a^3 + 399
*a^2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 42*a^2*b - 324
*a*b^2 - 96*b^3 + 7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cos
h(d*x + c)^3 + 3*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c))
*sinh(d*x + c)^4 + 18*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^3)*cosh(d*x
+ c)^3 + (35*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^4 +
945*a^3 + 1134*a^2*b - 108*a*b^2 - 1152*b^3 + 30*(175*a^3 + 42*a^2*b - 324
*a*b^2 - 96*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 + 63*a^2
*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 399*a^2*b + 162*a*
b^2 + 48*b^3)*cosh(d*x + c)^3 + 9*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^
3)*cosh(d*x + c))*sinh(d*x + c)^2 + 210*(35*a^3 + 93*a^2*b + 90*a*b^2 + 32
*b^3)*cosh(d*x + c) + (7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d
*x + c)^6 + 15*(175*a^3 + 42*a^2*b - 324*a*b^2 - 96*b^3)*cosh(d*x + c)^4 +
 525*a^3 + 882*a^2*b + 756*a*b^2 + 1344*b^3 + 27*(105*a^3 + 126*a^2*b - 12
*a*b^2 - 128*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11
*d*cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)
^9 + (55*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + ...
```

3.72.6 Sympy [F]

$$\int \operatorname{sech}^4(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right)^3 \, dx = \int \left(a+b\operatorname{sech}^2(c+dx)\right)^3 \operatorname{sech}^4(c+dx) \, dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. $2(100) = 200$.

Time = 0.20 (sec) , antiderivative size = 1245, normalized size of antiderivative = 11.53

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
256/315*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*
c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) +
 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^
(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e
^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*
d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x
 - 16*c) + e^(-18*d*x - 18*c) + 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x
- 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c)
+ 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) +
 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 126*e^(-8*d*x - 8*c)/(d
*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(
-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14
*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e
^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*
x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x
- 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))) + 96/35*a*b^2*(
7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6
*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x -
 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x -
2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) ...
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(100) = 200$.

Time = 0.31 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.33

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{4\left(315\,a^3e^{(14\,dx+14\,c)}+1995\,a^3e^{(12\,dx+12\,c)}+2520\,a^2be^{(12\,dx+12\,c)}+5355\,a^3e^{(10\,dx+10\,c)}+11340\,a^2be^{(10\,dx+1}\right.}{}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-4/315*(315*a^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*c) + 2520*a^2*
b*e^(12*d*x + 12*c) + 5355*a^3*e^(10*d*x + 10*c) + 11340*a^2*b*e^(10*d*x +
 10*c) + 7560*a*b^2*e^(10*d*x + 10*c) + 7875*a^3*e^(8*d*x + 8*c) + 20412*a
^2*b*e^(8*d*x + 8*c) + 19656*a*b^2*e^(8*d*x + 8*c) + 8064*b^3*e^(8*d*x + 8
*c) + 6825*a^3*e^(6*d*x + 6*c) + 18648*a^2*b*e^(6*d*x + 6*c) + 18144*a*b^2
*e^(6*d*x + 6*c) + 5376*b^3*e^(6*d*x + 6*c) + 3465*a^3*e^(4*d*x + 4*c) + 9
072*a^2*b*e^(4*d*x + 4*c) + 7776*a*b^2*e^(4*d*x + 4*c) + 2304*b^3*e^(4*d*x
 + 4*c) + 945*a^3*e^(2*d*x + 2*c) + 2268*a^2*b*e^(2*d*x + 2*c) + 1944*a*b^
2*e^(2*d*x + 2*c) + 576*b^3*e^(2*d*x + 2*c) + 105*a^3 + 252*a^2*b + 216*a*
b^2 + 64*b^3)/(d*(e^(2*d*x + 2*c) + 1)^9)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 1333, normalized size of antiderivative = 12.34

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

output
```
- ((16*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(315*d) + (4*a^3*exp(6*c +
6*d*x))/(9*d) + (4*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) +
(8*a^2*exp(4*c + 4*d*x)*(a + 2*b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c
 + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x)
+ 1) - ((32*exp(8*c + 8*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(9*d)
 + (8*a^3*exp(2*c + 2*d*x))/(9*d) + (8*a^3*exp(14*c + 14*d*x))/(9*d) + (8*
a*exp(6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(3*d) + (8*a*exp(10*c + 10*d
*x)*(16*a*b + 5*a^2 + 16*b^2))/(3*d) + (16*a^2*exp(4*c + 4*d*x)*(a + 2*b))
/(3*d) + (16*a^2*exp(12*c + 12*d*x)*(a + 2*b))/(3*d))/(9*exp(2*c + 2*d*x)
+ 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) + 126*exp(8*c + 8*d*x) + 126*e
xp(10*c + 10*d*x) + 84*exp(12*c + 12*d*x) + 36*exp(14*c + 14*d*x) + 9*exp(
16*c + 16*d*x) + exp(18*c + 18*d*x) + 1) - ((4*a^2*(a + 2*b))/(21*d) + (2*
a^3*exp(2*c + 2*d*x))/(9*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) + 1) - ((a*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (16*exp(2*c
+ 2*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(63*d) + (5*a^3*exp(8*c +
 8*d*x))/(9*d) + (10*a*exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d)
+ (40*a^2*exp(6*c + 6*d*x)*(a + 2*b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp
(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 1
0*d*x) + exp(12*c + 12*d*x) + 1) - (a^3/(9*d) + (16*exp(6*c + 6*d*x)*(24*a
*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(9*d) + (7*a^3*exp(12*c + 12*d*x))/(...
```

3.72. $\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

3.73 $\int \frac{\cosh^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

3.73.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\cosh^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx=\frac{(3a^2-4ab+8b^2)\,x}{8a^3}-\frac{b^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}d}+\frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d}+\frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad}$$

output
```
1/8*(3*a^2-4*a*b+8*b^2)*x/a^3+1/8*(3*a-4*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d+
1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d-b^(5/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b
)^(1/2))/a^3/d/(a+b)^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \frac{\cosh^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx=\frac{4(3a^2-4ab+8b^2)\,(c+dx)-\frac{32b^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+8a(a-b)\sinh(2(c+dx))+a^2\sinh(4(c+dx))}{32a^3d}$$

input
```
Integrate[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
(4*(3*a^2 - 4*a*b + 8*b^2)*(c + d*x) - (32*b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c
 + d*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sinh[2*(c + d*x)] + a^2*S
inh[4*(c + d*x)])/(32*a^3*d)
```

3.73.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4634, 316, 402, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^3\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-3b\tanh^2(c+dx)+3a-b}{\left(1-\tanh^2(c+dx)\right)^2\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{4a}+\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3a^2-ba+4b^2-(3a-4b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a}+\frac{(3a-4b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}}{4a}+\frac{\tanh(c+dx)}{4a\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{\frac{(3a^2-4ab+8b^2)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{8b^3\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a}+\frac{(3a-4b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}}{4a}+\frac{\tanh(c+dx)}{4a(1-\tanh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\frac{(3a^2-4ab+8b^2)\operatorname{arctanh}(\tanh(c+dx))}{a}-\frac{8b^3\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a}+\frac{(3a-4b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}}{4a}+\frac{\tanh(c+dx)}{4a(1-\tanh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\frac{(3a^2-4ab+8b^2)\operatorname{arctanh}(\tanh(c+dx))}{a}-\frac{8b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a}+\frac{(3a-4b)\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}}{4a}+\frac{\tanh(c+dx)}{4a(1-\tanh^2(c+dx))^2}}{d}$$

input
```
Int[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
(Tanh[c + d*x]/(4*a*(1 - Tanh[c + d*x]^2)^2) + ((((3*a^2 - 4*a*b + 8*b^2)*
ArcTanh[Tanh[c + d*x]])/a - (8*b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqr
t[a + b]])/(a*Sqrt[a + b]))/(2*a) + ((3*a - 4*b)*Tanh[c + d*x])/(2*a*(1 -
Tanh[c + d*x]^2)))/(4*a))/d
```

3.73.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(103) = 206$.

Time = 1.17 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.00

method	result
risch	$\frac{3x}{8a}-\frac{bx}{2a^2}+\frac{x\,b^2}{a^3}+\frac{\mathrm{e}^{4dx+4c}}{64da}+\frac{\mathrm{e}^{2dx+2c}}{8da}-\frac{\mathrm{e}^{2dx+2c}b}{8d\,a^2}-\frac{\mathrm{e}^{-2dx-2c}}{8da}+\frac{\mathrm{e}^{-2dx-2c}b}{8d\,a^2}-\frac{\mathrm{e}^{-4dx-4c}}{64da}+\frac{\sqrt{(a+b)b}\,b^2}{}$
derivativedivides	$\frac{\frac{1}{4a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{-7a+4b}{8a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-5a+4b}{8a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-3a^2+4ab-8b^2\right)\ln\left(\tanh\left(\right.\right.}{8a^3}}{}$
default	$\frac{\frac{1}{4a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{-7a+4b}{8a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-5a+4b}{8a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-3a^2+4ab-8b^2\right)\ln\left(\tanh\left(\right.\right.}{8a^3}}{}$

input
```
int(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/a-1/2*b*x/a^2+x/a^3*b^2+1/64/d/a*exp(4*d*x+4*c)+1/8/d/a*exp(2*d*x+2*
c)-1/8/d/a^2*exp(2*d*x+2*c)*b-1/8/d/a*exp(-2*d*x-2*c)+1/8/d/a^2*exp(-2*d*x
-2*c)*b-1/64/d/a*exp(-4*d*x-4*c)+1/2*((a+b)*b)^(1/2)/(a+b)*b^2/d/a^3*ln(ex
p(2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/a)-1/2*((a+b)*b)^(1/2)/(a+b)*b^2/d/
a^3*ln(exp(2*d*x+2*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a)
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(103) = 206$.

Time = 0.29 (sec) , antiderivative size = 1713, normalized size of antiderivative = 14.64

$$\int \frac{\cosh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sin
h(d*x + c)^8 + 8*(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 - a*
b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x +
c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*x + 60*(a^2
 - a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3
*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c) + 20*(a^2 - a*b)*cosh(d*x + c)^3)*
sinh(d*x + c)^3 - 8*(a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6
 + 12*(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 - a*b)*cosh(d*
x + c)^4 - 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 +
4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(b/(a + b))
*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh
(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 +
 a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)
^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x
+ c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x
+ c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*co
sh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c
)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x...
```

3.73.6 Sympy [F]

$$\int \frac{\cosh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\cosh^4{(c+dx)}}{a+b\,\text{sech}^2{(c+dx)}}\,dx$$

input
```
integrate(cosh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.73.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(103) = 206$.

Time = 0.31 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.50

$$\int \frac{\cosh^4(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \frac{3\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}ad} + \frac{3\,(dx+c)}{8\,ad} - \frac{\left(8\,be^{(-2\,dx-2\,c)}-a\right)e^{(4\,dx+4\,c)}}{64\,a^2d} + \frac{e^{(2\,dx+2\,c)}}{8\,ad} - \frac{e^{(-2\,dx-2\,c)}}{8\,ad} - \frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^2d} + \frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,a^2d} + \frac{(ab+2\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}a^2d} - \frac{(ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}a^2d} + \frac{(ab+2\,b^2)(dx+c)}{2\,a^3d} + \frac{8\,be^{(-2\,dx-2\,c)}-ae^{(-4\,dx-4\,c)}}{64\,a^2d} + \frac{(a^2b+8\,ab^2+8\,b^3)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}a^3d}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
3/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 3/8*(d*x +
 c)/(a*d) - 1/64*(8*b*e^(-2*d*x - 2*c) - a)*e^(4*d*x + 4*c)/(a^2*d) + 1/8*
e^(2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b*log(a*e^(4*d*x
+ 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) + 1/4*b*log(2*(a + 2*b)*
e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) + 1/8*(a*b + 2*b^2)*log
((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a
+ 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) - 1/8*(a*b + 2*b^2)*lo
g((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
 a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) + 1/2*(a*b + 2*b^2)
*(d*x + c)/(a^3*d) + 1/64*(8*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x - 4*c))/(a^2
*d) + 1/16*(a^2*b + 8*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2
*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqr
t((a + b)*b)*a^3*d)
```

3.73.8 Giac [F]

$$\int \frac{\cosh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^4}{b\,\text{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.73.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.22

$$\int \frac{\cosh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{x(3a^2-4ab+8b^2)}{8a^3} - \frac{e^{-4c-4dx}}{64ad} + \frac{e^{4c+4dx}}{64ad} - \frac{e^{-2c-2dx}(a-b)}{8a^2d} + \frac{e^{2c+2dx}(a-b)}{8a^2d} + \frac{b^{5/2}\ln\left(\frac{4b^3e^{2c+2dx}}{a^4} - \frac{2b^{5/2}(ad+ade^{2c+2dx}+2bde^{2c+2dx})}{a^4d\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}} - \frac{b^{5/2}\ln\left(\frac{4b^3e^{2c+2dx}}{a^4} + \frac{2b^{5/2}(ad+ade^{2c+2dx}+2bde^{2c+2dx})}{a^4d\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}}$$

input
```
int(cosh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

output
```
(x*(3*a^2 - 4*a*b + 8*b^2))/(8*a^3) - exp(- 4*c - 4*d*x)/(64*a*d) + exp(4*
c + 4*d*x)/(64*a*d) - (exp(- 2*c - 2*d*x)*(a - b))/(8*a^2*d) + (exp(2*c +
2*d*x)*(a - b))/(8*a^2*d) + (b^(5/2)*log((4*b^3*exp(2*c + 2*d*x))/a^4 - (2
*b^(5/2)*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(a^4*d*(a
+ b)^(1/2))))/(2*a^3*d*(a + b)^(1/2)) - (b^(5/2)*log((4*b^3*exp(2*c + 2*d*
x))/a^4 + (2*b^(5/2)*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x))
)/(a^4*d*(a + b)^(1/2))))/(2*a^3*d*(a + b)^(1/2))
```

3.74 $\int \frac{\cosh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.74.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\cosh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\frac{b^2\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}d}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}$$

output
```
(a-b)*sinh(d*x+c)/a^2/d+1/3*sinh(d*x+c)^3/a/d+b^2*arctan(sinh(d*x+c)*a^(1/
2)/(a+b)^(1/2))/a^(5/2)/d/(a+b)^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{\cosh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{-\frac{12b^2\arctan\left(\frac{\sqrt{a+b}\text{csch}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+3\sqrt{a}(3a-4b)\sinh(c+dx)+a^{3/2}\sinh(3(c+dx))}{12a^{5/2}d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
((-12*b^2*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/Sqrt[a + b] + 3*Sqr
t[a]*(3*a - 4*b)*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)])/(12*a^(5/2)*d)
```

3.74.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{b^2}{a^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh^2(c+dx)}{a}+\frac{a-b}{a^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\sinh(c+dx)}{a^2}+\frac{\sinh^3(c+dx)}{3a}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
((b^2*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) +
 ((a - b)*Sinh[c + d*x])/a^2 + Sinh[c + d*x]^3/(3*a))/d
```

3.74.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(66) = 132$.

Time = 0.85 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.75

method	result
derivativedivides	$\frac{-\frac{1}{3a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{a-b}{a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{1}{2a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{\cdots}{a^{2}\cdots}}{d}$
default	$\frac{-\frac{1}{3a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{a-b}{a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{1}{2a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{\cdots}{a^{2}\cdots}}{d}$
risch	$\frac{e^{3dx+3c}}{24da}+\frac{3\,e^{dx+c}}{8ad}-\frac{e^{dx+c}b}{2a^{2}d}-\frac{3\,e^{-dx-c}}{8ad}+\frac{e^{-dx-c}b}{2a^{2}d}-\frac{e^{-3dx-3c}}{24da}-\frac{b^{2}\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^{2}-ab}}-1\right)}{2\sqrt{-a^{2}-ab}\,d\,a^{2}}+\frac{b^{2}\cdots}{\cdots}$

input
```
int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/a/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/a/(tanh(1/2*d*x+1/2*c)-1)^2-(a-b
)/a^2/(tanh(1/2*d*x+1/2*c)-1)-1/3/a/(1+tanh(1/2*d*x+1/2*c))^3+1/2/a/(1+tan
h(1/2*d*x+1/2*c))^2-(a-b)/a^2/(1+tanh(1/2*d*x+1/2*c))+2*b^2/a^2*(1/2/(a+b)
^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^
(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*
c)+2*b^(1/2))/a^(1/2))))
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(66) = 132$.

Time = 0.29 (sec) , antiderivative size = 1616, normalized size of antiderivative = 21.26

$$\int \frac{\cosh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/24*((a^3 + a^2*b)*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*cosh(d*x + c)*sinh(
d*x + c)^5 + (a^3 + a^2*b)*sinh(d*x + c)^6 + 3*(3*a^3 - a^2*b - 4*a*b^2)*c
osh(d*x + c)^4 + 3*(3*a^3 - a^2*b - 4*a*b^2 + 5*(a^3 + a^2*b)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(3*a^3 - a^2
*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 3*(3*a^3 - a^
2*b - 4*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + a^2*b)*cosh(d*x + c)^4 - 3*a^
3 + a^2*b + 4*a*b^2 + 6*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) +
 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(-a^2 - a*
b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
 + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2
*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(
d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-
a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a
*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 +
a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))
*sinh(d*x + c) + a)) + 6*((a^3 + a^2*b)*cosh(d*x + c)^5 + 2*(3*a^3 - a^2*b
 - 4*a*b^2)*cosh(d*x + c)^3 - (3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sin
h(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^3 + 3*(a^4 + a^3*b)*d*cosh(d...
```

3.74.6 Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \int \frac{\cosh^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.74.7 Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^3}{b\,\mathrm{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/24*(3*(3*a*e^(4*c) - 4*b*e^(4*c))*e^(4*d*x) - 3*(3*a*e^(2*c) - 4*b*e^(2*
c))*e^(2*d*x) + a*e^(6*d*x + 6*c) - a)*e^(-3*d*x - 3*c)/(a^2*d) + 1/8*inte
grate(16*(b^2*e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^3*e^(4*d*x + 4*c) + a^
3 + 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)), x)
```

3.74.8 Giac [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^3}{b\,\mathrm{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.74.9 Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.37

$$\int \frac{\cosh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{e^{3c+3dx}}{24\,a\,d} - \frac{e^{-3c-3dx}}{24\,a\,d}$$
$$- \frac{\sqrt{b^4}\left(2\,\mathrm{atan}\left(\left(\left(e^{dx}\,e^{c}\left(\frac{2\,b^2}{a^8\,d\,(a+b)^2\,\sqrt{b^4}} - \frac{4\left(2\,a^2\,b^4\,d\,\sqrt{b^4}+2\,a^3\,b^3\,d\,\sqrt{b^4}\right)}{a^6\,b^5\,(a+b)\,\sqrt{a^6\,d^2+b\,a^5\,d^2}\,\sqrt{a^5\,d^2\,(a+b)}}\right) - \frac{2\,b^2\,e^{3\,c}\,e^{3\,d\,x}}{a^8\,d\,(a+b)^2\,\sqrt{b^4}}\right)\left(\frac{a^7\,\sqrt{a^6\,d^2+b\,a^5\,d}}{4}\right.\right.\right.\right.}{2\,\sqrt{a^6\,d^2+b\,a^5\,d^2}}$$
$$+ \frac{e^{c+dx}\,(3\,a-4\,b)}{8\,a^2\,d} - \frac{e^{-c-dx}\,(3\,a-4\,b)}{8\,a^2\,d}$$

input
```
int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

output
```
exp(3*c + 3*d*x)/(24*a*d) - exp(- 3*c - 3*d*x)/(24*a*d) - ((b^4)^(1/2)*(2*
atan((exp(d*x)*exp(c)*((2*b^2)/(a^8*d*(a + b)^2*(b^4)^(1/2)) - (4*(2*a^2*b
^4*d*(b^4)^(1/2) + 2*a^3*b^3*d*(b^4)^(1/2)))/(a^6*b^5*(a + b)*(a^6*d^2 + a
^5*b*d^2)^(1/2)*(a^5*d^2*(a + b))^(1/2))) - (2*b^2*exp(3*c)*exp(3*d*x))/(a
^8*d*(a + b)^2*(b^4)^(1/2)))*((a^7*(a^6*d^2 + a^5*b*d^2)^(1/2))/4 + (a^6*b
*(a^6*d^2 + a^5*b*d^2)^(1/2))/4)) - 2*atan((b^2*exp(d*x)*exp(c)*(a^5*d^2*(
a + b))^(1/2))/(2*a^2*d*(a + b)*(b^4)^(1/2)))))/(2*(a^6*d^2 + a^5*b*d^2)^(
1/2)) + (exp(c + d*x)*(3*a - 4*b))/(8*a^2*d) - (exp(- c - d*x)*(3*a - 4*b)
)/(8*a^2*d)
```

3.75 $\int \frac{\cosh^2(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

3.75.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{(a-2b)x}{2a^2}+\frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2ad}$$

output
```
1/2*(a-2*b)*x/a^2+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d+b^(3/2)*arctanh(b^(1/2)*
tanh(d*x+c)/(a+b)^(1/2))/a^2/d/(a+b)^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{2(a-2b)(c+dx)+\frac{4b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+a\sinh(2(c+dx))}{4a^2d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
(2*(a - 2*b)*(c + d*x) + (4*b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a
 + b]])/Sqrt[a + b] + a*Sinh[2*(c + d*x)])/(4*a^2*d)
```

3.75.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4634, 316, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)}dx$$
$$\Big\downarrow \text{4634}$$
$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{d}$$
$$\Big\downarrow \text{316}$$
$$\frac{\frac{\int \frac{-b\tanh^2(c+dx)+a-b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}}{d}$$
$$\Big\downarrow \text{397}$$
$$\frac{\frac{\frac{2b^2\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}+\frac{(a-2b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}}{d}$$
$$\Big\downarrow \text{219}$$
$$\frac{\frac{\frac{2b^2\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}+\frac{(a-2b)\text{arctanh}(\tanh(c+dx))}{a}}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}}{d}$$
$$\Big\downarrow \text{221}$$
$$\frac{\frac{\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-2b)\text{arctanh}(\tanh(c+dx))}{a}}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
((((a - 2*b)*ArcTanh[Tanh[c + d*x]])/a + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[
c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a) + Tanh[c + d*x]/(2*a*(1 - T
anh[c + d*x]^2)))/d
```

3.75.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(63) = 126$.

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.03

method	result
risch	$\frac{x}{2a} - \frac{bx}{a^2} + \frac{\mathrm{e}^{2dx+2c}}{8da} - \frac{\mathrm{e}^{-2dx-2c}}{8da} + \frac{\sqrt{(a+b)b}\, b \ln\left(\mathrm{e}^{2dx+2c} - \frac{2\sqrt{(a+b)b}-a-2b}{a}\right)}{2(a+b)d\,a^2} - \frac{\sqrt{(a+b)b}\, b \ln\left(\mathrm{e}^{2dx+2c} + \frac{2\sqrt{(a+ \cdots}{\cdots}\right)}{2(a+b)d\,a^2}$
derivativedivides	$\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{(-a+2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^2} - \frac{2b^2\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\cdots\right)\right)}{4\sqrt{b}\,\sqrt{a+b}}\cdots\right)}{\cdots}$
default	$\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{(-a+2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^2} - \frac{2b^2\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\cdots\right)\right)}{4\sqrt{b}\,\sqrt{a+b}}\cdots\right)}{\cdots}$

input
```
int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a-b*x/a^2+1/8/d/a*exp(2*d*x+2*c)-1/8/d/a*exp(-2*d*x-2*c)+1/2*((a+b)*
b)^(1/2)/(a+b)*b/d/a^2*ln(exp(2*d*x+2*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a)-1/2*
((a+b)*b)^(1/2)/(a+b)*b/d/a^2*ln(exp(2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/
a)
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(63) = 126$.

Time = 0.28 (sec) , antiderivative size = 829, normalized size of antiderivative = 11.05

$$\int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2 + a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(2*(a - 2*b)*d*x + 3*a*cosh(d*
x + c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(
d*x + c) + b*sinh(d*x + c)^2)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4
*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)
*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2
 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x +
 c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d
*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)
*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 +
 a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
+ a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c
))*sinh(d*x + c) + a)) + 4*(2*(a - 2*b)*d*x*cosh(d*x + c) + a*cosh(d*x + c
)^3)*sinh(d*x + c) - a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sin
h(d*x + c) + a^2*d*sinh(d*x + c)^2), 1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2
+ a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^
4 + 2*(2*(a - 2*b)*d*x + 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/
(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) +
 a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 4*(2*(a - 2*b)*d*x*...
```

3.75.6 Sympy [F]

$$\int \frac{\cosh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\cosh^2{(c+dx)}}{a+b\,\text{sech}^2{(c+dx)}}\,dx$$

input
```
integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.75.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(63) = 126.

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.69

$$\int \frac{\cosh^2(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}ad} + \frac{dx+c}{2\,ad} + \frac{e^{(2\,dx+2\,c)}}{8\,ad} - \frac{e^{(-2\,dx-2\,c)}}{8\,ad} - \frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^2d} + \frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,a^2d} + \frac{(ab+2\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}a^2d} - \frac{(ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}a^2d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 1/2*(d*x +
c)/(a*d) + 1/8*e^(2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b*
log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) + 1/4*b*l
og(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) + 1/8*(a
*b + 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*
d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) - 1/8*(
a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(
-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d)
```

3.75.8 Giac [F]

$$\int \frac{\cosh^2(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^2}{b\,\mathrm{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.75.9 Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.75

$$\int \frac{\cosh^2(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \frac{x\,(a-2\,b)}{2\,a^2} - \frac{\mathrm{e}^{-2\,c-2\,d\,x}}{8\,a\,d} + \frac{\mathrm{e}^{2\,c+2\,d\,x}}{8\,a\,d} + \frac{b^{3/2}\,\ln\left(-\frac{4\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{a^3} - \frac{2\,b^{3/2}\,\left(a\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^3\,d\,\sqrt{a+b}}\right)}{2\,a^2\,d\,\sqrt{a+b}} - \frac{b^{3/2}\,\ln\left(\frac{2\,b^{3/2}\,\left(a\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^3\,d\,\sqrt{a+b}} - \frac{4\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{a^3}\right)}{2\,a^2\,d\,\sqrt{a+b}}$$

input
```
int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

output
```
(x*(a - 2*b))/(2*a^2) - exp(- 2*c - 2*d*x)/(8*a*d) + exp(2*c + 2*d*x)/(8*a
*d) + (b^(3/2)*log(- (4*b^2*exp(2*c + 2*d*x))/a^3 - (2*b^(3/2)*(a*d + a*d*
exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(a^3*d*(a + b)^(1/2))))/(2*a^2
*d*(a + b)^(1/2)) - (b^(3/2)*log((2*b^(3/2)*(a*d + a*d*exp(2*c + 2*d*x) +
2*b*d*exp(2*c + 2*d*x)))/(a^3*d*(a + b)^(1/2)) - (4*b^2*exp(2*c + 2*d*x))/
a^3))/(2*a^2*d*(a + b)^(1/2))
```

3.76 $\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)} dx$

3.76.1 Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)} dx = -\frac{b \arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{ad}$$

output
```
sinh(d*x+c)/a/d-b*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/d/(a+b)^
(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)} dx = \frac{-\frac{b \arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \sqrt{a}\sinh(c+dx)}{a^{3/2}d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-((b*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*
Sinh[c + d*x])/(a^(3/2)*d)
```

3.76.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4635, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)\left(a+b\sec(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\sinh(c+dx)}{a}-\frac{b\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{a}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sinh(c+dx)}{a}-\frac{b\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-((b*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]))
+ Sinh[c + d*x]/a)/d
```

3.76.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(44) = 88$.

Time = 0.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{-\frac{2b\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\right)}{a}-\frac{1}{a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{1}{a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$
default	$\frac{-\frac{2b\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\right)}{a}-\frac{1}{a\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{1}{a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$
risch	$\frac{e^{dx+c}}{2ad}-\frac{e^{-dx-c}}{2ad}-\frac{b\ln\left(e^{2dx+2c}+\frac{2(a+b)e^{dx+c}}{\sqrt{-a^{2}-ab}}-1\right)}{2\sqrt{-a^{2}-ab}\,da}+\frac{b\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^{2}-ab}}-1\right)}{2\sqrt{-a^{2}-ab}\,da}$

input
```
int(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b/a*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*
x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2)))-1/a/(1+tanh(1/2*d*x+1/2*c))-
1/a/(tanh(1/2*d*x+1/2*c)-1))
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(44) = 88$.

Time = 0.28 (sec) , antiderivative size = 718, normalized size of antiderivative = 13.81

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$= \left[\frac{(a^2+ab)\cosh(dx+c)^2 + 2(a^2+ab)\cosh(dx+c)\sinh(dx+c) + (a^2+ab)\sinh(dx+c)^2 - \sqrt{-a^2-a\right.$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
 c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*
sinh(d*x + c))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
+ a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(
d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x
+ c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*
x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d
*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*co
sh(d*x + c))*sinh(d*x + c) + a)) - a^2 - a*b)/((a^3 + a^2*b)*d*cosh(d*x +
c) + (a^3 + a^2*b)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*
(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 - 2*
sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*(a*cosh(d*x
 + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4
*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(
a^2 + a*b)) - 2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan
(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - a^2 - a*b)
/((a^3 + a^2*b)*d*cosh(d*x + c) + (a^3 + a^2*b)*d*sinh(d*x + c))]
```

3.76.6 Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\cosh{(c+dx)}}{a+b\,\text{sech}^2{(c+dx)}}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.76.7 Maxima [F]

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\cosh{(dx+c)}}{b\,\text{sech}{(dx+c)}^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x
 + 3*c) + b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a
*b*e^(2*c))*e^(2*d*x)), x)
```

3.76.8 Giac [F]

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\cosh\left(dx+c\right)}{b\,\text{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.76.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 277, normalized size of antiderivative = 5.33

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{e^{c+d\,x}}{2\,a\,d} - \frac{e^{-c-d\,x}}{2\,a\,d} - \frac{\left(2\,\text{atan}\left(\frac{b^3\,e^{d\,x}\,e^{c}\,\sqrt{a^3\,d^2\,(a+b)}}{2\,a\,d\,(a+b)\,{(b^2)}^{3/2}}\right) - 2\,\text{atan}\left(\left(e^{d\,x}\,e^{c}\left(\frac{2\,b^3}{a^5\,d\,{(a+b)}^2\,{(b^2)}^{3/2}} - \frac{4\left(2\,a^2\,d\,{(b^2)}^{3/2}+2\,a\,b\,d\,{(b^2)}^{3/2}\right)}{a^4\,b^3\,(a+b)\,\sqrt{a^4\,d^2+b\,a^3\,d^2}\,\sqrt{a^3\,d^2\,(a+b)}}\right) - \frac{}{a^5 \cdots}\right.\right.}{2\,\sqrt{a^4\,d^2+b\,a^3\,d^2}}$$

input
```
int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

output
```
exp(c + d*x)/(2*a*d) - exp(- c - d*x)/(2*a*d) - ((2*atan((b^3*exp(d*x)*exp
(c)*(a^3*d^2*(a + b))^(1/2))/(2*a*d*(a + b)*(b^2)^(3/2))) - 2*atan((exp(d*
x)*exp(c)*((2*b^3)/(a^5*d*(a + b)^2*(b^2)^(3/2)) - (4*(2*a^2*d*(b^2)^(3/2)
 + 2*a*b*d*(b^2)^(3/2)))/(a^4*b^3*(a + b)*(a^4*d^2 + a^3*b*d^2)^(1/2)*(a^3
*d^2*(a + b))^(1/2))) - (2*b^3*exp(3*c)*exp(3*d*x))/(a^5*d*(a + b)^2*(b^2)
^(3/2)))*((a^5*(a^4*d^2 + a^3*b*d^2)^(1/2))/4 + (a^4*b*(a^4*d^2 + a^3*b*d^
2)^(1/2))/4))*(b^2)^(1/2))/(2*(a^4*d^2 + a^3*b*d^2)^(1/2))
```

3.77 $\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.77.1 Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

output
```
arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/d/a^(1/2)/(a+b)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)
```

3.77.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4635, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(ic+idx)}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

input
```
Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)
```

3.77.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.77.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(28) = 56$.

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.22

method	result	size
derivativedivides	$\frac{\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{\sqrt{a+b}\,\sqrt{a}}}{d}$	80
default	$\frac{\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{\sqrt{a+b}\,\sqrt{a}}}{d}$	80
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}$	106

input
```
int(sech(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2
*b^(1/2))/a^(1/2))+1/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/
2*d*x+1/2*c)+2*b^(1/2))/a^(1/2)))
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 487, normalized size of antiderivative = 13.53

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

$$= \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4-2\,(3\,a+2\,b)\cosh(dx+c)^2+2\,\left(3\,a\cosh(dx+c)^2-3\,a-2\,b\right.}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(d}\right.}{}\right.$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*
b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sin
h(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - c
osh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)
*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*
a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a +
 2*b)*cosh(d*x + c))*sinh(d*x + c) + a))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)
*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sin
h(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*
b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 +
 a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)))/((a^2 + a*b)*d)]
```

3.77.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(c+dx\right)}{a+b\operatorname{sech}^2\left(c+dx\right)}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.77.7 Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{b\operatorname{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate(sech(d*x + c)/(b*sech(d*x + c)^2 + a), x)
```

3.77.8 Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)}{b\operatorname{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.77.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{\ln\left(-\frac{4\left(b-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^2\,\left(a+b\right)}-\frac{8\,b\,\mathrm{e}^{c+d\,x}}{{\left(-a\right)}^{5/2}\,\sqrt{a+b}}\right)-\ln\left(\frac{8\,b\,\mathrm{e}^{c+d\,x}}{{\left(-a\right)}^{5/2}\,\sqrt{a+b}}-\frac{4\left(b-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^2\,\left(a+b\right)}\right)}{2\,\sqrt{-a}\,d\,\sqrt{a+b}}$$

input
```
int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)),x)
```

output
```
-(log(- (4*(b - b*exp(2*c + 2*d*x)))/(a^2*(a + b)) - (8*b*exp(c + d*x))/((
-a)^(5/2)*(a + b)^(1/2))) - log((8*b*exp(c + d*x))/((-a)^(5/2)*(a + b)^(1/
2)) - (4*(b - b*exp(2*c + 2*d*x)))/(a^2*(a + b))))/(2*(-a)^(1/2)*d*(a + b)
^(1/2))
```

3.78 $\int \frac{\text{sech}^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.78.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\text{sech}^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}d}$$

output
```
arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/d/b^(1/2)/(a+b)^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)
```

3.78.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4634, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}dx$$
$$\Big\downarrow \ \color{blue}{3042}$$
$$\int \frac{\sec(ic+idx)^2}{a+b\sec(ic+idx)^2}dx$$
$$\Big\downarrow \ \color{blue}{4634}$$
$$\frac{\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{d}$$
$$\Big\downarrow \ \color{blue}{221}$$
$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)
```

3.78.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.78.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{2\sqrt{b}\,\sqrt{a+b}}-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{2\sqrt{b}\,\sqrt{a+b}}}{d}$	102
default	$\frac{\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{2\sqrt{b}\,\sqrt{a+b}}-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{2\sqrt{b}\,\sqrt{a+b}}}{d}$	102
risch	$\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}-2ab-2b^2}{a\sqrt{ab+b^2}}\right)}{2\sqrt{ab+b^2}\,d}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}+2ab+2b^2}{a\sqrt{ab+b^2}}\right)}{2\sqrt{ab+b^2}\,d}$	144

input
```
int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1
/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/2/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 411, normalized size of antiderivative = 11.42

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

$$= \left[\frac{\log\left(\frac{a^2\cosh(dx+c)^4+4a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\left(a^2+2ab\right)\cosh(dx+c)^2+2\left(3a^2\cosh(dx+c)^2+a^2+2ab\right)\sinh(dx+c}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a+2b)\cosh(dx+}\right.}{}\right.$$

input
```
integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*
sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)
^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x
+ c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2
 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b
 + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b
)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d
*x + c) + a))/(sqrt(a*b + b^2)*d), sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x
 + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*s
qrt(-a*b - b^2)/(a*b + b^2))/((a*b + b^2)*d)]
```

3.78.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.78.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(28) = 56$.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*d)
```

3.78.8 Giac [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\int \frac{\operatorname{sech}(dx+c)^2}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.47

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{d\,(a+2\,b)}{2\,\sqrt{-b\,d^2\,(a+b)}}+\frac{a\,e^{2\,c}\,e^{2\,d\,x}\,\left(\frac{4}{a^2\,d}+\frac{(a+2\,b)\,(a\,d+2\,b\,d)}{a^2\,\sqrt{-b^2\,d^2-a\,b\,d^2}\,\sqrt{-b\,d^2\,(a+b)}}\right)\,\sqrt{-b^2\,d^2-a\,b\,d^2}}{2}\right)}{\sqrt{-b^2\,d^2-a\,b\,d^2}}$$

input
```
int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)),x)
```

output
```
atan((d*(a + 2*b))/(2*(-b*d^2*(a + b))^(1/2)) + (a*exp(2*c)*exp(2*d*x)*(4/
(a^2*d) + ((a + 2*b)*(a*d + 2*b*d))/(a^2*(- b^2*d^2 - a*b*d^2)^(1/2)*(-b*d
^2*(a + b))^(1/2)))*(- b^2*d^2 - a*b*d^2)^(1/2))/2)/(- b^2*d^2 - a*b*d^2)^
(1/2)
```

3.79 $\int \frac{\text{sech}^3(c+dx)}{a+b\text{sech}^2(c+dx)} \, dx$

3.79.1 Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{\text{sech}^3(c+dx)}{a+b\text{sech}^2(c+dx)} \, dx = \frac{\arctan(\sinh(c+dx))}{bd} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}d}$$

output
```
arctan(sinh(d*x+c))/b/d-arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b/
d/(a+b)^(1/2)
```

3.79.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 194 vs. 2(55) = 110.

Time = 0.82 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.53

$$\int \frac{\text{sech}^3(c+dx)}{a+b\text{sech}^2(c+dx)} \, dx = \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(\sqrt{a}\arctan\left(\frac{\sqrt{a+b}\text{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)\cosh}{2b\sqrt{a+b}d\,(a}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a]*ArcTan[(Sqrt[a +
 b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]
]*Cosh[c] + 2*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]*Sqrt[(Cosh[c] - Sinh[c
])^2] - Sqrt[a]*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])
^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*Sinh[c]))/(2*b*Sqrt[a + b]*d*(a + b*Sech
[c + d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2])
```

3.79.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4635, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^3}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{(\sinh^2(c+dx)+1)(a\sinh^2(c+dx)+a+b)}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{303}$$

$$\frac{\frac{\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}-\frac{a\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\arctan(\sinh(c+dx))}{b}-\frac{a\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan(\sinh(c+dx))}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
(ArcTan[Sinh[c + d*x]]/b - (Sqrt[a]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a
+ b]])/(b*Sqrt[a + b]))/d
```

3.79.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.79. $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.79.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(47) = 94$.

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.89

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{2a\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\right)}{b}}{d}$
default	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{2a\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\right)}{b}}{d}$
risch	$\frac{i\ln\left(\mathrm{e}^{dx+c}+i\right)}{db}-\frac{i\ln\left(\mathrm{e}^{dx+c}-i\right)}{db}+\frac{\sqrt{-a(a+b)}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{-a(a+b)}\,\mathrm{e}^{dx+c}}{a}-1\right)}{2(a+b)db}-\frac{\sqrt{-a(a+b)}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{\cdot}}{}\right.}{2(a+b)db}$

input
```
int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b*arctan(tanh(1/2*d*x+1/2*c))-2*a/b*(1/2/(a+b)^(1/2)/a^(1/2)*arctan
(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2
)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2)
)))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(47) = 94$.

Time = 0.30 (sec) , antiderivative size = 526, normalized size of antiderivative = 9.56

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{-\frac{a}{a+b}}\log\left(\frac{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4-2\,(3\,a+2\,b)\cosh(dx+c)^2+2\left(3\,a\cosh(dx+c)^2-3\,a-2\,b\right)\sinh(d}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+}\right.}{}\right.$$

$$-\frac{\sqrt{\frac{a}{a+b}}\arctan\left(\frac{1}{2}\sqrt{\frac{a}{a+b}}(\cosh(dx+c)+\sinh(dx+c))\right)+\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left(a\cosh(dx+c)^3+3\,a\cosh(dx+c)\sinh(dx\right.}{}\right.}{}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*
b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*c
osh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x
+ c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b))
 + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
+ c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*s
inh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a)) + 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d), -(sqrt(a/(a +
 b))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + sqrt(a/
(a + b))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 +
3*a + 4*b)*sinh(d*x + c))*sqrt(a/(a + b))/a) - 2*arctan(cosh(d*x + c) + si
nh(d*x + c)))/(b*d)]
```

3.79.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.79.7 Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\int \frac{\operatorname{sech}(dx+c)^3}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
2*arctan(e^(d*x + c))/(b*d) - 8*integrate(1/4*(a*e^(3*d*x + 3*c) + a*e^(d*
x + c))/(a*b*e^(4*d*x + 4*c) + a*b + 2*(a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*
d*x)), x)
```

3.79.8 Giac [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.79.9 Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 307, normalized size of antiderivative = 5.58

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\left(9\,a^2\sqrt{b^2\,d^2}+16\,b^2\sqrt{b^2\,d^2}+24\,a\,b\sqrt{b^2\,d^2}\right)}{9\,d\,a^2\,b+24\,d\,a\,b^2+16\,d\,b^3}\right)}{\sqrt{b^2\,d^2}} - \frac{\sqrt{a}\left(2\,\mathrm{atan}\left(\frac{\sqrt{a}\,e^{dx}\,e^{c}\sqrt{b^2\,d^2\,(a+b)}}{2\,b\,d\,(a+b)}\right)+2\,\mathrm{atan}\left(\frac{4\,b^4\,d^2\,e^{dx}\,e^{c}+4\,a^2\,b^2\,d^2\,e^{dx}\,e^{c}-a\,e^{dx}\,e^{c}\sqrt{b^3\,d^2+a\,b^2\,d^2}\sqrt{b^2\,d^2\,(a+b)}+8\,a\,b^3\,d^2\,e^{d}\cdots}{\sqrt{a}\,d\,(2\,b^2+2\,a\,b)\sqrt{b^2\,d^2\,(a+b)}}\right)\right)}{2\sqrt{b^3\,d^2+a\,b^2\,d^2}}$$

input
```
int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)),x)
```

output
```
(2*atan((exp(d*x)*exp(c)*(9*a^2*(b^2*d^2)^(1/2) + 16*b^2*(b^2*d^2)^(1/2) +
 24*a*b*(b^2*d^2)^(1/2)))/(16*b^3*d + 24*a*b^2*d + 9*a^2*b*d)))/(b^2*d^2)^
(1/2) - (a^(1/2)*(2*atan((a^(1/2)*exp(d*x)*exp(c)*(b^2*d^2*(a + b))^(1/2))
/(2*b*d*(a + b))) + 2*atan((4*b^4*d^2*exp(d*x)*exp(c) + 4*a^2*b^2*d^2*exp(
d*x)*exp(c) - a*exp(d*x)*exp(c)*(b^3*d^2 + a*b^2*d^2)^(1/2)*(b^2*d^2*(a +
b))^(1/2) + 8*a*b^3*d^2*exp(d*x)*exp(c) + a*exp(3*c)*exp(3*d*x)*(b^3*d^2 +
 a*b^2*d^2)^(1/2)*(b^2*d^2*(a + b))^(1/2))/(a^(1/2)*d*(2*a*b + 2*b^2)*(b^2
*d^2*(a + b))^(1/2)))))/(2*(b^3*d^2 + a*b^2*d^2)^(1/2))
```

3.80 $\int \frac{\text{sech}^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.80.1 Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\text{sech}^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}d}+\frac{\tanh(c+dx)}{bd}$$

output
```
-a*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/d/(a+b)^(1/2)+tanh(d*x
+c)/b/d
```

3.80.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(52) = 104.

Time = 2.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.50

$$\int \frac{\text{sech}^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(a\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(\cdots}{2b\sqrt{a+b}d\left(a+b\text{sech}^2(c+dx)\right)\sqrt{b(\cdots}}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(a*ArcTanh[(Sech[d*x]*(Co
sh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b
]*Sech[c]*Sech[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x]))/(2*b*Sqr
t[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

3.80.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^4}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\tanh(c+dx)}{b}-\frac{a\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\tanh(c+dx)}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-((a*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]))
 + Tanh[c + d*x]/b)/d
```

3.80.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(44) = 88$.

Time = 0.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.65

method	result
derivativedivides	$\frac{\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}+\frac{2a\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\ldots}\right.}{4\sqrt{b}\,\sqrt{a+b}}\right.}{b}}{d}$
default	$\frac{\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}+\frac{2a\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\ldots}\right.}{4\sqrt{b}\,\sqrt{a+b}}\right.}{b}}{d}$
risch	$-\frac{2}{bd\left(e^{2dx+2c}+1\right)}+\frac{a\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^{2}}+2b\sqrt{ab+b^{2}}+2ab+2b^{2}}{a\sqrt{ab+b^{2}}}\right)}{2\sqrt{ab+b^{2}}\,db}-\frac{a\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^{2}}+2b\sqrt{ab+b^{2}}-2ab-2b^{2}}{a\sqrt{ab+b^{2}}}\right.}{2\sqrt{ab+b^{2}}\,db}$

input
```
int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)+2*a/b*(-1/4/b^(1/2)
/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^
(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2
*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(44) = 88$.

Time = 0.28 (sec) , antiderivative size = 645, normalized size of antiderivative = 12.40

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

$$= \left[\frac{\left(a\cosh(dx+c)^2 + 2\,a\cosh(dx+c)\sinh(dx+c) + a\sinh(dx+c)^2 + a\right)\sqrt{ab+b^2}\log\left(\frac{a^2\cosh(dx+c)^4+4\,a^2\cdots}{\cdots}\right.}{2\,((\cdots}\right.$$

$$-\frac{\left(a\cosh(dx+c)^2 + 2\,a\cosh(dx+c)\sinh(dx+c) + a\sinh(dx+c)^2 + a\right)\sqrt{-ab-b^2}\arctan\left(\frac{\left(a\cosh(dx\cdots\right.}{\cdots}\right.}{(ab^2+b^3)d\cosh(dx+c)^2 + 2\,(ab^2+b^3)d\cosh(dx+c)\sinh(dx+c) + (ab\cdots}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*((a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x +
c)^2 + a)*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*s
inh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2
*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b
^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^
2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh
(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) + a)) - 4*a*b - 4*b^2)/((a*b^2 + b^3)*d*cosh(
d*x + c)^2 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 + b^3)
*d*sinh(d*x + c)^2 + (a*b^2 + b^3)*d), -((a*cosh(d*x + c)^2 + 2*a*cosh(d*x
 + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a*b - b^2)*arctan(1/2*(
a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 +
a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 2*a*b + 2*b^2)/((a*b^2 + b^3)*d*c
osh(d*x + c)^2 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 +
b^3)*d*sinh(d*x + c)^2 + (a*b^2 + b^3)*d)]
```

3.80.6 Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\int \frac{\operatorname{sech}^4{(c+dx)}}{a+b\operatorname{sech}^2{(c+dx)}}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.80.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(44) = 88$.

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\frac{a\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}bd}+\frac{2}{(be^{(-2\,dx-2\,c)}+b)d}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 2/((b*e^(-2
*d*x - 2*c) + b)*d)
```

3.80.8 Giac [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^4}{b\operatorname{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.19

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\frac{a\ln\left(\frac{4\,e^{2\,c+2\,d\,x}}{b}-\frac{2\,\left(a\,d+a\,d\,e^{2\,c+2\,d\,x}+2\,b\,d\,e^{2\,c+2\,d\,x}\right)}{b^{3/2}\,d\,\sqrt{a+b}}\right)}{2\,b^{3/2}\,d\,\sqrt{a+b}}-\frac{2}{b\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{a\ln\left(\frac{4\,e^{2\,c+2\,d\,x}}{b}+\frac{2\,\left(a\,d+a\,d\,e^{2\,c+2\,d\,x}+2\,b\,d\,e^{2\,c+2\,d\,x}\right)}{b^{3/2}\,d\,\sqrt{a+b}}\right)}{2\,b^{3/2}\,d\,\sqrt{a+b}}$$

input
```
int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)),x)
```

output
```
(a*log((4*exp(2*c + 2*d*x))/b - (2*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*d*exp
(2*c + 2*d*x)))/(b^(3/2)*d*(a + b)^(1/2))))/(2*b^(3/2)*d*(a + b)^(1/2)) -
2/(b*d*(exp(2*c + 2*d*x) + 1)) - (a*log((4*exp(2*c + 2*d*x))/b + (2*(a*d +
 a*d*exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(b^(3/2)*d*(a + b)^(1/2))
))/(2*b^(3/2)*d*(a + b)^(1/2))
```

3.81 $\int \frac{\text{sech}^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.81.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\text{sech}^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{(2a-b)\arctan(\sinh(c+dx))}{2b^2d} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}\,d} + \frac{\text{sech}(c+dx)\tanh(c+dx)}{2bd}$$

output
```
-1/2*(2*a-b)*arctan(sinh(d*x+c))/b^2/d+a^(3/2)*arctan(sinh(d*x+c)*a^(1/2)/
(a+b)^(1/2))/b^2/d/(a+b)^(1/2)+1/2*sech(d*x+c)*tanh(d*x+c)/b/d
```

3.81.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 213 vs. 2(86) = 172.

Time = 2.03 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.48

$$\int \frac{\text{sech}^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\cosh(c)(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(b\sqrt{a+b}\text{sech}^2(c)\text{sech}^2(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}\,\text{s}\right.}{}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

output
```
(Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*Sqrt[a + b]*Se
ch[c]^2*Sech[c + d*x]^2*Sqrt[(Cosh[c] - Sinh[c])^2]*Sinh[d*x] + 2*a^(3/2)*
ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + S
inh[c]))/Sqrt[a]]*(-1 + Tanh[c]) - Sqrt[a + b]*Sech[c]*Sqrt[(Cosh[c] - Sin
h[c])^2]*(2*(2*a - b)*ArcTan[Tanh[(c + d*x)/2]] - b*Sech[c + d*x]*Tanh[c])
))/(4*b^2*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2
])
```

3.81.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4635, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^5(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^5}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2\left(a\sinh^2(c+dx)+a+b\right)}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)}-\frac{\int \frac{-a\sinh^2(c+dx)+a-b}{\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}d\sinh(c+dx)}{2b}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)}-\frac{\frac{(2a-b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}-\frac{2a^2\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{2b}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\sinh(c+dx)}{2b(\sinh^2(c+dx)+1)}-\frac{\frac{(2a-b)\arctan(\sinh(c+dx))}{b}-\frac{2a^2\int\frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{2b}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sinh(c+dx)}{2b(\sinh^2(c+dx)+1)}-\frac{\frac{(2a-b)\arctan(\sinh(c+dx))}{b}-\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{2b}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-1/2*(((2*a - b)*ArcTan[Sinh[c + d*x]])/b - (2*a^(3/2)*ArcTan[(Sqrt[a]*Si
nh[c + d*x])/Sqrt[a + b]])/(b*Sqrt[a + b]))/b + Sinh[c + d*x]/(2*b*(1 + Si
nh[c + d*x]^2)))/d
```

3.81.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.81. $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(74) = 148$.

Time = 1.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+\frac{(2a-b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2a^2\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\sqrt{a}}+\frac{\arctan\left(\cdots\right.}{\cdots}\right.}{b^2}\cdots}{d}$
default	$\frac{-\frac{2\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+\frac{(2a-b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2a^2\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\sqrt{a}}+\frac{\arctan\left(\cdots\right.}{\cdots}\right.}{b^2}\cdots}{d}$
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{db\left(e^{2dx+2c}+1\right)^2}+\frac{i\ln\left(e^{dx+c}-i\right)a}{d\,b^2}-\frac{i\ln\left(e^{dx+c}-i\right)}{2db}-\frac{i\ln\left(e^{dx+c}+i\right)a}{d\,b^2}+\frac{i\ln\left(e^{dx+c}+i\right)}{2db}+\frac{\sqrt{-a(a+b)}\,a\ln\left(e^{\cdots}\right.}{\cdots}$

input
```
int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^2*((1/2*tanh(1/2*d*x+1/2*c)^3*b-1/2*b*tanh(1/2*d*x+1/2*c))/(tanh
(1/2*d*x+1/2*c)^2+1)^2+1/2*(2*a-b)*arctan(tanh(1/2*d*x+1/2*c)))+2*a^2/b^2*
(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b
^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/
2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))))
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 1518, normalized size of antiderivative = 17.65

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(d
*x + c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sin
h(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x
+ c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-
a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*
sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 -
 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x
+ c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)
*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c) + (3*(a
 + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/(a*cos
h(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(
a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)
^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) -
 2*((2*a - b)*cosh(d*x + c)^4 + 4*(2*a - b)*cosh(d*x + c)*sinh(d*x + c)^3
+ (2*a - b)*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)
*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*((2*a - b)*cosh(d*x + c)^3
 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + 2*a - b)*arctan(cosh(d*x + c)
+ sinh(d*x + c)) - 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*
x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b
^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cos...
```

3.81.6 Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2), x)
```

3.81.7 Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^5}{b\operatorname{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
(e^(3*d*x + 3*c) - e^(d*x + c))/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*
c) + b*d) - (2*a*e^c - b*e^c)*arctan(e^(d*x + c))*e^(-c)/(b^2*d) + 32*inte
grate(1/16*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(a*b^2*e^(4*d*x + 4*c)
+ a*b^2 + 2*(a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

3.81.8 Giac [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^5}{b\operatorname{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.81.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 946, normalized size of antiderivative = 11.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$
$$= \frac{\sqrt{a^3}\left(2\,\mathrm{atan}\left(\left(\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(\frac{64\left(6\,b^3\,d\,{\left(a^3\right)}^{3/2}+2\,b^6\,d\,\sqrt{a^3}+6\,a\,b^2\,d\,{\left(a^3\right)}^{3/2}-4\,a\,b^5\,d\,\sqrt{a^3}-6\,a^2\,b^4\,d\,\sqrt{a^3}\right)}{a^4\,b^4\,\left(a+b\right)\,\left(b^2+a\,b\right)\,\sqrt{b^5\,d^2+a\,b^4\,d^2}\,\sqrt{b^4\,d^2\,\left(a+b\right)}\,\left(3\,a^3-3\,a\,b^2+b^3\right)}-\frac{32\left(3\,a^5\,\sqrt{b^5\,d^2+a\,b^4\,d^2}+a^2\,b^3 \cdots\right.}{a^2\,b^6\,d\,{\left(a+b\right)}^2\,\left(b^2+a\,b\right)\,\sqrt{\cdots}}\cdots}{\cdots}$$
$$-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(18\,a^7\,\sqrt{b^4\,d^2}-b^7\,\sqrt{b^4\,d^2}-21\,a^2\,b^5\,\sqrt{b^4\,d^2}+12\,a^3\,b^4\,\sqrt{b^4\,d^2}+30\,a^4\,b^3\,\sqrt{b^4\,d^2}-36\,a^5\,b^2\,\sqrt{b^4\,d^2}+8\,a\,b^6\,\sqrt{b^4\,d^2}-9\,a^6\,b \cdots\right.}{b^8\,d\,\sqrt{4\,a^2-4\,a\,b+b^2}+9\,a^2\,b^6\,d\,\sqrt{4\,a^2-4\,a\,b+b^2}+6\,a^3\,b^5\,d\,\sqrt{4\,a^2-4\,a\,b+b^2}-18\,a^4\,b^4\,d\,\sqrt{4\,a^2-4\,a\,b+b^2}+9\,a^6\,b^2\,d\,\sqrt{4\,a^2-4\,a\,b+b^2}-6\,a\,b \cdots}\cdots\right)\cdots}{\sqrt{b^4\,d^2}\cdots}$$
$$+\frac{\mathrm{e}^{c+d\,x}}{b\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}}{b\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

input
```
int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)),x)
```

output
```
((a^3)^(1/2)*(2*atan((exp(d*x)*exp(c)*((64*(6*b^3*d*(a^3)^(3/2) + 2*b^6*d*
(a^3)^(1/2) + 6*a*b^2*d*(a^3)^(3/2) - 4*a*b^5*d*(a^3)^(1/2) - 6*a^2*b^4*d*
(a^3)^(1/2)))/(a^4*b^4*(a + b)*(a*b + b^2)*(b^5*d^2 + a*b^4*d^2)^(1/2)*(b^
4*d^2*(a + b))^(1/2)*(3*a^3 - 3*a*b^2 + b^3)) - (32*(3*a^5*(b^5*d^2 + a*b^
4*d^2)^(1/2) + a^2*b^3*(b^5*d^2 + a*b^4*d^2)^(1/2) - 3*a^3*b^2*(b^5*d^2 +
a*b^4*d^2)^(1/2)))/(a^2*b^6*d*(a + b)^2*(a*b + b^2)*(b^5*d^2 + a*b^4*d^2)^
(1/2)*(a^3)^(1/2)*(3*a^3 - 3*a*b^2 + b^3))) + (32*exp(3*c)*exp(3*d*x)*(3*a
^5*(b^5*d^2 + a*b^4*d^2)^(1/2) + a^2*b^3*(b^5*d^2 + a*b^4*d^2)^(1/2) - 3*a
^3*b^2*(b^5*d^2 + a*b^4*d^2)^(1/2)))/(a^2*b^6*d*(a + b)^2*(a*b + b^2)*(b^5
*d^2 + a*b^4*d^2)^(1/2)*(a^3)^(1/2)*(3*a^3 - 3*a*b^2 + b^3)))*((a^2*b^7*(b
^5*d^2 + a*b^4*d^2)^(1/2))/64 + (a^3*b^6*(b^5*d^2 + a*b^4*d^2)^(1/2))/32 +
 (a^4*b^5*(b^5*d^2 + a*b^4*d^2)^(1/2))/64)) + 2*atan((a^2*exp(d*x)*exp(c)*
(b^4*d^2*(a + b))^(1/2))/(2*b^2*d*(a + b)*(a^3)^(1/2)))))/(2*(b^5*d^2 + a*
b^4*d^2)^(1/2)) - (atan((exp(d*x)*exp(c)*(18*a^7*(b^4*d^2)^(1/2) - b^7*(b^
4*d^2)^(1/2) - 21*a^2*b^5*(b^4*d^2)^(1/2) + 12*a^3*b^4*(b^4*d^2)^(1/2) + 3
0*a^4*b^3*(b^4*d^2)^(1/2) - 36*a^5*b^2*(b^4*d^2)^(1/2) + 8*a*b^6*(b^4*d^2)
^(1/2) - 9*a^6*b*(b^4*d^2)^(1/2)))/(b^8*d*(4*a^2 - 4*a*b + b^2)^(1/2) + 9*
a^2*b^6*d*(4*a^2 - 4*a*b + b^2)^(1/2) + 6*a^3*b^5*d*(4*a^2 - 4*a*b + b^2)^
(1/2) - 18*a^4*b^4*d*(4*a^2 - 4*a*b + b^2)^(1/2) + 9*a^6*b^2*d*(4*a^2 - 4*
a*b + b^2)^(1/2) - 6*a*b^7*d*(4*a^2 - 4*a*b + b^2)^(1/2)))*(4*a^2 - 4*a...
```

3.81. $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.82 $\int \frac{\text{sech}^6(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.82.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\text{sech}^6(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}d}-\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}$$

output
```
a^2*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/d/(a+b)^(1/2)-(a-b)*t
anh(d*x+c)/b^2/d-1/3*tanh(d*x+c)^3/b/d
```

3.82.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 214 vs. 2(77) = 154.

Time = 3.64 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.78

$$\int \frac{\text{sech}^6(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$=\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(3a^2\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)\right.}{6b^2\sqrt{a+b}}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a^2*ArcTanh[(Sech[d*x]
*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqr
t[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]) + Sqrt[a
+ b]*Sech[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(Sech[c]*(-3*a + 2*b + b*
Sech[c + d*x]^2)*Sinh[d*x] + b*Sech[c + d*x]*Tanh[c])))/(6*b^2*Sqrt[a + b]
*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

3.82.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^6(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^6}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{a^2}{b^2\left(-b\tanh^2(c+dx)+a+b\right)}-\frac{\tanh^2(c+dx)}{b}-\frac{a-b}{b^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\tanh(c+dx)}{b^2}-\frac{\tanh^3(c+dx)}{3b}}{d}$$

input
```
Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a^2*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b])
- ((a - b)*Tanh[c + d*x])/b^2 - Tanh[c + d*x]^3/(3*b))/d
```

3.82.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(67) = 134$.

Time = 1.45 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.38

method	result
derivativedivides	$\frac{\frac{2(-a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+2\left(-2a+\frac{2b}{3}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+2(-a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^3}-\frac{2a^2\left(-\frac{\ln\left(\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}{4\sqrt{b}\sqrt{a+b}}\cdots\right.}{\cdots}}{d}$
default	$\frac{\frac{2(-a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+2\left(-2a+\frac{2b}{3}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+2(-a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^3}-\frac{2a^2\left(-\frac{\ln\left(\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}{4\sqrt{b}\sqrt{a+b}}\cdots\right.}{\cdots}}{d}$
risch	$\frac{2a\,e^{4dx+4c}+4\,e^{2dx+2c}a-4b\,e^{2dx+2c}+2a-\frac{4b}{3}}{b^2d\left(e^{2dx+2c}+1\right)^3}+\frac{a^2\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}-2ab-2b^2}{a\sqrt{ab+b^2}}\right)}{2\sqrt{ab+b^2}\,d\,b^2}-\frac{a^2\ln\left(e^{2dx+2c}+\cdots\right.}{2\cdots}$

input
```
int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*((-a+b)*tanh(1/2*d*x+1/2*c)^5+(-2*a+2/3*b)*tanh(1/2*d*x+1/2*c)^
3+(-a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^3-2*a^2/b^2*(-1/4/
b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/
2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 1905, normalized size of antiderivative = 24.74

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 4*a*b^
2 - 8*b^3 + 24*(a^2*b - b^3)*cosh(d*x + c)^2 + 24*(a^2*b - b^3 + 3*(a^2*b
+ a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(a^2*cosh(d*x + c)^6 + 6*a^2
*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)
^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)
^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2
 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sin
h(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)
*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8
*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)
 - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c
)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sin
h(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*co
sh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b
)*cosh(d*x + c))*sinh(d*x + c) + a)) + 48*((a^2*b + a*b^2)*cosh(d*x + c)^3
 + (a^2*b - b^3)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 + b^4)*d*cosh(d*x +
 c)^6 + 6*(a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a*b^3 + b^4)...
```

3.82.6 Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2), x)
```

3.82.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(67)=134$.

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.08

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=-\frac{a^2\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}b^2d}-\frac{2\,(6\,(a-b)e^{(-2\,dx-2\,c)}+3\,ae^{(-4\,dx-4\,c)}+3\,a-2\,b)}{3\,(3\,b^2e^{(-2\,dx-2\,c)}+3\,b^2e^{(-4\,dx-4\,c)}+b^2e^{(-6\,dx-6\,c)}+b^2)d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*a^2*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d
*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b^2*d) - 2/3*(6
*(a - b)*e^(-2*d*x - 2*c) + 3*a*e^(-4*d*x - 4*c) + 3*a - 2*b)/((3*b^2*e^(-
2*d*x - 2*c) + 3*b^2*e^(-4*d*x - 4*c) + b^2*e^(-6*d*x - 6*c) + b^2)*d)
```

3.82.8 Giac [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\int \frac{\operatorname{sech}(dx+c)^6}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.82.9 Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.34

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$
$$= \frac{8}{3\,b\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$- \frac{4}{b\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} + \frac{2\,a}{b^2\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$
$$- \frac{a^2\,\ln\left(\frac{4\,a^2\,\left(2\,a\,b+a^2+a^2\,e^{2\,c+2\,d\,x}+8\,b^2\,e^{2\,c+2\,d\,x}+8\,a\,b\,e^{2\,c+2\,d\,x}\right)}{b^5\,(a+b)} - \frac{8\,a^2\,\left(a+2\,a\,e^{2\,c+2\,d\,x}+4\,b\,e^{2\,c+2\,d\,x}\right)}{b^{9/2}\,\sqrt{a+b}}\right)}{2\,b^{5/2}\,d\,\sqrt{a+b}}$$
$$+ \frac{a^2\,\ln\left(\frac{8\,a^2\,\left(a+2\,a\,e^{2\,c+2\,d\,x}+4\,b\,e^{2\,c+2\,d\,x}\right)}{b^{9/2}\,\sqrt{a+b}} + \frac{4\,a^2\,\left(2\,a\,b+a^2+a^2\,e^{2\,c+2\,d\,x}+8\,b^2\,e^{2\,c+2\,d\,x}+8\,a\,b\,e^{2\,c+2\,d\,x}\right)}{b^5\,(a+b)}\right)}{2\,b^{5/2}\,d\,\sqrt{a+b}}$$

input
```
int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)),x)
```

output
```
8/(3*b*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
 - 4/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (2*a)/(b^2*d*(exp
(2*c + 2*d*x) + 1)) - (a^2*log((4*a^2*(2*a*b + a^2 + a^2*exp(2*c + 2*d*x)
+ 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*x)))/(b^5*(a + b)) - (8*a^2
*(a + 2*a*exp(2*c + 2*d*x) + 4*b*exp(2*c + 2*d*x)))/(b^(9/2)*(a + b)^(1/2)
)))/(2*b^(5/2)*d*(a + b)^(1/2)) + (a^2*log((8*a^2*(a + 2*a*exp(2*c + 2*d*x
) + 4*b*exp(2*c + 2*d*x)))/(b^(9/2)*(a + b)^(1/2)) + (4*a^2*(2*a*b + a^2 +
 a^2*exp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*x)))/
(b^5*(a + b))))/(2*b^(5/2)*d*(a + b)^(1/2))
```

3.83 $\int \frac{\cosh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

3.83.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{b^2(6a+5b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}d} + \frac{(a-2b)\sinh(c+dx)}{a^3d} + \frac{\sinh^3(c+dx)}{3a^2d} - \frac{b^3\sinh(c+dx)}{2a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
1/2*b^2*(6*a+5*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/(a+b)^(3
/2)/d+(a-2*b)*sinh(d*x+c)/a^3/d+1/3*sinh(d*x+c)^3/a^2/d-1/2*b^3*sinh(d*x+c
)/a^3/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{-\frac{6b^2(6a+5b)\arctan\left(\frac{\sqrt{a+b}\text{csch}(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + 3\sqrt{a}\left(3a-8b-\frac{4b^3}{(a+b)(a+2b+a\cosh(2(c+dx)))}\right)\sinh(c+dx) + a^{3/2}\sinh(3(c-}{12a^{7/2}d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((-6*b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/(a + b)^
(3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*b^3)/((a + b)*(a + 2*b + a*Cosh[2*(c + d
*x)]))*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)])/(12*a^(7/2)*d)
```

3.83.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^3}{\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{\sinh^2(c+dx)}{a^2}+\frac{a-2b}{a^3}+\frac{3a\sinh^2(c+dx)b^2+(3a+2b)b^2}{a^3\left(a\sinh^2(c+dx)+a+b\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(6a+5b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}}-\frac{b^3\sinh(c+dx)}{2a^3(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{(a-2b)\sinh(c+dx)}{a^3}+\frac{\sinh^3(c+dx)}{3a^2}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((b^2*(6*a + 5*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(7/2)*
(a + b)^(3/2)) + ((a - 2*b)*Sinh[c + d*x])/a^3 + Sinh[c + d*x]^3/(3*a^2) -
 (b^3*Sinh[c + d*x])/(2*a^3*(a + b)*(a + b + a*Sinh[c + d*x]^2)))/d
```

3.83.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.83.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(111) = 222$.

Time = 1.80 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.57

method	result
derivativedivides	$\frac{-\frac{1}{3a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a-2b}{a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b^2\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2a+2b}-\frac{b\tan\cdots}{\cdots}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{d\cdots}{2}\cdots\right.}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{1}{3a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a-2b}{a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b^2\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2a+2b}-\frac{b\tan\cdots}{\cdots}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{d\cdots}{2}\cdots\right.}\cdots\right.}{\cdots}}{\cdots}$
risch	$\frac{e^{3dx+3c}}{24a^2d}+\frac{3\,e^{dx+c}}{8a^2d}-\frac{e^{dx+c}b}{a^3d}-\frac{3\,e^{-dx-c}}{8a^2d}+\frac{e^{-dx-c}b}{a^3d}-\frac{e^{-3dx-3c}}{24a^2d}-\frac{b^3e^{dx+c}\left(e^{2dx+2c}-1\right)}{a^3(a+b)d\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx\cdots}\right.}$

input

```
int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/3/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/a^2/(tanh(1/2*d*x+1/2*c)-1)^2-
(a-2*b)/a^3/(tanh(1/2*d*x+1/2*c)-1)+2/a^3*b^2*((1/2*b/(a+b)*tanh(1/2*d*x+1
/2*c)^3-1/2*b/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2
*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1
/2*(6*a+5*b)/(a+b)*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh
(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))))-1/3/a^2/(1+tanh(1/2*
d*x+1/2*c))^3+1/2/a^2/(1+tanh(1/2*d*x+1/2*c))^2-(a-2*b)/a^3/(1+tanh(1/2*d*
x+1/2*c)))
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3084 vs. $2(111) = 222$.

Time = 0.33 (sec) , antiderivative size = 5842, normalized size of antiderivative = 46.74

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.83.6 Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh^3(c+dx)}{(a+b\,\text{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(cosh(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

3.83.7 Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)^3}{(b\,\text{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*(a^3 + a^2*b - (a^3*e^(10*c) + a^2*b*e^(10*c))*e^(10*d*x) - (11*a^3*
e^(8*c) - 9*a^2*b*e^(8*c) - 20*a*b^2*e^(8*c))*e^(8*d*x) - 2*(5*a^3*e^(6*c)
 + 11*a^2*b*e^(6*c) - 42*a*b^2*e^(6*c) - 60*b^3*e^(6*c))*e^(6*d*x) + 2*(5*
a^3*e^(4*c) + 11*a^2*b*e^(4*c) - 42*a*b^2*e^(4*c) - 60*b^3*e^(4*c))*e^(4*d
*x) + (11*a^3*e^(2*c) - 9*a^2*b*e^(2*c) - 20*a*b^2*e^(2*c))*e^(2*d*x))/((a
^5*d*e^(7*c) + a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^(5*c) + 3*a^4*b*d*e
^(5*c) + 2*a^3*b^2*d*e^(5*c))*e^(5*d*x) + (a^5*d*e^(3*c) + a^4*b*d*e^(3*c)
)*e^(3*d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c) + 5*b^3*e^(3*c))*e^(3*d*x
) + (6*a*b^2*e^c + 5*b^3*e^c)*e^(d*x))/(a^5 + a^4*b + (a^5*e^(4*c) + a^4*b
*e^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 3*a^4*b*e^(2*c) + 2*a^3*b^2*e^(2*c)
)*e^(2*d*x)), x)
```

3.83.8 Giac [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx=\int \frac{\cosh(dx+c)^3}{(b\,\text{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx=\int \frac{\cosh(c+d\,x)^3}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2, x)
```

3.84 $\int \frac{\cosh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

3.84.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b(a+2b)\tanh(c+dx)}{2a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
1/2*(a-4*b)*x/a^3+1/2*b^(3/2)*(5*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^
(1/2))/a^3/(a+b)^(3/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c
)^2)+1/2*b*(a+2*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

3.84.2 Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.72

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{2(a-4b)(c+dx) + \frac{2b^{3/2}(5a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \left(a+\frac{2ab^2}{(a+b)(a+2b+a\cosh(2(c+dx)))}\right)\sinh(2(c+dx))}{4a^3d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(2*(a - 4*b)*(c + d*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*C
osh[2*(c + d*x)])))*Sinh[2*(c + d*x)])/(4*a^3*d)
```

3.84.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4634, 316, 402, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-3b\tanh^2(c+dx)+a-b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{b(a+2b)\tanh(c+dx)}{a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\int -\frac{2\left(a^2-2ba-2b^2-b(a+2b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{a^2-2ba-2b^2-b(a+2b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a(a+b)}+\frac{b(a+2b)\tanh(c+dx)}{a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{b^2(5a+4b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}+\frac{(a-4b)(a+b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}}{a(a+b)}+\frac{b(a+2b)\tanh(c+dx)}{a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\frac{b^2(5a+4b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}+\frac{(a-4b)(a+b)\text{arctanh}(\tanh(c+dx))}{a}}{a(a+b)}+\frac{b(a+2b)\tanh(c+dx)}{a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{\frac{b^{3/2}(5a+4b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-4b)(a+b)\text{arctanh}(\tanh(c+dx))}{a}}{a(a+b)}+\frac{b(a+2b)\tanh(c+dx)}{a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{2a}+\frac{\tanh(c+dx)}{2a\left(1-\tanh^2(c+dx)\right)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]/(2*a*(1 - Tanh[c + d*x]^2)*(a + b - b*Tanh[c + d*x]^2)) + (
(((a - 4*b)*(a + b)*ArcTanh[Tanh[c + d*x]])/a + (b^(3/2)*(5*a + 4*b)*ArcTa
nh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b
*(a + 2*b)*Tanh[c + d*x])/(a*(a + b)*(a + b - b*Tanh[c + d*x]^2)))/(2*a))/
d
```

3.84. $\int \frac{\cosh^2(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.84.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(128) = 256$.

Time = 1.26 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.35

method	result
risch	$\frac{x}{2a^2} - \frac{2xb}{a^3} + \frac{e^{2dx+2c}}{8a^2d} - \frac{e^{-2dx-2c}}{8a^2d} - \frac{b^2\left(e^{2dx+2c}a+2b\,e^{2dx+2c}+a\right)}{a^3(a+b)d\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)} + \frac{5\sqrt{(a+b)b}\,b\ln\left(e^{2dx+\dots}\right.}{4(a+b\dots}$
derivativedivides	$\frac{-\frac{1}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a-4b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^3}-\frac{2b^2\left(\frac{-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)}\dots}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tan\dots}\right.}{\dots}}{\dots}$
default	$\frac{-\frac{1}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a-4b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^3}-\frac{2b^2\left(\frac{-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)}\dots}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tan\dots}\right.}{\dots}}{\dots}$

input
```
int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a^2-2*x/a^3*b+1/8/a^2/d*exp(2*d*x+2*c)-1/8/a^2/d*exp(-2*d*x-2*c)-b^2
*(exp(2*d*x+2*c)*a+2*b*exp(2*d*x+2*c)+a)/a^3/(a+b)/d/(a*exp(4*d*x+4*c)+2*e
xp(2*d*x+2*c)*a+4*b*exp(2*d*x+2*c)+a)+5/4*((a+b)*b)^(1/2)/(a+b)^2*b/d/a^2*
ln(exp(2*d*x+2*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a)+((a+b)*b)^(1/2)/(a+b)^2*b^2
/d/a^3*ln(exp(2*d*x+2*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a)-5/4*((a+b)*b)^(1/2)/
(a+b)^2*b/d/a^2*ln(exp(2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/a)-((a+b)*b)^(
1/2)/(a+b)^2*b^2/d/a^3*ln(exp(2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/a)
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1731 vs. $2(134) = 268$.

Time = 0.32 (sec) , antiderivative size = 3739, normalized size of antiderivative = 25.97

$$\int \frac{\cosh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*((a^3 + a^2*b)*cosh(d*x + c)^8 + 8*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d
*x + c)^7 + (a^3 + a^2*b)*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2
*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 + 3*a^2*b + 2*a*b
^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x + 14*(a^3 + a^2*b)*cosh(d*x + c)^2)*s
inh(d*x + c)^6 + 4*(14*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b +
2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5
- 8*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^4
 + 2*(35*(a^3 + a^2*b)*cosh(d*x + c)^4 - 4*a*b^2 - 8*b^3 + 4*(a^3 - a^2*b
- 10*a*b^2 - 8*b^3)*d*x + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b -
 4*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + a^2*b)*cosh(
d*x + c)^5 + 5*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)
*cosh(d*x + c)^3 - 4*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x
)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 2*(a^3 + 3*a^2*b + 6*a*b^
2 - 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + a^2*b)
*cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^
2)*d*x)*cosh(d*x + c)^4 - a^3 - 3*a^2*b - 6*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a
*b^2)*d*x - 24*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 2*((5*a^2*b + 4*a*b^2)*cosh(d*x + c)^6 + 6*
(5*a^2*b + 4*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 4*a*b^2)*si
nh(d*x + c)^6 + 2*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + (10*a^...
```

3.84.6 Sympy [F]

$$\int \frac{\cosh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh^2(c+dx)}{(a+b\,\text{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(cosh(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.84.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(134) = 268$.

Time = 0.31 (sec) , antiderivative size = 696, normalized size of antiderivative = 4.83

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$$
$$= \frac{(3\,a^2b+12\,ab^2+8\,b^3)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^4+a^3b)\sqrt{(a+b)b}d} - \frac{(3\,a^2b+12\,ab^2+8\,b^3)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^4+a^3b)\sqrt{(a+b)b}d} + \frac{(3\,ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,(a^3+a^2b)\sqrt{(a+b)b}d} - \frac{a^2b+2\,ab^2+(a^2b+8\,ab^2+8\,b^3)e^{(2\,dx+2\,c)}}{4\,(a^5+a^4b+(a^5+a^4b)e^{(4\,dx+4\,c)}+2\,(a^5+3\,a^4b+2\,a^3b^2)e^{(2\,dx+2\,c)})d} + \frac{a^2b+2\,ab^2+(a^2b+8\,ab^2+8\,b^3)e^{(-2\,dx-2\,c)}}{4\,(a^5+a^4b+2\,(a^5+3\,a^4b+2\,a^3b^2)e^{(-2\,dx-2\,c)}+(a^5+a^4b)e^{(-4\,dx-4\,c)})d} - \frac{ab+(ab+2\,b^2)e^{(-2\,dx-2\,c)}}{2\,(a^4+a^3b+2\,(a^4+3\,a^3b+2\,a^2b^2)e^{(-2\,dx-2\,c)}+(a^4+a^3b)e^{(-4\,dx-4\,c)})d} + \frac{dx+c}{2\,a^2d} + \frac{e^{(2\,dx+2\,c)}}{8\,a^2d} - \frac{e^{(-2\,dx-2\,c)}}{8\,a^2d} - \frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{2\,a^3d} + \frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^3d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqr
t((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a
^3*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(-2*
d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 1/8*(3*a*b + 2*b^2
)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*
c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) - 1/4
*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(2*d*x + 2*c))/((a^5 + a^4
*b + (a^5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(2*d*
x + 2*c))*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(-2*d*x
- 2*c))/((a^5 + a^4*b + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(-2*d*x - 2*c) + (
a^5 + a^4*b)*e^(-4*d*x - 4*c))*d) - 1/2*(a*b + (a*b + 2*b^2)*e^(-2*d*x - 2
*c))/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4
 + a^3*b)*e^(-4*d*x - 4*c))*d) + 1/2*(d*x + c)/(a^2*d) + 1/8*e^(2*d*x + 2*
c)/(a^2*d) - 1/8*e^(-2*d*x - 2*c)/(a^2*d) - 1/2*b*log(a*e^(4*d*x + 4*c) +
2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3*d) + 1/2*b*log(2*(a + 2*b)*e^(-2*d*x
 - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

3.84.8 Giac [F]

$$\int \frac{\cosh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)^2}{(b\,\text{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)^2}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2, x)
```

3.85 $\int \frac{\cosh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

3.85.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\cosh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = -\frac{b(4a+3b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}d}+\frac{\sinh(c+dx)}{a^2d}+\frac{b^2\sinh(c+dx)}{2a^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
-1/2*b*(4*a+3*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+b)^(3/
2)/d+sinh(d*x+c)/a^2/d+1/2*b^2*sinh(d*x+c)/a^2/(a+b)/d/(a+b+a*sinh(d*x+c)^
2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{-\frac{b(4a+3b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\sqrt{a}\sinh(c+dx)\left(2+\frac{b^2}{(a+b)\left(a+b+a\sinh^2(c+dx)\right)}\right)}{2a^{5/2}d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(-((b*(4*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a + b)^(3/
2)) + Sqrt[a]*Sinh[c + d*x]*(2 + b^2/((a + b)*(a + b + a*Sinh[c + d*x]^2))
))/(2*a^(5/2)*d)
```

3.85.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(ic+idx)\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(\frac{1}{a^2}-\frac{2ab\sinh^2(c+dx)+b(2a+b)}{a^2\left(a\sinh^2(c+dx)+a+b\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b(4a+3b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}}+\frac{b^2\sinh(c+dx)}{2a^2(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{a^2}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(-1/2*(b*(4*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(5/2)
*(a + b)^(3/2)) + Sinh[c + d*x]/a^2 + (b^2*Sinh[c + d*x])/(2*a^2*(a + b)*(
a + b + a*Sinh[c + d*x]^2)))/d
```

3.85.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(88) = 176$.

Time = 0.93 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.37

method	result
derivativedivides	$\frac{-\frac{1}{a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{1}{a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2b\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2a+2b}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a \cdots}\right.}{\cdots}}{d}$
default	$\frac{-\frac{1}{a^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{1}{a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2b\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2a+2b}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a \cdots}\right.}{\cdots}}{d}$
risch	$\frac{e^{dx+c}}{2a^2d}-\frac{e^{-dx-c}}{2a^2d}+\frac{b^2 e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\,a^2(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}-\frac{b\ln\left(e^{2dx+2c}+\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{\sqrt{-a^2-ab}\,(a+b)da}-\frac{3b^2\ln \cdots}{\cdots}$

input
```
int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2/(1+tanh(1/2*d*x+1/2*c))-1/a^2/(tanh(1/2*d*x+1/2*c)-1)-2/a^2*b*
((1/2*b/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/2*b/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh
(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)+1/2*(4*a+3*b)/(a+b)*(1/2/(a+b)^(1/2)/a^(1/2)*ar
ctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/(a+b)^
(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(
1/2)))))
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(88) = 176$.

Time = 0.30 (sec) , antiderivative size = 3154, normalized size of antiderivative = 31.54

$$\int \frac{\cosh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 12*(a^4 + 2*a^3*b + a^
2*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4 + 2*a^3*b + a^2*b^2)*sinh(d*
x + c)^6 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 2*(a
^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 - 2*a^4 - 4*a^3*b - 2*a^2*b^2 + 8*(5*(a^4 + 2*a^
3*b + a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*co
sh(d*x + c))*sinh(d*x + c)^3 - 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*co
sh(d*x + c)^2 + 2*(15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 - 6*
a^3*b - 11*a^2*b^2 - 6*a*b^3 + 6*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 - ((4*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 5*
(4*a^2*b + 3*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 3*a*b^2)*si
nh(d*x + c)^5 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^3 + 2*(4*a^2*
b + 11*a*b^2 + 6*b^3 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c
)^3 + 2*(5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^2*b + 11*a*b^2 + 6
*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a^2*b + 3*a*b^2)*cosh(d*x + c) +
 (5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 4*a^2*b + 3*a*b^2 + 6*(4*a^2*b +
 11*a*b^2 + 6*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a
*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 -
 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x...
```

3.85.6 Sympy [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(c+dx\right)}{\left(a+b\,\text{sech}^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(cosh(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

3.85.7 Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)}{(b\,\text{sech}\,(dx+c)^2+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) + 5
*a*b*e^(4*c) + 6*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) + 5*a*b*e^(2*c) + 6
*b^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + a^3*b*d*e^(5*c))*e^(5*d*x) + 2*
(a^4*d*e^(3*c) + 3*a^3*b*d*e^(3*c) + 2*a^2*b^2*d*e^(3*c))*e^(3*d*x) + (a^4
*d*e^c + a^3*b*d*e^c)*e^(d*x)) - 1/2*integrate(2*((4*a*b*e^(3*c) + 3*b^2*e
^(3*c))*e^(3*d*x) + (4*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^4 + a^3*b + (a^4*e
^(4*c) + a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 2*a
^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

3.85.8 Giac [F]

$$\int \frac{\cosh(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)}{(b\,\text{sech}\,(dx+c)^2+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

input
```
int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^2, x)
```

3.86 $\int \frac{\mathrm{sech}(c+dx)}{(a+b\mathrm{sech}^2(c+dx))^2}\,dx$

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\mathrm{sech}(c+dx)}{(a+b\mathrm{sech}^2(c+dx))^2}\,dx = \frac{(2a+b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}d} - \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
1/2*(2*a+b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)/d-
1/2*b*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int \frac{\mathrm{sech}(c+dx)}{(a+b\mathrm{sech}^2(c+dx))^2}\,dx = \frac{(2a^2+3ab+b^2)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right) - \sqrt{a}b\sqrt{a+b}\sinh(c+dx) + a(2a+b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)\sinh^2}{a^{3/2}(a+b)^{3/2}d(a+2b+a\cosh(2(c+dx)))}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((2*a^2 + 3*a*b + b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]] - Sqrt[
a]*b*Sqrt[a + b]*Sinh[c + d*x] + a*(2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x]
)/Sqrt[a + b]]*Sinh[c + d*x]^2)/(a^(3/2)*(a + b)^(3/2)*d*(a + 2*b + a*Cosh
[2*(c + d*x)]))
```

3.86.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4635, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(2a+b)\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{2a(a+b)}-\frac{b\sinh(c+dx)}{2a(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(2a+b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{2a(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(((2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b
)^(3/2)) - (b*Sinh[c + d*x])/(2*a*(a + b)*(a + b + a*Sinh[c + d*x]^2)))/d
```

3.86.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.86.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(70) = 140$.

Time = 0.55 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.45

method	result
derivativedivides	$\frac{\frac{\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a(a+b)}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\frac{(2a+b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\cdots\right.}{a(a+b)}}{d}$
default	$\frac{\frac{\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a(a+b)}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\frac{(2a+b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\cdots\right.}{a(a+b)}}{d}$
risch	$-\frac{e^{dx+c}b\left(e^{2dx+2c}-1\right)}{ad(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^{2}-ab}}-1\right)}{2\sqrt{-a^{2}-ab}\,(a+b)d}-\frac{b\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^{2}-ab}}\cdots\right.}{4\sqrt{-a^{2}-ab}\,(a+b)da}$

input
```
int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(1/2*b/a/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/2*b/a/(a+b)*tanh(1/2*d*x+1/2
*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+(2*a+b)/a/(a+b)*(1/2/(a+b)^(1/2)/a^(1
/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/
(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2
))/a^(1/2))))
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 1856, normalized size of antiderivative = 22.63

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 + ((2*a^2 + a*b)*cos
h(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*
b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a
^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2
 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d
*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh
(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c
)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x
+ c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3
*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)
*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 +
 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(
d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh
(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(a^2*b + a*
b^2)*cosh(d*x + c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*
a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^
2)*d*sinh(d*x + c)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*
x + c)^2 + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*...
```

3.86.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

3.86.7 Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}(dx+c)}{\left(b\operatorname{sech}(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*d + a^2*b*d + (a^3*d*e^(4*c) + a
^2*b*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) + 3*a^2*b*d*e^(2*c) + 2*a*b^2
*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/2*((2*a*e^(3*c) + b*e^(3*c))*e^(3*d
*x) + (2*a*e^c + b*e^c)*e^(d*x))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(4*
c))*e^(4*d*x) + 2*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c))*e^(2*d
*x)), x)
```

3.86.8 Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}(dx+c)}{\left(b\operatorname{sech}(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2), x)
```

3.87 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx$

3.87.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}d} + \frac{\tanh(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
1/2*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/(a+b)^(3/2)/d/b^(1/2)+1/2*tan
h(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

3.87.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 187 vs. 2(74) = 148.

Time = 2.45 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.53

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^4(c+dx)\left(\frac{\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\,\mathrm{c}}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{8(a+b)d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((ArcTanh[(Sech[d*x]*(Cos
h[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[
2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + Sech[2*c]
*Sinh[2*d*x] - ((a + 2*b)*Tanh[2*c])/a))/(8*(a + b)*d*(a + b*Sech[c + d*x]
^2)^2)
```

3.87.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^2}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}}+\frac{\tanh(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)) +
Tanh[c + d*x]/(2*(a + b)*(a + b - b*Tanh[c + d*x]^2)))/d
```

3.87.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.87.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(62) = 124$.

Time = 0.51 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.78

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a+b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}-\frac{-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}\cdots\right.}{4\sqrt{b}\,\sqrt{a+b}}\cdots}{\cdots}}{d}$
default	$-\frac{\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a+b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}-\frac{-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}\cdots\right.}{4\sqrt{b}\,\sqrt{a+b}}\cdots}{\cdots}}{d}$
risch	$-\frac{e^{2dx+2c}a+2b\,e^{2dx+2c}+a}{ad(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^{2}}+2b\sqrt{ab+b^{2}}-2ab-2b^{2}}{a\sqrt{ab+b^{2}}}\right)}{4\sqrt{ab+b^{2}}\,(a+b)d}-\frac{\ln\left(e^{2dx+2c}+\cdots\right.}{\cdots}$

input
```
int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/2/(a+b)*tanh(1/2*d*x+1/2*c))/(
tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^
(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4
/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1
/2*c)*b^(1/2)+(a+b)^(1/2))))
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 1489, normalized size of antiderivative = 20.12

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8
*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^
2 + 2*b^3)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*si
nh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*
(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(
d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*l
og((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d
*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a
^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3
 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a
*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2
))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c
)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh
(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a
^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a
^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*
cosh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (
a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 2*a
^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^...
```

3.87.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(65) = 130$.

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.03

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \frac{(a+2\,b)e^{(-2\,dx-2\,c)}+a}{(a^3+a^2b+2\,(a^3+3\,a^2b+2\,ab^2)e^{(-2\,dx-2\,c)}+(a^3+a^2b)e^{(-4\,dx-4\,c)})d} - \frac{\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}(a+b)d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a + 2*b)*e^(-2*d*x - 2*c) + a)/((a^3 + a^2*b + 2*(a^3 + 3*a^2*b + 2*a*b^
2)*e^(-2*d*x - 2*c) + (a^3 + a^2*b)*e^(-4*d*x - 4*c))*d) - 1/4*log((a*e^(-
2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
+ 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a + b)*d)
```

3.87.8 Giac [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2), x)
```

3.88 $\int \frac{\text{sech}^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2} dx$

3.88.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{\text{sech}^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2} dx = \frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
1/2*sinh(d*x+c)/(a+b)/d/(a+b+a*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*a^(1/
2)/(a+b)^(1/2))/(a+b)^(3/2)/d/a^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.55

$$\int \frac{\text{sech}^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2} dx$$
$$= \frac{(a+2b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right) + a\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)\cosh(2(c+dx)) + 2\sqrt{a}\sqrt{a+b}\sinh(c+dx)}{2\sqrt{a}(a+b)^{3/2}d(a+2b+a\cosh(2(c+dx)))}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]] + a*ArcTan[(Sqrt[a]
*Sinh[c + d*x])/Sqrt[a + b]]*Cosh[2*(c + d*x)] + 2*Sqrt[a]*Sqrt[a + b]*Sin
h[c + d*x])/(2*Sqrt[a]*(a + b)^(3/2)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))
```

3.88.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4635, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^3}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{2(a+b)}+\frac{\sinh(c+dx)}{2(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}}+\frac{\sinh(c+dx)}{2(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)) + S
inh[c + d*x]/(2*(a + b)*(a + b + a*Sinh[c + d*x]^2)))/d
```

3.88.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(61) = 122$.

Time = 0.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.48

method	result
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{d(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^{2}-ab}}-1\right)}{4\sqrt{-a^{2}-ab}\,(a+b)d}+\frac{\ln\left(e^{2dx+2c}+\frac{2(a+b)e^{dx+c}}{\sqrt{-a^{2}-ab}}-1\right)}{4\sqrt{-a^{2}-ab}\,(a+b)d}$
derivativedivides	$\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a+b}+\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a+2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\frac{\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan(\ldots}{}}{a+b}}{d}$
default	$\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a+b}+\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a+2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\frac{\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan(\ldots}{}}{a+b}}{d}$

input
```
int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/(a+b)/(a*exp(4*d*x+4*c)+2*exp(2*d*x+2*c)*a
+4*b*exp(2*d*x+2*c)+a)-1/4/(-a^2-a*b)^(1/2)/(a+b)/d*ln(exp(2*d*x+2*c)-2*(a
+b)/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/4/(-a^2-a*b)^(1/2)/(a+b)/d*ln(exp(2*d
*x+2*c)+2*(a+b)/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(61) = 122$.

Time = 0.27 (sec) , antiderivative size = 1570, normalized size of antiderivative = 21.51

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - (a*cosh(d*x + c)^4 + 4*a*cosh(d
*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2
+ 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3
 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-a^2 - a*b)*log((a*cos
h(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(
3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x
+ c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) -
 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) +
a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c
)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh
(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) - 4*(a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*x + c)^2 -
 a^2 - a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 +
4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3
*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*
d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^
4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a
^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a...
```

3.88.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

3.88.7 Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(e^(3*d*x + 3*c) - e^(d*x + c))/(a^2*d + a*b*d + (a^2*d*e^(4*c) + a*b*d*e^
(4*c))*e^(4*d*x) + 2*(a^2*d*e^(2*c) + 3*a*b*d*e^(2*c) + 2*b^2*d*e^(2*c))*e
^(2*d*x)) + 8*integrate(1/8*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (
a^2*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) + 3*a*b*e^(2*c) + 2*
b^2*e^(2*c))*e^(2*d*x)), x)
```

3.88.8 Giac [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2), x)
```

3.89 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx$

3.89.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx = \frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d} - \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
1/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/d
-1/2*a*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx = 4\left(\frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{3/2}d} - \frac{a\sinh(2(c+dx))}{8b(a+b)d(a+2b+a\cosh(2(c+dx)))}\right)$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
4*(((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a
+ b)^(3/2)*d) - (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(
c + d*x)])))
```

3.89.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(ic+idx)^4}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{(a+2b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{2b(a+b)}-\frac{a\tanh(c+dx)}{2b(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}}-\frac{a\tanh(c+dx)}{2b(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a +
b)^(3/2)) - (a*Tanh[c + d*x])/(2*b*(a + b)*(a + b - b*Tanh[c + d*x]^2)))/d
```

3.89.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.89.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(71) = 142$.

Time = 0.56 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.67

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)b}+\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)b}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}-\frac{(a+2b)\left(-\frac{\ln\left(\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}\cdots\right.\right.}{4\sqrt{b}\sqrt{a+b}}\cdots\right.}{\cdots}}{d}$
default	$\frac{-\frac{2\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)b}+\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)b}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}-\frac{(a+2b)\left(-\frac{\ln\left(\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}\cdots\right.\right.}{4\sqrt{b}\sqrt{a+b}}\cdots\right.}{\cdots}}{d}$
risch	$\frac{e^{2dx+2c}a+2b\,e^{2dx+2c}+a}{db(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}-2ab-2b^2}{a\sqrt{ab+b^2}}\right)a}{4\sqrt{ab+b^2}\,(a+b)db}+\frac{\ln\left(e^{2dx+2c}+\cdots\right.}{2\cdots}$

input
```
int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(1/2*a/(a+b)/b*tanh(1/2*d*x+1/2*c)^3+1/2*a/(a+b)/b*tanh(1/2*d*x+1/
2*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-(a+2*b)/(a+b)/b*(-1/4/b^(1/2)/(a+b)^
(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(
a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2
*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 1569, normalized size of antiderivative = 18.90

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8*
(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^2
 + 2*b^3)*sinh(d*x + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*
b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2
+ 4*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^
2 + 4*a*b + 4*b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d
*x + c)^3 + (a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b +
 b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2
*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c
)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x
 + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*
b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*
b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(
d*x + c) + a)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*
b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 + 2*a^
2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^
5)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^
2 + (a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d)*sinh(d*x + c)^2 + (a^3*b...
```

3.89.6 Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(74) = 148$.

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.99

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = -\frac{(a+2b)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{4\sqrt{(a+b)b}(ab+b^2)d} - \frac{(a+2b)e^{(-2dx-2c)}+a}{(a^2b+ab^2+2(a^2b+3ab^2+2b^3)e^{(-2dx-2c)}+(a^2b+ab^2)e^{(-4dx-4c)})d}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(a + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e
^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^
2)*d) - ((a + 2*b)*e^(-2*d*x - 2*c) + a)/((a^2*b + a*b^2 + 2*(a^2*b + 3*a*
b^2 + 2*b^3)*e^(-2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d)
```

3.89.8 Giac [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^4}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^4\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2), x)
```

3.90 $\int \frac{\text{sech}^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} dx$

3.90.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \frac{\text{sech}^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} dx = \frac{\arctan(\sinh(c+dx))}{b^2d} - \frac{\sqrt{a}(2a+3b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}d} - \frac{a\sinh(c+dx)}{2b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
arctan(sinh(d*x+c))/b^2/d-1/2*a*sinh(d*x+c)/b/(a+b)/d/(a+b+a*sinh(d*x+c)^2
)-1/2*(2*a+3*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^2/(a+b)^
(3/2)/d
```

3.90.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 282 vs. 2(101) = 202.

Time = 2.77 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.79

$$\int \frac{\text{sech}^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} dx = \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^3(c+dx)\left(\sqrt{a}(2a+3b)\arctan\left(\frac{\sqrt{a+b}\text{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}(\cosh(c)+\sinh}{\sqrt{a}}\right.\right.}{}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(Sqrt[a]*(2*a + 3*b)*ArcT
an[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[
c]))/Sqrt[a]]*Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x] - (a +
 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*(-4*(a + b)^(3/2)*ArcTan[Tanh[(c
 + d*x)/2]]*Sqrt[(Cosh[c] - Sinh[c])^2] + Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt
[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqr
t[a]]*Sinh[c]) - 2*a*b*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[c + d*
x]))/(8*b^2*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)^2*Sqrt[(Cosh[c] - Sinh
[c])^2])
```

3.90.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4635, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^5}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-a\sinh^2(c+dx)+a+2b}{\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}d\sinh(c+dx)}{2b(a+b)}-\frac{a\sinh(c+dx)}{2b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2(a+b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}-\frac{a(2a+3b)\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{2b(a+b)}-\frac{a\sinh(c+dx)}{2b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{d}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{\frac{2(a+b)\arctan(\sinh(c+dx))}{b}-\frac{a(2a+3b)\int\frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{2b(a+b)}-\frac{a\sinh(c+dx)}{2b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{d}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{\frac{\frac{2(a+b)\arctan(\sinh(c+dx))}{b}-\frac{\sqrt{a}(2a+3b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{2b(a+b)}-\frac{a\sinh(c+dx)}{2b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(((2*(a + b)*ArcTan[Sinh[c + d*x]])/b - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[
a]*Sinh[c + d*x])/Sqrt[a + b]])/(b*Sqrt[a + b]))/(2*b*(a + b)) - (a*Sinh[c
 + d*x])/(2*b*(a + b)*(a + b + a*Sinh[c + d*x]^2)))/d
```

3.90.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

3.90. $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.90.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(89) = 178$.

Time = 1.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.14

method	result
derivativedivides	$-\frac{\frac{2a\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a+2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\frac{(2a+3b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cdots}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\cdots\right)}{2\cdots}\right)}{b^{2}}\cdots}{d}$
default	$-\frac{\frac{2a\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a+2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\frac{(2a+3b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cdots}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\cdots\right)}{2\cdots}\right)}{b^{2}}\cdots}{d}$
risch	$-\frac{a\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{bd(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}+\frac{i\ln\left(e^{dx+c}+i\right)}{d\,b^{2}}-\frac{i\ln\left(e^{dx+c}-i\right)}{d\,b^{2}}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2dx+2c}-2\cdots\right.}{2(a+b)^{2}\cdots}$

input
```
int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a/b^2*((-1/2*b/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/(a+b)*tanh(1/2*d*
x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+
1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2*(2*a+3*b)/(a+b)*(1/2/(a+b)^(
1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1
/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)
+2*b^(1/2))/a^(1/2))))+2/b^2*arctan(tanh(1/2*d*x+1/2*c)))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(89) = 178$.

Time = 0.31 (sec) , antiderivative size = 2069, normalized size of antiderivative = 20.49

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*
b*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - ((2*a^2 + 3*a*b)*cosh(d*x + c)^4
 + 4*(2*a^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(
d*x + c)^4 + 2*(2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a
*b)*cosh(d*x + c)^2 + 2*a^2 + 7*a*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a
*b + 4*((2*a^2 + 3*a*b)*cosh(d*x + c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x
 + c))*sinh(d*x + c))*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d
*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^
2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x +
c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)
^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (
a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*
sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c
)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x
 + c))*sinh(d*x + c) + a)) - 8*((a^2 + a*b)*cosh(d*x + c)^4 + 4*(a^2 + a*b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*x + c)^4 + 2*(a^2 + 3
*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 + a^2 + 3
*a*b + 2*b^2)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)^3
 + (a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x ...
```

3.90.6 Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)
```

3.90.7 Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-(a*e^(3*d*x + 3*c) - a*e^(d*x + c))/(a^2*b*d + a*b^2*d + (a^2*b*d*e^(4*c)
 + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c) + 3*a*b^2*d*e^(2*c) + 2
*b^3*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integrate(
1/32*((2*a^2*e^(3*c) + 3*a*b*e^(3*c))*e^(3*d*x) + (2*a^2*e^c + 3*a*b*e^c)*
e^(d*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*e^(4*d*x) +
2*(a^2*b^2*e^(2*c) + 3*a*b^3*e^(2*c) + 2*b^4*e^(2*c))*e^(2*d*x)), x)
```

3.90.8 Giac [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^5\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2), x)
```

3.91 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} \, dx$

3.91.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} \, dx = -\frac{a(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d} + \frac{\tanh(c+dx)}{b^2d} + \frac{a^2\tanh(c+dx)}{2b^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
-1/2*a*(3*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3
/2)/d+tanh(d*x+c)/b^2/d+1/2*a^2*tanh(d*x+c)/b^2/(a+b)/d/(a+b-b*tanh(d*x+c)
^2)
```

3.91.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(101) = 202.

Time = 5.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.27

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} \, dx = \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^4(c+dx)\left(-\frac{a(3a+4b)\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)}{(a+b)^{3/2}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(-((a*(3*a + 4*b)*ArcTanh
[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*
x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c
 + d*x)])*(Cosh[2*c] - Sinh[2*c]))/((a + b)^(3/2)*Sqrt[b*(Cosh[c] - Sinh[c
])^4])) + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]*Sinh[d*x
] + (a*(a*Sech[2*c]*Sinh[2*d*x] - (a + 2*b)*Tanh[2*c]))/(a + b)))/(8*b^2*d
*(a + b*Sech[c + d*x]^2)^2)
```

3.91.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^6}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{b^2}-\frac{a(a+2b)-2ab\tanh^2(c+dx)}{b^2\left(-b\tanh^2(c+dx)+a+b\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\tanh(c+dx)}{2b^2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}}+\frac{\tanh(c+dx)}{b^2}}{d}$$

input
```
Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(-1/2*(a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(5/2
)*(a + b)^(3/2)) + Tanh[c + d*x]/b^2 + (a^2*Tanh[c + d*x])/(2*b^2*(a + b)*
(a + b - b*Tanh[c + d*x]^2)))/d
```

3.91.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.91.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(89) = 178$.

Time = 1.92 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.48

method	result
derivativedivides	$\frac{\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}+\frac{2a\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{2a+2b}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{2a+2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\frac{(3a+4b)\left(-\frac{\ln\left(\sqrt{a-\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{d}$
default	$\frac{\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}+\frac{2a\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{2a+2b}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{2a+2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b}+\frac{(3a+4b)\left(-\frac{\ln\left(\sqrt{a-\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{d}$
risch	$-\frac{3a^{2}e^{4dx+4c}+4ab\,e^{4dx+4c}+6a^{2}e^{2dx+2c}+14ab\,e^{2dx+2c}+8\,e^{2dx+2c}b^{2}+3a^{2}+2ab}{b^{2}d\left(e^{2dx+2c}+1\right)(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}+\frac{3a^{2}\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^{2}}+2b\sqrt{ab-\cdots}}{a\sqrt{ab+\cdots}}\right.}{4\sqrt{ab+b^{2}}\,(a+b)d\,b^{2}\cdots}$

input
```
int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)+2*a/b^2*((1/2*a/(
a+b)*tanh(1/2*d*x+1/2*c)^3+1/2*a/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+
1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)+1/2*(3*a+4*b)/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/
b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/
2*c)*b^(1/2)+(a+b)^(1/2)))))
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. $2(92) = 184$.

Time = 0.31 (sec) , antiderivative size = 2958, normalized size of antiderivative = 29.29

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 7
*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 7*a^2*b^2
 + 4*a*b^3)*sinh(d*x + c)^4 + 12*a^3*b + 20*a^2*b^2 + 8*a*b^3 + 8*(3*a^3*b
 + 10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d*x + c)^2 + 8*(3*a^3*b + 10*a^2*b^
2 + 11*a*b^3 + 4*b^4 + 3*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 - ((3*a^3 + 4*a^2*b)*cosh(d*x + c)^6 + 6*(3*a^3 + 4*a^2*b)
*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 4*a^2*b)*sinh(d*x + c)^6 + (9*a^
3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^4 + (9*a^3 + 24*a^2*b + 16*a*b^2 +
15*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 4*a^
2*b)*cosh(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c))*sinh(d
*x + c)^3 + 3*a^3 + 4*a^2*b + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^
2 + (15*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^4 + 9*a^3 + 24*a^2*b + 16*a*b^2 +
6*(9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3
*a^3 + 4*a^2*b)*cosh(d*x + c)^5 + 2*(9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x
 + c)^3 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt
(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3
 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d
*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*co
sh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x
 + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b...
```

3.91.6 Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2)**2, x)
```

3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(92) = 184.

Time = 0.36 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.42

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \frac{(3a+4b)a\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{4(ab^2+b^3)\sqrt{(a+b)b}d} + \frac{3a^2+2ab+2(3a^2+7ab+4b^2)e^{(-2dx-2c)}+(3a^2+4ab)e^{(-4dx-4c)}}{(a^2b^2+ab^3+(3a^2b^2+7ab^3+4b^4)e^{(-2dx-2c)}+(3a^2b^2+7ab^3+4b^4)e^{(-4dx-4c)}+(a^2b^2+ab^3)e^{(-6dx-}}$$

input
```
integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(3*a + 4*b)*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(
a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a*b^2 + b^3)*sqrt((a
+ b)*b)*d) + (3*a^2 + 2*a*b + 2*(3*a^2 + 7*a*b + 4*b^2)*e^(-2*d*x - 2*c) +
 (3*a^2 + 4*a*b)*e^(-4*d*x - 4*c))/((a^2*b^2 + a*b^3 + (3*a^2*b^2 + 7*a*b^
3 + 4*b^4)*e^(-2*d*x - 2*c) + (3*a^2*b^2 + 7*a*b^3 + 4*b^4)*e^(-4*d*x - 4*
c) + (a^2*b^2 + a*b^3)*e^(-6*d*x - 6*c))*d)
```

3.91.8 Giac [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^6}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^6\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2), x)
```

3.92 $\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} dx$

3.92.1 Optimal result

Integrand size = 23, antiderivative size = 153

$$\begin{aligned}\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} dx = & -\frac{(4a-b)\arctan(\sinh(c+dx))}{2b^3d} \\ & + \frac{a^{3/2}(4a+5b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3(a+b)^{3/2}d} \\ & + \frac{a(2a+b)\sinh(c+dx)}{2b^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} \\ & + \frac{\text{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)}\end{aligned}$$

output
```
-1/2*(4*a-b)*arctan(sinh(d*x+c))/b^3/d+1/2*a^(3/2)*(4*a+5*b)*arctan(sinh(d
*x+c)*a^(1/2)/(a+b)^(1/2))/b^3/(a+b)^(3/2)/d+1/2*a*(2*a+b)*sinh(d*x+c)/b^2
/(a+b)/d/(a+b+a*sinh(d*x+c)^2)+1/2*sech(d*x+c)*tanh(d*x+c)/b/d/(a+b+a*sinh
(d*x+c)^2)
```

3.92.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 489 vs. $2(153) = 306$.

Time = 5.71 (sec) , antiderivative size = 489, normalized size of antiderivative = 3.20

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$
$$= \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(c)\operatorname{sech}^3(c+dx)\left(-a^{3/2}(4a+5b)\arctan\left(\frac{\sqrt{a+b}\operatorname{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}(}{\sqrt{a}}\right.\right.}{}$$

input
```
Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]^3*(-(a^(3/2)*(4*a +
 5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[
c] + Sinh[c]))/Sqrt[a]]*Cosh[c]^2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c +
 d*x]) + b*(a + b)^(3/2)*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*S
qrt[(Cosh[c] - Sinh[c])^2]*Sinh[c] - Cosh[c]*Sech[c + d*x]*(2*Sqrt[a + b]*
(4*a^2 + 3*a*b - b^2)*ArcTan[Tanh[(c + d*x)/2]]*(a + 2*b + a*Cosh[2*(c + d
*x)])*Sqrt[(Cosh[c] - Sinh[c])^2] - a^(5/2)*b*ArcTan[(Sqrt[a + b]*Csch[c +
 d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(13 + 5*Co
sh[2*(c + d*x)])*Sinh[c]) + a^(3/2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt
[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(2*a^2 + 5*b^2 + 2*a
^2*Cosh[2*(c + d*x)])*Sech[c + d*x]*Sinh[2*c] + b*(a + b)^(3/2)*(a + 2*b +
 a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*Sqrt[(Cosh[c] - Sinh[c])^2]*Sinh[d*x
] + 2*a^2*b*Sqrt[a + b]*Cosh[c]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[c + d*x])
)/(8*b^3*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)^2*Sqrt[(Cosh[c] - Sinh[c]
)^2])
```

3.92.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4635, 316, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\sec(ic+idx)^7}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$\Big\downarrow$ 4635

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{d}$$

$\Big\downarrow$ 316

$$\frac{\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{\int \frac{-3a\sinh^2(c+dx)+a-b}{\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{2b}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{\frac{\int \frac{2\left(2a^2-(2a+b)\sinh^2(c+dx)a+2ba-b^2\right)}{\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}d\sinh(c+dx)}{2b(a+b)}-\frac{a(2a+b)\sinh(c+dx)}{b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{2b}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{\frac{\int \frac{2a^2-(2a+b)\sinh^2(c+dx)a+2ba-b^2}{\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}d\sinh(c+dx)}{b(a+b)}-\frac{a(2a+b)\sinh(c+dx)}{b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{2b}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{\frac{\frac{(4a-b)(a+b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}-\frac{a^2(4a+5b)\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{b(a+b)}-\frac{a(2a+b)\sinh(c+dx)}{b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{2b}}{d}$$

$\Big\downarrow$ 216

$$\frac{\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{\frac{\frac{(4a-b)(a+b)\arctan(\sinh(c+dx))}{b}-\frac{a^2(4a+5b)\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{b(a+b)}-\frac{a(2a+b)\sinh(c+dx)}{b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{2b}}{d}$$

$\Big\downarrow$ 218

3.92. $\displaystyle\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$

$$\frac{\frac{\sinh(c+dx)}{2b(\sinh^2(c+dx)+1)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{\frac{\frac{(4a-b)(a+b)\arctan(\sinh(c+dx))}{b}-\frac{a^{3/2}(4a+5b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{b(a+b)}-\frac{a(2a+b)\sinh(c+dx)}{b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{2b}}{d}$$

input
```
Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(Sinh[c + d*x]/(2*b*(1 + Sinh[c + d*x]^2)*(a + b + a*Sinh[c + d*x]^2)) - (
(((4*a - b)*(a + b)*ArcTan[Sinh[c + d*x]])/b - (a^(3/2)*(4*a + 5*b)*ArcTan
[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(b*Sqrt[a + b]))/(b*(a + b)) - (a*(
2*a + b)*Sinh[c + d*x])/(b*(a + b)*(a + b + a*Sinh[c + d*x]^2)))/(2*b))/d
```

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.92.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b}{2}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)^{2}}+\frac{(4a-b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^{3}}+\frac{2a^{2}\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a+b)}+\frac{b\tanh \cdots}{2\cdots}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\cdots\right.}\cdots\right.}{\cdots}}{d}$
default	$\frac{-\frac{2\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b}{2}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)^{2}}+\frac{(4a-b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^{3}}+\frac{2a^{2}\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2(a+b)}+\frac{b\tanh \cdots}{2\cdots}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\cdots\right.}\cdots\right.}{\cdots}}{d}$
risch	$\frac{e^{dx+c}\left(2a^{2}e^{6dx+6c}+ab\,e^{6dx+6c}+2a^{2}e^{4dx+4c}+5ab\,e^{4dx+4c}+4\,e^{4dx+4c}b^{2}-2a^{2}e^{2dx+2c}-5ab\,e^{2dx+2c}-4\,e^{2dx+2c}b^{2}-2a^{2}-\cdots\right.}{b^{2}d(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)\left(e^{2dx+2c}+1\right)^{2}}$

input
```
int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^3*((1/2*tanh(1/2*d*x+1/2*c)^3*b-1/2*b*tanh(1/2*d*x+1/2*c))/(tanh
(1/2*d*x+1/2*c)^2+1)^2+1/2*(4*a-b)*arctan(tanh(1/2*d*x+1/2*c)))+2/b^3*a^2*
((-1/2*b/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/(a+b)*tanh(1/2*d*x+1/2*c))/(tan
h(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)+1/2*(5*b+4*a)/(a+b)*(1/2/(a+b)^(1/2)/a^(1/2)*a
rctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/(a+b)
^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^
(1/2)))))
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3476 vs. $2(137) = 274$.

Time = 0.35 (sec) , antiderivative size = 6499, normalized size of antiderivative = 42.48

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.92.6 Sympy [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

input
```
integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**7/(a + b*sech(c + d*x)**2)**2, x)
```

3.92.7 Maxima [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^7}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((2*a^2*e^(7*c) + a*b*e^(7*c))*e^(7*d*x) + (2*a^2*e^(5*c) + 5*a*b*e^(5*c)
+ 4*b^2*e^(5*c))*e^(5*d*x) - (2*a^2*e^(3*c) + 5*a*b*e^(3*c) + 4*b^2*e^(3*c
))*e^(3*d*x) - (2*a^2*e^c + a*b*e^c)*e^(d*x))/(a^2*b^2*d + a*b^3*d + (a^2*
b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x) + 4*(a^2*b^2*d*e^(6*c) + 2*a*b^
3*d*e^(6*c) + b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^2*b^2*d*e^(4*c) + 7*a*b^3*
d*e^(4*c) + 4*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^2*b^2*d*e^(2*c) + 2*a*b^3*d*
e^(2*c) + b^4*d*e^(2*c))*e^(2*d*x)) - (4*a*e^c - b*e^c)*arctan(e^(d*x + c)
)*e^(-c)/(b^3*d) + 128*integrate(1/128*((4*a^3*e^(3*c) + 5*a^2*b*e^(3*c))*
e^(3*d*x) + (4*a^3*e^c + 5*a^2*b*e^c)*e^(d*x))/(a^2*b^3 + a*b^4 + (a^2*b^3
*e^(4*c) + a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^2*b^3*e^(2*c) + 3*a*b^4*e^(2*c)
 + 2*b^5*e^(2*c))*e^(2*d*x)), x)
```

3.92.8 Giac [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^7}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{1}{\cosh(c+d\,x)^7\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2), x)
```

3.93 $\int \frac{\cosh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.93.1 Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}(35a^2+56ab+24b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(4a+3b)(a+4b)\tanh(c+dx)}{8a^3(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
1/2*(a-6*b)*x/a^4+1/8*b^(3/2)*(35*a^2+56*a*b+24*b^2)*arctanh(b^(1/2)*tanh(
d*x+c)/(a+b)^(1/2))/a^4/(a+b)^(5/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b
-b*tanh(d*x+c)^2)^2+1/4*b*(2*a+3*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*
x+c)^2)^2+1/8*b*(4*a+3*b)*(a+4*b)*tanh(d*x+c)/a^3/(a+b)^2/d/(a+b-b*tanh(d*
x+c)^2)
```

3.93.2 Mathematica [A] (verified)

Time = 4.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.76

$$\int \frac{\cosh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx$$
$$= \frac{4(a-6b)(c+dx)+\frac{b^{3/2}(35a^2+56ab+24b^2)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+a\left(2+\frac{13ab^2}{(a+b)^2(a+2b+a\cosh(2(c+dx)))}+\frac{2b^3(3a+8b+}{(a+b)^2(a+2b}\right.}{8a^4d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(4*(a - 6*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b
)^2*(a + 2*b + a*Cosh[2*(c + d*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cosh[2*(c +
 d*x)]))/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2))*Sinh[2*(c + d*x)])
/(8*a^4*d)
```

3.93.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4634, 316, 402, 27, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(1-\tanh^2(c+dx))^2(-b\tanh^2(c+dx)+a+b)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-5b\tanh^2(c+dx)+a-b}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^3}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))(a-b\tanh^2(c+dx)+b)^2}}{d}$$

$\downarrow$ 402

$$\frac{\frac{\frac{b(2a+3b)\tanh(c+dx)}{2a(a+b)(a-b\tanh^2(c+dx)+b)^2}-\frac{\int -\frac{2(2a^2-4ba-3b^2-3b(2a+3b)\tanh^2(c+dx))}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^2}d\tanh(c+dx)}{4a(a+b)}}{2a}+\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))(a-b\tanh^2(c+dx)+b)^2}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{2a^2-4ba-3b^2-3b(2a+3b)\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^2}d\tanh(c+dx)}{2a(a+b)}+\frac{b(2a+3b)\tanh(c+dx)}{2a(a+b)(a-b\tanh^2(c+dx)+b)^2}}{2a}+\frac{\tanh(c+dx)}{2a(1-\tanh^2(c+dx))(a-b\tanh^2(c+dx)+b)^2}}{d}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{2a(a+b)(a-b\tanh^2(c+dx)+b)}-\frac{\int -\frac{4a^3-12ba^2-25b^2a-12b^3-b(4a+3b)(a+4b)\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)}d\tanh(c+dx)}{2a(a+b)}}{2a(a+b)}+\frac{b(2a+3b)\tanh(c+dx)}{2a(a+b)(a-b\tanh^2(c+dx)+b)^2}}{2a}+\frac{}{2a(1-\tanh^2(}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\frac{\int \frac{4a^3-12ba^2-25b^2a-12b^3-b(4a+3b)(a+4b)\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)}d\tanh(c+dx)}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{2a(a+b)(a-b\tanh^2(c+dx)+b)}}{2a(a+b)}+\frac{b(2a+3b)\tanh(c+dx)}{2a(a+b)(a-b\tanh^2(c+dx)+b)^2}}{2a}+\frac{}{2a(1-\tanh^2(c}}{d}$$

$\downarrow$ 397

$$\frac{\frac{\frac{\frac{\frac{b^2(35a^2+56ab+24b^2)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}+\frac{4(a-6b)(a+b)^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{2a(a+b)(a-b\tanh^2(c+dx)+b)}}{2a(a+b)}+\frac{b(2a+3b)}{2a(a+b)(a-b\,t}}{2a}}{d}$$

$\downarrow$ 219

3.93. $\int \frac{\cosh^2(c+dx)}{(a+b\,\text{sech}^2(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\frac{b^2(35a^2+56ab+24b^2)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}+\frac{4(a-6b)(a+b)^2\text{arctanh}(\tanh(c+dx))}{a}}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{2a(a+b)}+\frac{b(2a+3b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+\right.}}{2a}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{\frac{b^{3/2}(35a^2+56ab+24b^2)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{4(a-6b)(a+b)^2\text{arctanh}(\tanh(c+dx))}{a}}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{2a(a+b)}+\frac{b(2a+3b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+\right.}}{2a}}{d}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(2*a*(1 - Tanh[c + d*x]^2)*(a + b - b*Tanh[c + d*x]^2)^2) +
 ((b*(2*a + 3*b)*Tanh[c + d*x])/(2*a*(a + b)*(a + b - b*Tanh[c + d*x]^2)^2
) + (((4*(a - 6*b)*(a + b)^2*ArcTanh[Tanh[c + d*x]])/a + (b^(3/2)*(35*a^2
+ 56*a*b + 24*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a
 + b]))/(2*a*(a + b)) + (b*(4*a + 3*b)*(a + 4*b)*Tanh[c + d*x])/(2*a*(a +
b)*(a + b - b*Tanh[c + d*x]^2)))/(2*a*(a + b)))/(2*a))/d
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.93. $\int \frac{\cosh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.93.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(186) = 372$.

Time = 3.59 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.15

method	result
derivativedivides	$-\frac{1}{2a^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2a^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a-6b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^4}-\frac{2b^2\left(\frac{-\frac{a(13a+8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)}-\frac{a\left(39a^2+\cdots\right.}{\cdots}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cdots\right.}\cdots\right.}{\cdots}$
default	$-\frac{1}{2a^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2a^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a-6b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^4}-\frac{2b^2\left(\frac{-\frac{a(13a+8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)}-\frac{a\left(39a^2+\cdots\right.}{\cdots}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cdots\right.}\cdots\right.}{\cdots}$
risch	$\frac{x}{2a^3}-\frac{3xb}{a^4}+\frac{e^{2dx+2c}}{8a^3d}-\frac{e^{-2dx-2c}}{8a^3d}-\frac{b^2\left(13a^3e^{6dx+6c}+40a^2b\,e^{6dx+6c}+24a\,b^2e^{6dx+6c}+39a^3e^{4dx+4c}+134a^2b\,e^{4dx}\cdots\right.}{4a^4(a+b)^2d(a\cdots}$

input
```
int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a^3/(1+tanh(1/2*d*x+1/2*c))^2+1/2/a^3/(1+tanh(1/2*d*x+1/2*c))+1/
2*(a-6*b)/a^4*ln(1+tanh(1/2*d*x+1/2*c))-2*b^2/a^4*((-1/8*a*(13*a+8*b)/(a+b
)*tanh(1/2*d*x+1/2*c)^7-1/8*a*(39*a^2+19*a*b-8*b^2)/(a+b)^2*tanh(1/2*d*x+1
/2*c)^5-1/8*a*(39*a^2+19*a*b-8*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-1/8*a*(1
3*a+8*b)/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+
1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/8*
(35*a^2+56*a*b+24*b^2)/(a^2+2*a*b+b^2)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^
(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4
/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1
/2*c)*b^(1/2)+(a+b)^(1/2))))+1/2/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/a^3/(ta
nh(1/2*d*x+1/2*c)-1)+1/2/a^4*(-a+6*b)*ln(tanh(1/2*d*x+1/2*c)-1))
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5729 vs. $2(195) = 390$.

Time = 0.40 (sec) , antiderivative size = 11740, normalized size of antiderivative = 57.55

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.93.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(195) = 390$.

Time = 0.37 (sec) , antiderivative size = 1373, normalized size of antiderivative = 6.73

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
3/64*(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a
 + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*
b)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) - 3/64*(5*a^3*b + 30*a^
2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a +
 b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*
b + a^4*b^2)*sqrt((a + b)*b)*d) + 1/32*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((
a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
+ 2*b + 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d)
- 1/16*(9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a
^2*b^3 + 32*a*b^4)*e^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3
 + 832*a*b^4 + 320*b^5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^
2*b^3 + 160*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a
^7*b + a^6*b^2)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3
)*e^(6*d*x + 6*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*
b^4)*e^(4*d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x
+ 2*c))*d) + 1/16*(9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3
*b^2 + 336*a^2*b^3 + 160*a*b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2
 + 740*a^2*b^3 + 832*a*b^4 + 320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a
^3*b^2 + 64*a^2*b^3 + 32*a*b^4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^
2 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a...
```

3.93.8 Giac [F]

$$\int \frac{\cosh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\cosh(dx+c)^2}{\left(b\,\text{sech}\,(dx+c)^2+a\right)^3}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^2}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^3}\,dx$$

input
```
int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3, x)
```

3.94 $\int \frac{\cosh(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

3.94.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\cosh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = -\frac{3b(4(a+b)^2+(2a+b)^2)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}d} + \frac{\sinh(c+dx)}{a^3d} - \frac{b^3\sinh(c+dx)}{4a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3b^2(4a+3b)\sinh(c+dx)}{8a^3(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
-3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(7/
2)/(a+b)^(5/2)/d+sinh(d*x+c)/a^3/d-1/4*b^3*sinh(d*x+c)/a^3/(a+b)/d/(a+b+a*
sinh(d*x+c)^2)^2+3/8*b^2*(4*a+3*b)*sinh(d*x+c)/a^3/(a+b)^2/d/(a+b+a*sinh(d
*x+c)^2)
```

3.94.2 Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{\cosh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{-\frac{3b(8a^2+12ab+5b^2)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\sqrt{a}\sinh(c+dx)\left(8+\frac{12b^2}{(a+b)(a+b+a\sinh^2(c+dx))}-\frac{b^3(5(a+b)+3a\sinh^2(c+dx))}{(a+b)^2(a+b+a\sinh^2(c+dx))^2}\right)}{8a^{7/2}d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]
])/(a + b)^(5/2) + Sqrt[a]*Sinh[c + d*x]*(8 + (12*b^2)/((a + b)*(a + b + a
*Sinh[c + d*x]^2)) - (b^3*(5*(a + b) + 3*a*Sinh[c + d*x]^2))/((a + b)^2*(a
 + b + a*Sinh[c + d*x]^2)^2)))/(8*a^(7/2)*d)
```

3.94.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^3}{\left(a\sinh^2(c+dx)+a+b\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{a^3}-\frac{3a^2b\sinh^4(c+dx)+3ab(2a+b)\sinh^2(c+dx)+b(3a^2+3ba+b^2)}{a^3\left(a\sinh^2(c+dx)+a+b\right)^3}\right)d\sinh(c+dx)}{d}$$

$\downarrow$ 2009

$$\frac{-\frac{3b(4(a+b)^2+(2a+b)^2)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}}-\frac{b^3\sinh(c+dx)}{4a^3(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sinh(c+dx)}{8a^3(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{a^3}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a +
 b]])/(8*a^(7/2)*(a + b)^(5/2)) + Sinh[c + d*x]/a^3 - (b^3*Sinh[c + d*x])/
(4*a^3*(a + b)*(a + b + a*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sinh[c
+ d*x])/(8*a^3*(a + b)^2*(a + b + a*Sinh[c + d*x]^2)))/d
```

3.94.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.94.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(140) = 280$.

Time = 2.29 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.18

method	result
derivativedivides	$-\frac{2b\left(\frac{\frac{b(12a+7b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a+8b}+\frac{3b(4a^2-7ab-7b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2}-\frac{3b(4a^2-7ab-7b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a+b)^2}-\frac{b(12a+7b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}\right.}{a^3}$
default	$-\frac{2b\left(\frac{\frac{b(12a+7b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a+8b}+\frac{3b(4a^2-7ab-7b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2}-\frac{3b(4a^2-7ab-7b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a+b)^2}-\frac{b(12a+7b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}\right.}{a^3}$
risch	$\frac{e^{dx+c}}{2a^3d}-\frac{e^{-dx-c}}{2a^3d}+\frac{b^2e^{dx+c}(12a^2e^{6dx+6c}+9ab\,e^{6dx+6c}+12a^2e^{4dx+4c}+49ab\,e^{4dx+4c}+28\,e^{4dx+4c}b^2-12a^2e^{2dx+2c}-}{4a^3d(a+b)^2(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a)^2}$

input
```
int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^3*b*((1/8*b*(12*a+7*b)/(a+b)*tanh(1/2*d*x+1/2*c)^7+3/8*b*(4*a^2-
7*a*b-7*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/8*b*(4*a^2-7*a*b-7*b^2)/(a+b)
^2*tanh(1/2*d*x+1/2*c)^3-1/8*b*(12*a+7*b)/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh
(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2+3/8*(8*a^2+12*a*b+5*b^2)/(a^2+2*a*b+b^2)*(1/2
/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/
2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*
x+1/2*c)+2*b^(1/2))/a^(1/2))))-1/a^3/(tanh(1/2*d*x+1/2*c)-1)-1/a^3/(1+tanh
(1/2*d*x+1/2*c)))
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5414 vs. $2(140) = 280$.

Time = 0.37 (sec) , antiderivative size = 9856, normalized size of antiderivative = 64.00

$$\int \frac{\cosh(c + dx)}{(a + b\text{sech}^2(c + dx))^3} \, dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c + dx)}{(a + b\text{sech}^2(c + dx))^3} \, dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.94.7 Maxima [F]

$$\int \frac{\cosh(c + dx)}{(a + b\text{sech}^2(c + dx))^3} \, dx = \int \frac{\cosh(dx + c)}{(b\,\text{sech}(dx + c)^2 + a)^3} \, dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - 2*(a^4*e^(10*c) + 2*a^3*b*e^(10*c) + a
^2*b^2*e^(10*c))*e^(10*d*x) - (6*a^4*e^(8*c) + 28*a^3*b*e^(8*c) + 50*a^2*b
^2*e^(8*c) + 25*a*b^3*e^(8*c))*e^(8*d*x) - (4*a^4*e^(6*c) + 24*a^3*b*e^(6*
c) + 80*a^2*b^2*e^(6*c) + 129*a*b^3*e^(6*c) + 60*b^4*e^(6*c))*e^(6*d*x) +
(4*a^4*e^(4*c) + 24*a^3*b*e^(4*c) + 80*a^2*b^2*e^(4*c) + 129*a*b^3*e^(4*c)
 + 60*b^4*e^(4*c))*e^(4*d*x) + (6*a^4*e^(2*c) + 28*a^3*b*e^(2*c) + 50*a^2*
b^2*e^(2*c) + 25*a*b^3*e^(2*c))*e^(2*d*x))/((a^7*d*e^(9*c) + 2*a^6*b*d*e^(
9*c) + a^5*b^2*d*e^(9*c))*e^(9*d*x) + 4*(a^7*d*e^(7*c) + 4*a^6*b*d*e^(7*c)
 + 5*a^5*b^2*d*e^(7*c) + 2*a^4*b^3*d*e^(7*c))*e^(7*d*x) + 2*(3*a^7*d*e^(5*
c) + 14*a^6*b*d*e^(5*c) + 27*a^5*b^2*d*e^(5*c) + 24*a^4*b^3*d*e^(5*c) + 8*
a^3*b^4*d*e^(5*c))*e^(5*d*x) + 4*(a^7*d*e^(3*c) + 4*a^6*b*d*e^(3*c) + 5*a^
5*b^2*d*e^(3*c) + 2*a^4*b^3*d*e^(3*c))*e^(3*d*x) + (a^7*d*e^c + 2*a^6*b*d*
e^c + a^5*b^2*d*e^c)*e^(d*x)) - 1/2*integrate(3/2*((8*a^2*b*e^(3*c) + 12*a
*b^2*e^(3*c) + 5*b^3*e^(3*c))*e^(3*d*x) + (8*a^2*b*e^c + 12*a*b^2*e^c + 5*
b^3*e^c)*e^(d*x))/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^(4*c) + 2*a^5*b*e^(4*c
) + a^4*b^2*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + 4*a^5*b*e^(2*c) + 5*a^4*
b^2*e^(2*c) + 2*a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

3.94.8 Giac [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\cosh\left(dx+c\right)}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\cosh(c+d\,x)}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^3}\,dx$$

input
```
int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^3, x)
```

3.95 $\int \frac{\text{sech}(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.95.1 Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}d} - \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a
+b)^(5/2)/d-1/4*b*cosh(d*x+c)^2*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2
)^2-3/8*b*(2*a+b)*sinh(d*x+c)/a^2/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)
```

3.95.2 Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.51

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$$
$$= \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^5(c+dx)\left(\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\,\text{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)(a+}{\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}}\right.}{64a^{5/}}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*(((8*a^2 + 8*a*b + 3*b^2)
*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] +
Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(-Cosh[
c] + Sinh[c]))/(Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]) + 8*Sqrt[a]*b^2*(
a + b)*Tanh[c + d*x] - 2*Sqrt[a]*b*(8*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*
x)])*Tanh[c + d*x]))/(64*a^(5/2)*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

3.95.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4635, 315, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{\left(a\sinh^2(c+dx)+a+b\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{\int \frac{(4a+3b)\sinh^2(c+dx)+4a+b}{\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{4a(a+b)}-\frac{b\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4a(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{\frac{\left(8a^2+8ab+3b^2\right)\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{2a(a+b)}-\frac{3b(2a+b)\sinh(c+dx)}{2a(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{4a(a+b)}-\frac{b\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4a(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{3b(2a+b)\sinh(c+dx)}{2a(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{4a(a+b)}-\frac{b\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4a(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/4*(b*Sinh[c + d*x]*(1 + Sinh[c + d*x]^2))/(a*(a + b)*(a + b + a*Sinh[c
 + d*x]^2)^2) + (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/S
qrt[a + b]])/(2*a^(3/2)*(a + b)^(3/2)) - (3*b*(2*a + b)*Sinh[c + d*x])/(2*
a*(a + b)*(a + b + a*Sinh[c + d*x]^2)))/(4*a*(a + b)))/d
```

3.95.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(128) = 256$.

Time = 1.27 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.17

method	result
derivativedivides	$\frac{\frac{\frac{b(8a+3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a^{2}(a+b)}+\frac{b\left(8a^{2}-13ab-9b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4(a+b)^{2}a^{2}}-\frac{b\left(8a^{2}-13ab-9b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4(a+b)^{2}a^{2}}-\frac{b(8a+3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}+\frac{\left(8a^{2}\right.}{}}{d}$
default	$\frac{\frac{\frac{b(8a+3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a^{2}(a+b)}+\frac{b\left(8a^{2}-13ab-9b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4(a+b)^{2}a^{2}}-\frac{b\left(8a^{2}-13ab-9b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4(a+b)^{2}a^{2}}-\frac{b(8a+3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^{2}(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}+\frac{\left(8a^{2}\right.}{}}{d}$
risch	$-\frac{e^{dx+c}b\left(8a^{2}e^{6dx+6c}+5ab\,e^{6dx+6c}+8a^{2}e^{4dx+4c}+29ab\,e^{4dx+4c}+12\,e^{4dx+4c}b^{2}-8a^{2}e^{2dx+2c}-29ab\,e^{2dx+2c}-12\,e^{2dx+2c}\right.}{4a^{2}d(a+b)^{2}\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^{2}}$

input
```
int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(1/8*b*(8*a+3*b)/a^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+1/8*b*(8*a^2-13*a*
b-9*b^2)/(a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^5-1/8*b*(8*a^2-13*a*b-9*b^2)/(a+b
)^2/a^2*tanh(1/2*d*x+1/2*c)^3-1/8*b*(8*a+3*b)/a^2/(a+b)*tanh(1/2*d*x+1/2*c
))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/4*(8*a^2+8*a*b+3*b^2)/a^2/(a^2+2*a*
b+b^2)*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2
*c)-2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))))
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3753 vs. $2(128) = 256$.

Time = 0.35 (sec) , antiderivative size = 6806, normalized size of antiderivative = 47.93

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.95.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

3.95.7 Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(b\operatorname{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((8*a^2*b*e^(7*c) + 5*a*b^2*e^(7*c))*e^(7*d*x) + (8*a^2*b*e^(5*c) + 2
9*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) - (8*a^2*b*e^(3*c) + 29*a*b^2*
e^(3*c) + 12*b^3*e^(3*c))*e^(3*d*x) - (8*a^2*b*e^c + 5*a*b^2*e^c)*e^(d*x))
/(a^6*d + 2*a^5*b*d + a^4*b^2*d + (a^6*d*e^(8*c) + 2*a^5*b*d*e^(8*c) + a^4
*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 4*a^5*b*d*e^(6*c) + 5*a^4*b
^2*d*e^(6*c) + 2*a^3*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d*e^(4*c) + 14*a^
5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) + 24*a^3*b^3*d*e^(4*c) + 8*a^2*b^4*d*
e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 4*a^5*b*d*e^(2*c) + 5*a^4*b^2*d*e^
(2*c) + 2*a^3*b^3*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/8*((8*a^2*e^(3*c)
+ 8*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (8*a^2*e^c + 8*a*b*e^c + 3*b^
2*e^c)*e^(d*x))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5*e^(4*c) + 2*a^4*b*e^(4*c)
+ a^3*b^2*e^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 4*a^4*b*e^(2*c) + 5*a^3*b^
2*e^(2*c) + 2*a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

3.95.8 Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)\,\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3), x)
```

3.96 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3} dx$

3.96.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3} dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}(a+b)^{5/2}d} + \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
3/8*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/(a+b)^(5/2)/d/b^(1/2)+1/4*tan
h(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/(a+b)^2/d/(a+b-b*
tanh(d*x+c)^2)
```

3.96.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 258 vs. 2(108) = 216.

Time = 3.96 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.39

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^6(c+dx)\left(\frac{3\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{64(}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((3*ArcTanh[(Sech[d*x]*(C
osh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(C
osh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*b*
(a + b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a^2 - ((a + 2*b +
 a*Cosh[2*(c + d*x)])*Sech[2*c]*((5*a^2 + 16*a*b + 8*b^2)*Sinh[2*c] - a*(5
*a + 2*b)*Sinh[2*d*x]))/a^2))/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

3.96.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4634, 215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^2}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\int \frac{1}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}}+\frac{\tanh(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(4*(a + b)*(a + b - b*Tanh[c + d*x]^2)^2) + (3*(ArcTanh[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)) + Tanh[c + d*x
]/(2*(a + b)*(a + b - b*Tanh[c + d*x]^2))))/(4*(a + b)))/d
```

3.96.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.96. $\int \frac{\text{sech}^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(94) = 188$.

Time = 1.22 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.41

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8(a+b)}-\frac{3(5a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8(a+b)^{2}}-\frac{3(5a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8(a+b)^{2}}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}-\frac{3\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\right.}{4\sqrt{b}\,\sqrt{}}\right.}{}}{d}$
default	$\frac{-\frac{2\left(-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8(a+b)}-\frac{3(5a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8(a+b)^{2}}-\frac{3(5a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8(a+b)^{2}}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}-\frac{3\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\right.}{4\sqrt{b}\,\sqrt{}}\right.}{}}{d}$
risch	$-\frac{5a^{3}e^{6dx+6c}+16a^{2}b\,e^{6dx+6c}+8a\,b^{2}e^{6dx+6c}+15a^{3}e^{4dx+4c}+46a^{2}b\,e^{4dx+4c}+56a\,b^{2}e^{4dx+4c}+16\,e^{4dx+4c}b^{3}+15a^{3}e^{2dx+2}}{4a^{2}(a+b)^{2}d\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^{2}}$

input
```
int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-5/8/(a+b)*tanh(1/2*d*x+1/2*c)^7-3/8*(5*a+b)/(a+b)^2*tanh(1/2*d*x
+1/2*c)^5-3/8*(5*a+b)/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-5/8/(a+b)*tanh(1/2*d*x
+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1
/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-3/4/(a^2+2*a*b+b^2)*(-1/4/b^(1/
2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*
b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1
/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2434 vs. 2(100) = 200.

Time = 0.32 (sec) , antiderivative size = 5109, normalized size of antiderivative = 47.31

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.96.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(100) = 200.

Time = 0.34 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.27

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$
$$= \frac{5a^3 + 2a^2b + (15a^3 + 32a^2b + 8ab^2)e^{(-2dx-2c)} + (15a^3 + 46a^2b + 56}{4(a^6 + 2a^5b + a^4b^2 + 4(a^6 + 4a^5b + 5a^4b^2 + 2a^3b^3)e^{(-2dx-2c)} + 2(3a^6 + 14a^5b + 27a^4b^2 + 24a^3b^3 + 8}$$
$$- \frac{3\log\left(\frac{ae^{(-2dx-2c)} + a + 2b - 2\sqrt{(a+b)b}}{ae^{(-2dx-2c)} + a + 2b + 2\sqrt{(a+b)b}}\right)}{16(a^2 + 2ab + b^2)\sqrt{(a+b)b}d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(5*a^3 + 2*a^2*b + (15*a^3 + 32*a^2*b + 8*a*b^2)*e^(-2*d*x - 2*c) + (1
5*a^3 + 46*a^2*b + 56*a*b^2 + 16*b^3)*e^(-4*d*x - 4*c) + (5*a^3 + 16*a^2*b
 + 8*a*b^2)*e^(-6*d*x - 6*c))/((a^6 + 2*a^5*b + a^4*b^2 + 4*(a^6 + 4*a^5*b
 + 5*a^4*b^2 + 2*a^3*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^6 + 14*a^5*b + 27*a^4*
b^2 + 24*a^3*b^3 + 8*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^6 + 4*a^5*b + 5*a^4*
b^2 + 2*a^3*b^3)*e^(-6*d*x - 6*c) + (a^6 + 2*a^5*b + a^4*b^2)*e^(-8*d*x -
8*c))*d) - 3/16*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*
e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt
((a + b)*b)*d)
```

3.96.8 Giac [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)^2}{(b\operatorname{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{1}{\cosh(c+d\,x)^2\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3), x)
```

3.97 $\int \frac{\text{sech}^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx$

3.97.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=\frac{(4a+b)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}d}-\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}+\frac{(4a+b)\sinh(c+dx)}{8a(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
1/8*(4*a+b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(5/2)/d-
1/4*b*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2+1/8*(4*a+b)*sinh(d*x+c
)/a/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.29

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=-\frac{(a+2b+a\cosh(2(c+dx)))^3\text{sech}^6(c+dx)\left(\frac{8\sinh(c+dx)}{(a+b+a\sinh^2(c+dx))^2}-(4a+b)\left(\frac{3\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{5(a+b)}{(a+}\right.\right.}{192ad\left(a+b\text{sech}^2(c+dx)\right)^3}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/192*((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((8*Sinh[c + d*x
])/(a + b + a*Sinh[c + d*x]^2)^2 - (4*a + b)*((3*ArcTan[(Sqrt[a]*Sinh[c +
d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (5*(a + b)*Sinh[c + d*x] + 3
*a*Sinh[c + d*x]^3)/((a + b)^2*(a + b + a*Sinh[c + d*x]^2)^2))))/(a*d*(a +
 b*Sech[c + d*x]^2)^3)
```

3.97.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4635, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^3}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{\left(a\sinh^2(c+dx)+a+b\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(4a+b)\int \frac{1}{\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{4a(a+b)}-\frac{b\sinh(c+dx)}{4a(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{(4a+b)\left(\frac{\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{2(a+b)}+\frac{\sinh(c+dx)}{2(a+b)\left(a\sinh^2(c+dx)+a+b\right)}\right)}{4a(a+b)}-\frac{b\sinh(c+dx)}{4a(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(4a+b)\left(\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}}+\frac{\sinh(c+dx)}{2(a+b)\left(a\sinh^2(c+dx)+a+b\right)}\right)}{4a(a+b)}-\frac{b\sinh(c+dx)}{4a(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/4*(b*Sinh[c + d*x])/(a*(a + b)*(a + b + a*Sinh[c + d*x]^2)^2) + ((4*a
+ b)*(ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)
) + Sinh[c + d*x]/(2*(a + b)*(a + b + a*Sinh[c + d*x]^2))))/(4*a*(a + b)))
/d
```

3.97.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(109) = 218$.

Time = 1.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.39

method	result
derivativedivides	$\frac{\frac{-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a(a+b)}-\frac{\left(4a^{2}-5ab+3b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4(a+b)^{2}a}+\frac{\left(4a^{2}-5ab+3b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4(a+b)^{2}a}+\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}+\frac{(4a+b)\left(\text{ar}\cdots\right.}{\cdots}}{d}$
default	$\frac{\frac{-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a(a+b)}-\frac{\left(4a^{2}-5ab+3b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4(a+b)^{2}a}+\frac{\left(4a^{2}-5ab+3b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4(a+b)^{2}a}+\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b+a+b\right)^{2}}+\frac{(4a+b)\left(\text{ar}\cdots\right.}{\cdots}}{d}$
risch	$\frac{e^{dx+c}\left(4a^{2}e^{6dx+6c}+ab\,e^{6dx+6c}+4a^{2}e^{4dx+4c}+9ab\,e^{4dx+4c}-4\,e^{4dx+4c}b^{2}-4a^{2}e^{2dx+2c}-9ab\,e^{2dx+2c}+4\,e^{2dx+2c}b^{2}-4a^{2}-\cdots\right.}{4a(a+b)^{2}d\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^{2}}$

input
```
int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/8*(4*a-b)/a/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/8*(4*a^2-5*a*b+3*b^2)
/(a+b)^2/a*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^2-5*a*b+3*b^2)/(a+b)^2/a*tanh(1/
2*d*x+1/2*c)^3+1/8*(4*a-b)/a/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*
c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2+1/4*(4*a+b)/a/(a^2+2*a*b+b^2)*(1/2/(a+b)^(1/2)/a^(1/2)*arc
tan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/(a+b)^(
1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1
/2))))
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. $2(109) = 218$.

Time = 0.32 (sec) , antiderivative size = 6037, normalized size of antiderivative = 49.08

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.97.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx=\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

3.97.7 Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx=\int \frac{\operatorname{sech}(dx+c)^3}{(b\operatorname{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((4*a^2*e^(7*c) + a*b*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) + 9*a*b*e^(5
*c) - 4*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) + 9*a*b*e^(3*c) - 4*b^2*e^
(3*c))*e^(3*d*x) - (4*a^2*e^c + a*b*e^c)*e^(d*x))/(a^5*d + 2*a^4*b*d + a^3
*b^2*d + (a^5*d*e^(8*c) + 2*a^4*b*d*e^(8*c) + a^3*b^2*d*e^(8*c))*e^(8*d*x)
 + 4*(a^5*d*e^(6*c) + 4*a^4*b*d*e^(6*c) + 5*a^3*b^2*d*e^(6*c) + 2*a^2*b^3*
d*e^(6*c))*e^(6*d*x) + 2*(3*a^5*d*e^(4*c) + 14*a^4*b*d*e^(4*c) + 27*a^3*b^
2*d*e^(4*c) + 24*a^2*b^3*d*e^(4*c) + 8*a*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^5
*d*e^(2*c) + 4*a^4*b*d*e^(2*c) + 5*a^3*b^2*d*e^(2*c) + 2*a^2*b^3*d*e^(2*c)
)*e^(2*d*x)) + 8*integrate(1/32*((4*a*e^(3*c) + b*e^(3*c))*e^(3*d*x) + (4*
a*e^c + b*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*
e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 4*a^3*b*e^(2*c) +
5*a^2*b^2*e^(2*c) + 2*a*b^3*e^(2*c))*e^(2*d*x)), x)
```

3.97.8 Giac [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3), x)
```

3.98 $\int \frac{\text{sech}^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.98.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\text{sech}^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{(a+4b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}d} - \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a+4b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
1/8*(a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/(a+b)^(5/2)/d
-1/4*a*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*(a+4*b)*tanh(d*x+
c)/b/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

3.98.2 Mathematica [A] (warning: unable to verify)

Time = 5.05 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.00

$$\int \frac{\text{sech}^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^6(c+dx)\left(\frac{(a+4b)\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+}{b\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{64(a\ -}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(((a + 4*b)*ArcTanh[(Sech
[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(
2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x
)])^2*(Cosh[2*c] - Sinh[2*c]))/(b*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4
]) - (4*(a + b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a + ((a +
 2*b + a*Cosh[2*(c + d*x)])*Sech[2*c]*((a + 4*b)*Sinh[2*c] - (a - 2*b)*Sin
h[2*d*x]))/b))/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

3.98.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^4}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(a+4b)\int \frac{1}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4b(a+b)}-\frac{a\tanh(c+dx)}{4b(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{(a+4b)\left(\frac{\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4b(a+b)}-\frac{a\tanh(c+dx)}{4b(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\downarrow \text{221}$$

$$\frac{\frac{(a+4b)\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}}+\frac{\tanh(c+dx)}{2(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4b(a+b)}-\frac{a\tanh(c+dx)}{4b(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/4*(a*Tanh[c + d*x])/(b*(a + b)*(a + b - b*Tanh[c + d*x]^2)^2) + ((a +
4*b)*(ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2
)) + Tanh[c + d*x]/(2*(a + b)*(a + b - b*Tanh[c + d*x]^2))))/(4*b*(a + b))
)/d
```

3.98.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.98. $\int \frac{\text{sech}^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.98.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(111) = 222$.

Time = 1.30 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.48

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)b}+\frac{(3a^2-5ab+4b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2b}+\frac{(3a^2-5ab+4b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a+b)^2b}+\frac{(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)b}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}-\frac{(a+4b}{}}{d}$
default	$\frac{-\frac{2\left(\frac{(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)b}+\frac{(3a^2-5ab+4b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2b}+\frac{(3a^2-5ab+4b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a+b)^2b}+\frac{(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)b}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}-\frac{(a+4b}{}}{d}$
risch	$\frac{a^3e^{6dx+6c}+4a^2b\,e^{6dx+6c}+3a^3e^{4dx+4c}+2a^2b\,e^{4dx+4c}-8a\,b^2e^{4dx+4c}-16\,e^{4dx+4c}b^3+3a^3e^{2dx+2c}-4a^2b\,e^{2dx+2c}-16\,e^{2dx}}{4adb(a+b)^2\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^2}$

input
```
int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(1/8*(a-4*b)/(a+b)/b*tanh(1/2*d*x+1/2*c)^7+1/8*(3*a^2-5*a*b+4*b^2)
/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^5+1/8*(3*a^2-5*a*b+4*b^2)/(a+b)^2/b*tanh(1/
2*d*x+1/2*c)^3+1/8*(a-4*b)/(a+b)/b*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*
c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2-1/4*(a+4*b)/b/(a^2+2*a*b+b^2)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln
((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/
2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/
2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2603 vs. $2(117) = 234$.

Time = 0.31 (sec) , antiderivative size = 5447, normalized size of antiderivative = 43.58

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.98.6 Sympy [F]

$$\int \frac{\mathrm{sech}^4(c+dx)}{(a+b\mathrm{sech}^2(c+dx))^3}\,dx = \int \frac{\mathrm{sech}^4\,(c+dx)}{\left(a+b\,\mathrm{sech}^2\,(c+dx)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(117) = 234$.

Time = 0.36 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.95

$$\int \frac{\mathrm{sech}^4(c+dx)}{(a+b\mathrm{sech}^2(c+dx))^3}\,dx = -\frac{(a+4\,b)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^2b+2\,ab^2+b^3)\sqrt{(a+b)b}d} - \frac{a^3-2\,a^2b+(3\,a^3-4\,a^2b-16\,ab^2)e^{(-2\,dx-2\,c)}+(3\,a^3+}{4\,(a^5b+2\,a^4b^2+a^3b^3+4\,(a^5b+4\,a^4b^2+5\,a^3b^3+2\,a^2b^4)e^{(-2\,dx-2\,c)}+2\,(3\,a^5b+14\,a^4b^2+27\,a^3b^3+24\,}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(a + 4*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*
e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2*a*b^2 + b^3)*
sqrt((a + b)*b)*d) - 1/4*(a^3 - 2*a^2*b + (3*a^3 - 4*a^2*b - 16*a*b^2)*e^(
-2*d*x - 2*c) + (3*a^3 + 2*a^2*b - 8*a*b^2 - 16*b^3)*e^(-4*d*x - 4*c) + (a
^3 + 4*a^2*b)*e^(-6*d*x - 6*c))/((a^5*b + 2*a^4*b^2 + a^3*b^3 + 4*(a^5*b +
 4*a^4*b^2 + 5*a^3*b^3 + 2*a^2*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^5*b + 14*a^4
*b^2 + 27*a^3*b^3 + 24*a^2*b^4 + 8*a*b^5)*e^(-4*d*x - 4*c) + 4*(a^5*b + 4*
a^4*b^2 + 5*a^3*b^3 + 2*a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 2*a^4*b^2 + a
^3*b^3)*e^(-8*d*x - 8*c))*d)
```

3.98.8 Giac [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^4}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^4\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3), x)
```

3.99 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3} dx$

3.99.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3} dx = \frac{3\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}(a+b)^{5/2}d} + \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
1/4*sinh(d*x+c)/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/(a+b)^2/d/
(a+b+a*sinh(d*x+c)^2)+3/8*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/(a+b)^(5
/2)/d/a^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3} dx = \frac{(a+2b+a\cosh(2(c+dx)))^3\operatorname{sech}^5(c+dx)\left(\frac{3\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)\operatorname{sech}(c+dx)}{\sqrt{a}(a+b)^{5/2}} + \frac{(5(a+b)+3a\sinh^2(c+dx))\tanh(c+dx)}{(a+b)^2\left(a+b+a\sinh^2(c+dx)\right)^2}\right)}{64d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^5*((3*ArcTan[(Sqrt[a]*Sin
h[c + d*x])/Sqrt[a + b]]*Sech[c + d*x])/(Sqrt[a]*(a + b)^(5/2)) + ((5*(a +
 b) + 3*a*Sinh[c + d*x]^2)*Tanh[c + d*x])/((a + b)^2*(a + b + a*Sinh[c + d
*x]^2)^2)))/(64*d*(a + b*Sech[c + d*x]^2)^3)
```

3.99.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4635, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^5}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{\left(a\sinh^2(c+dx)+a+b\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\int \frac{1}{\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{4(a+b)}+\frac{\sinh(c+dx)}{4(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{2(a+b)}+\frac{\sinh(c+dx)}{2(a+b)\left(a\sinh^2(c+dx)+a+b\right)}\right)}{4(a+b)}+\frac{\sinh(c+dx)}{4(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}}+\frac{\sinh(c+dx)}{2(a+b)\left(a\sinh^2(c+dx)+a+b\right)}\right)}{4(a+b)}+\frac{\sinh(c+dx)}{4(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Sinh[c + d*x]/(4*(a + b)*(a + b + a*Sinh[c + d*x]^2)^2) + (3*(ArcTan[(Sqr
t[a]*Sinh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)) + Sinh[c + d*x]
/(2*(a + b)*(a + b + a*Sinh[c + d*x]^2))))/(4*(a + b)))/d
```

3.99.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.99.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(92) = 184.

Time = 1.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.22

method	result
risch	$\frac{e^{dx+c}\left(3a\,e^{6dx+6c}+11a\,e^{4dx+4c}+20b\,e^{4dx+4c}-11\,e^{2dx+2c}a-20b\,e^{2dx+2c}-3a\right)}{4d(a+b)^2\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^2}-\frac{3\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{16\sqrt{-a^2-ab}\,(a+b)^2 d}+$
derivativedivides	$\frac{\frac{-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4(a+b)}+\frac{3(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4(a+b)^2}-\frac{3(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4(a+b)^2}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{\frac{3\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{8\sqrt{a+b}\,\sqrt{a}}}{a^2+}}{d}$
default	$\frac{\frac{-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4(a+b)}+\frac{3(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4(a+b)^2}-\frac{3(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4(a+b)^2}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{\frac{3\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{8\sqrt{a+b}\,\sqrt{a}}}{a^2+}}{d}$

input
```
int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(d*x+c)*(3*a*exp(6*d*x+6*c)+11*a*exp(4*d*x+4*c)+20*b*exp(4*d*x+4*c)
-11*exp(2*d*x+2*c)*a-20*b*exp(2*d*x+2*c)-3*a)/d/(a+b)^2/(a*exp(4*d*x+4*c)+
2*exp(2*d*x+2*c)*a+4*b*exp(2*d*x+2*c)+a)^2-3/16/(-a^2-a*b)^(1/2)/(a+b)^2/d
*ln(exp(2*d*x+2*c)-2*(a+b)/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+3/16/(-a^2-a*b)^
(1/2)/(a+b)^2/d*ln(exp(2*d*x+2*c)+2*(a+b)/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2638 vs. 2(92) = 184.

Time = 0.32 (sec) , antiderivative size = 5006, normalized size of antiderivative = 47.23

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.99.6 Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2)**3, x)
```

3.99.7 Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{\left(b\operatorname{sech}(dx+c)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((11*a*e^(5*c) + 20*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) + 20*b*e^(3*c
))*e^(3*d*x) + 3*a*e^(7*d*x + 7*c) - 3*a*e^(d*x + c))/(a^4*d + 2*a^3*b*d +
 a^2*b^2*d + (a^4*d*e^(8*c) + 2*a^3*b*d*e^(8*c) + a^2*b^2*d*e^(8*c))*e^(8*
d*x) + 4*(a^4*d*e^(6*c) + 4*a^3*b*d*e^(6*c) + 5*a^2*b^2*d*e^(6*c) + 2*a*b^
3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^4*d*e^(4*c) + 14*a^3*b*d*e^(4*c) + 27*a^2*
b^2*d*e^(4*c) + 24*a*b^3*d*e^(4*c) + 8*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^4*d
*e^(2*c) + 4*a^3*b*d*e^(2*c) + 5*a^2*b^2*d*e^(2*c) + 2*a*b^3*d*e^(2*c))*e^
(2*d*x)) + 32*integrate(3/128*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^3 + 2*a^2
*b + a*b^2 + (a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2
*(a^3*e^(2*c) + 4*a^2*b*e^(2*c) + 5*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*
x)), x)
```

3.99.8 Giac [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^5}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^5\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3), x)
```

3.100 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.100.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{(3a^2+8ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}d} - \frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{3a(a+2b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
1/8*(3*a^2+8*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/(
a+b)^(5/2)/d-1/4*a*sech(d*x+c)^2*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^
2)^2-3/8*a*(a+2*b)*tanh(d*x+c)/b^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\frac{(3a^2+8ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a+b)^2(a+2b+a\cosh(2(c+dx)))^2}}{8b^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a
 + b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(
c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2
))/(8*b^(5/2)*d)
```

3.100.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4634, 315, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^6(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^6}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{-\frac{\int -\frac{-\left((3a+4b)\tanh^2(c+dx)\right)+a+4b}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4b(a+b)}-\frac{a\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-\left((3a+4b)\tanh^2(c+dx)\right)+a+4b}{\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4b(a+b)}-\frac{a\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{\frac{\left(3a^2+8ab+8b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{2b(a+b)}-\frac{3a(a+2b)\tanh(c+dx)}{2b(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4b(a+b)}-\frac{a\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{\left(3a^2+8ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}}-\frac{3a(a+2b)\tanh(c+dx)}{2b(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4b(a+b)}-\frac{a\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/4*(a*Tanh[c + d*x]*(1 - Tanh[c + d*x]^2))/(b*(a + b)*(a + b - b*Tanh[c
 + d*x]^2)^2) + (((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(2*b^(3/2)*(a + b)^(3/2)) - (3*a*(a + 2*b)*Tanh[c + d*x])/(2
*b*(a + b)*(a + b - b*Tanh[c + d*x]^2)))/(4*b*(a + b)))/d
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.100.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(130) = 260$.

Time = 1.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.28

method	result
derivativedivides	$-\frac{2\left(\frac{a(3a+8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)b^2}+\frac{a\left(9a^2+13ab-8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2b^2}+\frac{a\left(9a^2+13ab-8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a+b)^2b^2}+\frac{a(3a+8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)b^2}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$
default	$-\frac{2\left(\frac{a(3a+8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)b^2}+\frac{a\left(9a^2+13ab-8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2b^2}+\frac{a\left(9a^2+13ab-8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a+b)^2b^2}+\frac{a(3a+8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)b^2}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$
risch	$\frac{3a^3e^{6dx+6c}+8a^2b\,e^{6dx+6c}+8a\,b^2e^{6dx+6c}+9a^3e^{4dx+4c}+42a^2b\,e^{4dx+4c}+72a\,b^2e^{4dx+4c}+48\,e^{4dx+4c}b^3+9a^3e^{2dx+2c}+40a}{4d\,b^2(a+b)^2\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^2}$

input
```
int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(1/8*a*(3*a+8*b)/(a+b)/b^2*tanh(1/2*d*x+1/2*c)^7+1/8*a*(9*a^2+13*a
*b-8*b^2)/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^5+1/8*a*(9*a^2+13*a*b-8*b^2)/(a+
b)^2/b^2*tanh(1/2*d*x+1/2*c)^3+1/8*a*(3*a+8*b)/(a+b)/b^2*tanh(1/2*d*x+1/2*
c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-1/4*(3*a^2+8*a*b+8*b^2)/b^2/(a^2+2*a
*b+b^2)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*t
anh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2823 vs. 2(136) = 272.

Time = 0.33 (sec) , antiderivative size = 5887, normalized size of antiderivative = 40.88

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.100.6 Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2)**3, x)
```

3.100.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(136) = 272.

Time = 0.41 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.74

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = -\frac{(3a^2+8ab+8b^2)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16(a^2b^2+2ab^3+b^4)\sqrt{(a+b)b}d} - \frac{3a^3+6a^2b+(9a^3+40a^2b+40ab^2)e^{(-2dx-2c)}+3(3a^3+14}{4(a^4b^2+2a^3b^3+a^2b^4+4(a^4b^2+4a^3b^3+5a^2b^4+2ab^5)e^{(-2dx-2c)}+2(3a^4b^2+14a^3b^3+27a^2b^4+2}$$

input
```
integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(3*a^2 + 8*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((
a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 +
 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) - 1/4*(3*a^3 + 6*a^2*b + (9*a^3 + 40*a^
2*b + 40*a*b^2)*e^(-2*d*x - 2*c) + 3*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3
)*e^(-4*d*x - 4*c) + (3*a^3 + 8*a^2*b + 8*a*b^2)*e^(-6*d*x - 6*c))/((a^4*b
^2 + 2*a^3*b^3 + a^2*b^4 + 4*(a^4*b^2 + 4*a^3*b^3 + 5*a^2*b^4 + 2*a*b^5)*e
^(-2*d*x - 2*c) + 2*(3*a^4*b^2 + 14*a^3*b^3 + 27*a^2*b^4 + 24*a*b^5 + 8*b^
6)*e^(-4*d*x - 4*c) + 4*(a^4*b^2 + 4*a^3*b^3 + 5*a^2*b^4 + 2*a*b^5)*e^(-6*
d*x - 6*c) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d)
```

3.100.8 Giac [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx=\int \frac{\operatorname{sech}(dx+c)^6}{\left(b\operatorname{sech}(dx+c)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx=\int \frac{1}{\cosh(c+d\,x)^6\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3), x)
```

3.101 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.101.1 Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\arctan(\sinh(c+dx))}{b^3d} - \frac{\sqrt{a}(8a^2+20ab+15b^2)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3(a+b)^{5/2}d} - \frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{a(4a+7b)\sinh(c+dx)}{8b^2(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)}$$

output
```
arctan(sinh(d*x+c))/b^3/d-1/4*a*sinh(d*x+c)/b/(a+b)/d/(a+b+a*sinh(d*x+c)^2
)^2-1/8*a*(4*a+7*b)*sinh(d*x+c)/b^2/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)-1/8*(8
*a^2+20*a*b+15*b^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^3/(a
+b)^(5/2)/d
```

3.101.2 Mathematica [A] (verified)

Time = 4.96 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.61

$$\int \frac{\text{sech}^7(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^5(c+dx)\left(16\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)(a+2b+a\cosh(2(c+dx)))^2\text{se}\right.}{}$$

input
```
Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*(16*ArcTan[Tanh[(c + d*x)
/2]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x] + (Sqrt[a]*(8*a^2 + 2
0*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])
^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c
 + d*x]*(Cosh[c] - Sinh[c]))/((a + b)^(5/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) -
 (8*a*b^2*Tanh[c + d*x])/(a + b) - (2*a*b*(4*a + 7*b)*(a + 2*b + a*Cosh[2*
(c + d*x)])*Tanh[c + d*x])/(a + b)^2))/(64*b^3*d*(a + b*Sech[c + d*x]^2)^3
)
```

3.101.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4635, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^7(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^7}{(a+b\sec(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \ 316$$

$$\frac{\frac{\int \frac{-3a\sinh^2(c+dx)+a+4b}{\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)^2}d\sinh(c+dx)}{4b(a+b)}-\frac{a\sinh(c+dx)}{4b(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{\int \frac{4a^2-(4a+7b)\sinh^2(c+dx)a+9ba+8b^2}{\left(\sinh^2(c+dx)+1\right)\left(a\sinh^2(c+dx)+a+b\right)}d\sinh(c+dx)}{2b(a+b)}-\frac{a(4a+7b)\sinh(c+dx)}{2b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{4b(a+b)}-\frac{a\sinh(c+dx)}{4b(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\int\frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}-\frac{a\left(8a^2+20ab+15b^2\right)\int\frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{2b(a+b)}-\frac{a(4a+7b)\sinh(c+dx)}{2b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{4b(a+b)}-\frac{a\sinh(c+dx)}{4b(a+b)\left(a\sinh^2(c+dx)\right.}}{d}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\arctan(\sinh(c+dx))}{b}-\frac{a\left(8a^2+20ab+15b^2\right)\int\frac{1}{a\sinh^2(c+dx)+a+b}d\sinh(c+dx)}{b}}{2b(a+b)}-\frac{a(4a+7b)\sinh(c+dx)}{2b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{4b(a+b)}-\frac{a\sinh(c+dx)}{4b(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\arctan(\sinh(c+dx))}{b}-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{2b(a+b)}-\frac{a(4a+7b)\sinh(c+dx)}{2b(a+b)\left(a\sinh^2(c+dx)+a+b\right)}}{4b(a+b)}-\frac{a\sinh(c+dx)}{4b(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/4*(a*Sinh[c + d*x])/(b*(a + b)*(a + b + a*Sinh[c + d*x]^2)^2) + (((8*(
a + b)^2*ArcTan[Sinh[c + d*x]])/b - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*Arc
Tan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(b*Sqrt[a + b]))/(2*b*(a + b)) -
 (a*(4*a + 7*b)*Sinh[c + d*x])/(2*b*(a + b)*(a + b + a*Sinh[c + d*x]^2)))/
(4*b*(a + b)))/d
```

3.101. $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.101.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.101. $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.101.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(139) = 278$.

Time = 2.96 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.05

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3} - \frac{2a\left(\frac{-\frac{b(9b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)} - \frac{b\left(4a^2-11ab-27b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2} + \frac{b\left(4a^2-11ab-27b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots}{8(a+b)^2}\cdots}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\cdots}\cdots\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3} - \frac{2a\left(\frac{-\frac{b(9b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)} - \frac{b\left(4a^2-11ab-27b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2} + \frac{b\left(4a^2-11ab-27b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots}{8(a+b)^2}\cdots}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\cdots}\cdots\right)}{\cdots}}{\cdots}$
risch	$-\frac{e^{dx+c}a\left(4a^2e^{6dx+6c}+7ab\,e^{6dx+6c}+4a^2e^{4dx+4c}+31ab\,e^{4dx+4c}+36\,e^{4dx+4c}b^2-4a^2e^{2dx+2c}-31ab\,e^{2dx+2c}-36\,e^{2dx+2c}\cdots\right.}{4b^2(a+b)^2d\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^2}$

input
```
int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^3*arctan(tanh(1/2*d*x+1/2*c))-2*a/b^3*((-1/8*b*(9*b+4*a)/(a+b)*ta
nh(1/2*d*x+1/2*c)^7-1/8*b*(4*a^2-11*a*b-27*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c
)^5+1/8*b*(4*a^2-11*a*b-27*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+1/8*b*(9*b+4
*a)/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c
)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/8*(8*a^
2+20*a*b+15*b^2)/(a^2+2*a*b+b^2)*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*
arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2)))))
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4335 vs. $2(139) = 278$.

Time = 0.39 (sec) , antiderivative size = 7993, normalized size of antiderivative = 52.24

$$\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.101.6 Sympy [F]

$$\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\text{sech}^7\left(c+dx\right)}{\left(a+b\,\text{sech}^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**7/(a + b*sech(c + d*x)**2)**3, x)
```

3.101.7 Maxima [F]

$$\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\text{sech}\left(dx+c\right)^7}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((4*a^3*e^(7*c) + 7*a^2*b*e^(7*c))*e^(7*d*x) + (4*a^3*e^(5*c) + 31*a^
2*b*e^(5*c) + 36*a*b^2*e^(5*c))*e^(5*d*x) - (4*a^3*e^(3*c) + 31*a^2*b*e^(3
*c) + 36*a*b^2*e^(3*c))*e^(3*d*x) - (4*a^3*e^c + 7*a^2*b*e^c)*e^(d*x))/(a^
4*b^2*d + 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) + 2*a^3*b^3*d*e^(8*
c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^4*b^2*d*e^(6*c) + 4*a^3*b^3*d*e^(
6*c) + 5*a^2*b^4*d*e^(6*c) + 2*a*b^5*d*e^(6*c))*e^(6*d*x) + 2*(3*a^4*b^2*d
*e^(4*c) + 14*a^3*b^3*d*e^(4*c) + 27*a^2*b^4*d*e^(4*c) + 24*a*b^5*d*e^(4*c
) + 8*b^6*d*e^(4*c))*e^(4*d*x) + 4*(a^4*b^2*d*e^(2*c) + 4*a^3*b^3*d*e^(2*c
) + 5*a^2*b^4*d*e^(2*c) + 2*a*b^5*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x
+ c))/(b^3*d) - 128*integrate(1/512*((8*a^3*e^(3*c) + 20*a^2*b*e^(3*c) + 1
5*a*b^2*e^(3*c))*e^(3*d*x) + (8*a^3*e^c + 20*a^2*b*e^c + 15*a*b^2*e^c)*e^(
d*x))/(a^3*b^3 + 2*a^2*b^4 + a*b^5 + (a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c)
+ a*b^5*e^(4*c))*e^(4*d*x) + 2*(a^3*b^3*e^(2*c) + 4*a^2*b^4*e^(2*c) + 5*a*
b^5*e^(2*c) + 2*b^6*e^(2*c))*e^(2*d*x)), x)
```

3.101.8 Giac [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^7}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^7\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3), x)
```

3.102 $\int (a+b\text{sech}^2(c+dx)) \tanh^4(c+dx) \, dx$

3.102.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a+b\text{sech}^2(c+dx)) \tanh^4(c+dx) \, dx = ax - \frac{a\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} + \frac{b\tanh^5(c+dx)}{5d}$$

output
```
a*x-a*tanh(d*x+c)/d-1/3*a*tanh(d*x+c)^3/d+1/5*b*tanh(d*x+c)^5/d
```

3.102.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a+b\text{sech}^2(c+dx)) \tanh^4(c+dx) \, dx = \frac{a\text{arctanh}(\tanh(c+dx))}{d} - \frac{a\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} + \frac{b\tanh^5(c+dx)}{5d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4,x]
```

output
```
(a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(
3*d) + (b*Tanh[c + d*x]^5)/(5*d)
```

3.102.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4629, 2075, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{363}$$

$$\frac{a\int \frac{\tanh^4(c+dx)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{1}{5}b\tanh^5(c+dx)}{d}$$

$$\Big\downarrow \text{254}$$

$$\frac{a\int \left(-\tanh^2(c+dx)+\frac{1}{1-\tanh^2(c+dx)}-1\right)d\tanh(c+dx)+\frac{1}{5}b\tanh^5(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\text{arctanh}(\tanh(c+dx))-\frac{1}{3}\tanh^3(c+dx)-\tanh(c+dx)\right)+\frac{1}{5}b\tanh^5(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4,x]
```

output
```
((b*Tanh[c + d*x]^5)/5 + a*(ArcTanh[Tanh[c + d*x]] - Tanh[c + d*x] - Tanh[
c + d*x]^3/3))/d
```

3.102.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.102.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

method	result
parts	$\frac{a\left(-\frac{\tanh(dx+c)^{3}}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\tanh(dx+c)^{5}}{5d}$
derivativedivides	$\frac{a\left(dx+c-\tanh(dx+c)-\frac{\tanh(dx+c)^{3}}{3}\right)+b\left(-\frac{\sinh(dx+c)^{3}}{2\cosh(dx+c)^{5}}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^{5}}+\frac{3\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^{4}}{5}+\frac{4\,\mathrm{sech}(dx+c)^{2}}{15}\right)\tanh(dx+}{8}\right.}{d}$
default	$\frac{a\left(dx+c-\tanh(dx+c)-\frac{\tanh(dx+c)^{3}}{3}\right)+b\left(-\frac{\sinh(dx+c)^{3}}{2\cosh(dx+c)^{5}}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^{5}}+\frac{3\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^{4}}{5}+\frac{4\,\mathrm{sech}(dx+c)^{2}}{15}\right)\tanh(dx+}{8}\right.}{d}$
risch	$ax+\frac{4a\,\mathrm{e}^{8dx+8c}-2b\,\mathrm{e}^{8dx+8c}+12a\,\mathrm{e}^{6dx+6c}+\frac{44a\,\mathrm{e}^{4dx+4c}}{3}-4b\,\mathrm{e}^{4dx+4c}+\frac{28\,\mathrm{e}^{2dx+2c}a}{3}+\frac{8a}{3}-\frac{2b}{5}}{d\left(\mathrm{e}^{2dx+2c}+1\right)^{5}}$

input
```
int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
a/d*(-1/3*tanh(d*x+c)^3-tanh(d*x+c)-1/2*ln(tanh(d*x+c)-1)+1/2*ln(tanh(d*x+
c)+1))+1/5*b*tanh(d*x+c)^5/d
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 327, normalized size of antiderivative = 6.81

$$\int \left(a+b\mathrm{sech}^{2}(c+dx)\right)\tanh^{4}(c+dx)\,dx$$
$$=\frac{(15\,adx+20\,a-3\,b)\cosh{(dx+c)}^{5}+5\,(15\,adx+20\,a-3\,b)\cosh{(dx+c)}\sinh{(dx+c)}^{4}-(20\,a-3\,b)\,\mathrm{si}}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/15*((15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^5 + 5*(15*a*d*x + 20*a - 3*b)*
cosh(d*x + c)*sinh(d*x + c)^4 - (20*a - 3*b)*sinh(d*x + c)^5 + 5*(15*a*d*x
 + 20*a - 3*b)*cosh(d*x + c)^3 - 5*(2*(20*a - 3*b)*cosh(d*x + c)^2 + 8*a +
 3*b)*sinh(d*x + c)^3 + 5*(2*(15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^3 + 3*(
15*a*d*x + 20*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a*d*x + 20*
a - 3*b)*cosh(d*x + c) - 5*((20*a - 3*b)*cosh(d*x + c)^4 + 3*(8*a + 3*b)*c
osh(d*x + c)^2 + 4*a - 6*b)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d
*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3
*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

3.102.6 Sympy [F]

$$\int \left(a + b\text{sech}^2(c + dx)\right) \tanh^4(c + dx)\, dx = \int \left(a + b\,\text{sech}^2\,(c + dx)\right) \tanh^4\,(c + dx)\, dx$$

input
```
integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**4,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**4, x)
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.92

$$\int \left(a + b\text{sech}^2(c + dx)\right) \tanh^4(c + dx)\, dx = \frac{b \tanh\left(dx + c\right)^5}{5\,d} + \frac{1}{3}\,a\left(3\,x + \frac{3\,c}{d} - \frac{4\left(3\,e^{(-2\,dx - 2\,c)} + 3\,e^{(-4\,dx - 4\,c)} + 2\right)}{d(3\,e^{(-2\,dx - 2\,c)} + 3\,e^{(-4\,dx - 4\,c)} + e^{(-6\,dx - 6\,c)} + 1)}\right)$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/5*b*tanh(d*x + c)^5/d + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e
^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d
*x - 6*c) + 1)))
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(44) = 88.

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.25

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^4(c+dx)\,dx = \frac{15\,(dx+c)a+\frac{2\,\left(30\,ae^{(8\,dx+8\,c)}-15\,be^{(8\,dx+8\,c)}+90\,ae^{(6\,dx+6\,c)}+110\,ae^{(4\,dx+4\,c)}-30\,be^{(4\,dx+4\,c)}+70\,ae^{(2\,dx+2\,c)}+20\,a-3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)*a + 2*(30*a*e^(8*d*x + 8*c) - 15*b*e^(8*d*x + 8*c) + 90
*a*e^(6*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) - 30*b*e^(4*d*x + 4*c) + 70*a*e
^(2*d*x + 2*c) + 20*a - 3*b)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.102.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 433, normalized size of antiderivative = 9.02

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^4(c+dx)\,dx = a\,x+\frac{\frac{2\,(2\,a-3\,b)}{15\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(a+b)}{5\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(2\,a-b)}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1} + \frac{\frac{2\,(2\,a-b)}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(a+b)}{5\,d}+\frac{8\,e^{6\,c+6\,d\,x}\,(a+b)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(2\,a-3\,b)}{5\,d}+\frac{2\,e^{8\,c+8\,d\,x}\,(2\,a-b)}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1} + \frac{\frac{2\,(a+b)}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(2\,a-b)}{5\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1} + \frac{\frac{2\,(a+b)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(a+b)}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(2\,a-3\,b)}{5\,d}+\frac{2\,e^{6\,c+6\,d\,x}\,(2\,a-b)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}+\frac{2\,(2\,a-b)}{5\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

output
```
a*x + ((2*(2*a - 3*b))/(15*d) + (4*exp(2*c + 2*d*x)*(a + b))/(5*d) + (2*ex
p(4*c + 4*d*x)*(2*a - b))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
+ exp(6*c + 6*d*x) + 1) + ((2*(2*a - b))/(5*d) + (8*exp(2*c + 2*d*x)*(a +
b))/(5*d) + (8*exp(6*c + 6*d*x)*(a + b))/(5*d) + (4*exp(4*c + 4*d*x)*(2*a
- 3*b))/(5*d) + (2*exp(8*c + 8*d*x)*(2*a - b))/(5*d))/(5*exp(2*c + 2*d*x)
+ 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*
c + 10*d*x) + 1) + ((2*(a + b))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a - b))/(5*
d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(a + b))/(5*d) + (6*
exp(4*c + 4*d*x)*(a + b))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a - 3*b))/(5*d) +
 (2*exp(6*c + 6*d*x)*(2*a - b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4
*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + (2*(2*a - b))/(5*d*(e
xp(2*c + 2*d*x) + 1))
```

3.102. $\int (a + b\operatorname{sech}^2(c + dx)) \tanh^4(c + dx)\, dx$

3.103 $\int (a + b\text{sech}^2(c + dx)) \tanh^3(c + dx) \, dx$

3.103.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int (a + b\text{sech}^2(c + dx)) \tanh^3(c + dx) \, dx = \frac{a \log(\cosh(c + dx))}{d} + \frac{(a - b)\text{sech}^2(c + dx)}{2d} + \frac{b\text{sech}^4(c + dx)}{4d}$$

output
```
a*ln(cosh(d*x+c))/d+1/2*(a-b)*sech(d*x+c)^2/d+1/4*b*sech(d*x+c)^4/d
```

3.103.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int (a + b\text{sech}^2(c + dx)) \tanh^3(c + dx) \, dx = \frac{a \log(\cosh(c + dx))}{d} - \frac{a \tanh^2(c + dx)}{2d} + \frac{b \tanh^4(c + dx)}{4d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^3,x]
```

output
```
(a*Log[Cosh[c + d*x]])/d - (a*Tanh[c + d*x]^2)/(2*d) + (b*Tanh[c + d*x]^4)
/(4*d)
```

3.103.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \left(b\sec(ic+idx)^2+a\right)\tan(ic+idx)^3dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)\text{sech}^5(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)\text{sech}^3(c+dx)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{85}$$

$$-\frac{\int \left(b\text{sech}^3(c+dx)+(a-b)\text{sech}^2(c+dx)-a\text{sech}(c+dx)\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-(a-b)\text{sech}(c+dx)-a\log\left(\cosh^2(c+dx)\right)-\frac{1}{2}b\text{sech}^2(c+dx)}{2d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^3,x]
```

output
```
-1/2*(-(a*Log[Cosh[c + d*x]^2]) - (a - b)*Sech[c + d*x] - (b*Sech[c + d*x]
^2)/2)/d
```

3.103.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.103.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\operatorname{sech}(dx+c)^4 b}{4}+\frac{\operatorname{sech}(dx+c)^2 a}{2}-\frac{b\operatorname{sech}(dx+c)^2}{2}-a\ln(\operatorname{sech}(dx+c))}{d}$	49
default	$\frac{\frac{\operatorname{sech}(dx+c)^4 b}{4}+\frac{\operatorname{sech}(dx+c)^2 a}{2}-\frac{b\operatorname{sech}(dx+c)^2}{2}-a\ln(\operatorname{sech}(dx+c))}{d}$	49
parts	$\frac{a\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\tanh(dx+c)^4}{4d}$	54
risch	$-ax-\frac{2ac}{d}+\frac{2\,\mathrm{e}^{2dx+2c}\left(a\,\mathrm{e}^{4dx+4c}-b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+a-b\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^4}+\frac{a\ln\left(\mathrm{e}^{2dx+2c}+1\right)}{d}$	97

input
```
int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*sech(d*x+c)^4*b+1/2*sech(d*x+c)^2*a-1/2*b*sech(d*x+c)^2-a*ln(sech
(d*x+c)))
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1072 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 1072, normalized size of antiderivative = 21.88

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)\tanh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="fricas")
```

output
```
-(a*d*x*cosh(d*x + c)^8 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a*d*x*si
nh(d*x + c)^8 + 2*(2*a*d*x - a + b)*cosh(d*x + c)^6 + 2*(14*a*d*x*cosh(d*x
 + c)^2 + 2*a*d*x - a + b)*sinh(d*x + c)^6 + 4*(14*a*d*x*cosh(d*x + c)^3 +
 3*(2*a*d*x - a + b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*co
sh(d*x + c)^4 + 2*(35*a*d*x*cosh(d*x + c)^4 + 3*a*d*x + 15*(2*a*d*x - a +
b)*cosh(d*x + c)^2 - 2*a)*sinh(d*x + c)^4 + 8*(7*a*d*x*cosh(d*x + c)^5 + 5
*(2*a*d*x - a + b)*cosh(d*x + c)^3 + (3*a*d*x - 2*a)*cosh(d*x + c))*sinh(d
*x + c)^3 + a*d*x + 2*(2*a*d*x - a + b)*cosh(d*x + c)^2 + 2*(14*a*d*x*cosh
(d*x + c)^6 + 15*(2*a*d*x - a + b)*cosh(d*x + c)^4 + 2*a*d*x + 6*(3*a*d*x
- 2*a)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^8 + 8*a
*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 + 4*a*cosh(d*x + c)^6 +
 4*(7*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^6 + 8*(7*a*cosh(d*x + c)^3 + 3*
a*cosh(d*x + c))*sinh(d*x + c)^5 + 6*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d*x
+ c)^4 + 30*a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x + c
)^5 + 10*a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a*cosh
(d*x + c)^2 + 4*(7*a*cosh(d*x + c)^6 + 15*a*cosh(d*x + c)^4 + 9*a*cosh(d*x
 + c)^2 + a)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 + 3*a*cosh(d*x + c)^5
+ 3*a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*cosh(d*x
 + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a*d*x*cosh(d*x + c)^7 + 3*(2
*a*d*x - a + b)*cosh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d*x + c)^3 + (...
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int (a+b\text{sech}^2(c+dx))\tanh^3(c+dx)\,dx = \begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} - \frac{a\tanh^2(c+dx)}{2d} - \frac{b\tanh^2(c+dx)\,\text{sech}^2(c+dx)}{4d} - \frac{b\,\text{sech}^2(c+dx)}{4d} & \text{for } d\neq 0 \\ x(a+b\,\text{sech}^2(c))\tanh^3(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**3,x)
```

output
```
Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) - b
*tanh(c + d*x)**2*sech(c + d*x)**2/(4*d) - b*sech(c + d*x)**2/(4*d), Ne(d,
 0)), (x*(a + b*sech(c)**2)*tanh(c)**3, True))
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int (a + b\text{sech}^2(c+dx))\tanh^3(c+dx)\,dx$$
$$= \frac{b\tanh(dx+c)^4}{4\,d} + a\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*b*tanh(d*x + c)^4/d + a*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(
-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

3.103.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(45) = 90.

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.43

$$\int (a + b\text{sech}^2(c+dx))\tanh^3(c+dx)\,dx =$$
$$-\frac{12\,(dx+c)a - 12\,a\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{25\,ae^{(8\,dx+8\,c)}+76\,ae^{(6\,dx+6\,c)}+24\,be^{(6\,dx+6\,c)}+102\,ae^{(4\,dx+4\,c)}+76\,ae^{(2\,dx+2\,c)}+24\,\cdots}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{12\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/12*(12*(d*x + c)*a - 12*a*log(e^(2*d*x + 2*c) + 1) + (25*a*e^(8*d*x + 8
*c) + 76*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x + 4*c)
+ 76*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c) + 1
)^4)/d
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.53

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)\tanh^3(c+dx)\,dx$$
$$=\frac{2\,(a-b)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-a\,x-\frac{2\,(a-3\,b)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{8\,b}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}+\frac{4\,b}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{a\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)}{d}$$

input
```
int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

output
```
(2*(a - b))/(d*(exp(2*c + 2*d*x) + 1)) - a*x - (2*(a - 3*b))/(d*(2*exp(2*c
 + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8*b)/(d*(3*exp(2*c + 2*d*x) + 3*exp(
4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*b)/(d*(4*exp(2*c + 2*d*x) + 6*e
xp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (a*log(exp
(2*c)*exp(2*d*x) + 1))/d
```

3.104 $\int (a+b\text{sech}^2(c+dx)) \tanh^2(c+dx) \, dx$

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int (a+b\text{sech}^2(c+dx)) \tanh^2(c+dx) \, dx = ax - \frac{a\tanh(c+dx)}{d} + \frac{b\tanh^3(c+dx)}{3d}$$

output
```
a*x-a*tanh(d*x+c)/d+1/3*b*tanh(d*x+c)^3/d
```

3.104.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int (a+b\text{sech}^2(c+dx)) \tanh^2(c+dx) \, dx = \frac{a\text{arctanh}(\tanh(c+dx))}{d} - \frac{a\tanh(c+dx)}{d} + \frac{b\tanh^3(c+dx)}{3d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]
```

output
```
(a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(
3*d)
```

3.104.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 25, 4629, 25, 2075, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\left(\tan(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \left(b\sec(ic+idx)^2+a\right)\tan(ic+idx)^2dx$$

$$\Big\downarrow \; 4629$$

$$-\frac{\int -\frac{\tanh^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 363$$

$$-\frac{-a\int \frac{\tanh^2(c+dx)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{1}{3}b\tanh^3(c+dx)}{d}$$

$$\Big\downarrow \; 262$$

$$-\frac{-a\left(\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\tanh(c+dx)\right)-\frac{1}{3}b\tanh^3(c+dx)}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{-a(\text{arctanh}(\tanh(c+dx))-\tanh(c+dx))-\frac{1}{3}b\tanh^3(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]
```

output
```
-((-(a*(ArcTanh[Tanh[c + d*x]] - Tanh[c + d*x])) - (b*Tanh[c + d*x]^3)/3)/
d)
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.104.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

method	result	size
parts	$\frac{a\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\tanh(dx+c)^3}{3d}$	52
derivativedivides	$\frac{a(dx+c-\tanh(dx+c))+b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$	60
default	$\frac{a(dx+c-\tanh(dx+c))+b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$	60
risch	$ax+\frac{2a\,e^{4dx+4c}-2b\,e^{4dx+4c}+4\,e^{2dx+2c}a+2a-\frac{2b}{3}}{d\left(e^{2dx+2c}+1\right)^3}$	66

input
```
int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d*(-tanh(d*x+c)-1/2*ln(tanh(d*x+c)-1)+1/2*ln(tanh(d*x+c)+1))+1/3*b*tanh(
d*x+c)^3/d
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(30) = 60.

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.84

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)\tanh^2(c+dx)\,dx$$
$$=\frac{(3\,adx+3\,a-b)\cosh\left(dx+c\right)^3+3\,(3\,adx+3\,a-b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-(3\,a-b)\sinh\left(dx+c\right.}{3\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(d\right.\right.}$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*((3*a*d*x + 3*a - b)*cosh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*cosh(d*x
+ c)*sinh(d*x + c)^2 - (3*a - b)*sinh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*c
osh(d*x + c) - 3*((3*a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/(d*cos
h(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

3.104.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^2(c+dx)\,dx=\int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)\tanh^2\left(c+dx\right)dx$$

input
```
integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**2, x)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^2(c+dx)\,dx=\frac{b\tanh\left(dx+c\right)^3}{3\,d}+a\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*b*tanh(d*x + c)^3/d + a*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(30) = 60.

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.25

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^2(c+dx)\,dx =\frac{3\left(dx+c\right)a+\frac{2\left(3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+3\,a-b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a + 2*(3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 6*a*e^
(2*d*x + 2*c) + 3*a - b)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.104.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.09

$$\int (a+b\operatorname{sech}^2(c+dx))\tanh^2(c+dx)\,dx = \frac{\frac{2\,(a+b)}{3\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(a-b)}{3\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}+a\,x + \frac{\frac{2\,(a-b)}{3\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(a+b)}{3\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(a-b)}{3\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1} + \frac{2\,(a-b)}{3\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

output
```
((2*(a + b))/(3*d) + (2*exp(2*c + 2*d*x)*(a - b))/(3*d))/(2*exp(2*c + 2*d*
x) + exp(4*c + 4*d*x) + 1) + a*x + ((2*(a - b))/(3*d) + (4*exp(2*c + 2*d*x
)*(a + b))/(3*d) + (2*exp(4*c + 4*d*x)*(a - b))/(3*d))/(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (2*(a - b))/(3*d*(exp(2*c
+ 2*d*x) + 1))
```

3.105 $\int (a + b\text{sech}^2(c + dx)) \tanh(c + dx) \, dx$

3.105.1 Optimal result

Integrand size = 19, antiderivative size = 29

$$\int (a + b\text{sech}^2(c + dx)) \tanh(c + dx) \, dx = \frac{a \log(\cosh(c + dx))}{d} - \frac{b\text{sech}^2(c + dx)}{2d}$$

output
```
a*ln(cosh(d*x+c))/d-1/2*b*sech(d*x+c)^2/d
```

3.105.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a + b\text{sech}^2(c + dx)) \tanh(c + dx) \, dx = \frac{a \log(\cosh(c + dx))}{d} - \frac{b\text{sech}^2(c + dx)}{2d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]
```

output
```
(a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)
```

3.105.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 26, 4626, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\tan(ic+idx)\left(a+b\sec(ic+idx)^2\right)dx$$

$$\Big\downarrow \; 26$$

$$-i\int \left(b\sec(ic+idx)^2+a\right)\tan(ic+idx)dx$$

$$\Big\downarrow \; 4626$$

$$\frac{\int \left(a\cosh^2(c+dx)+b\right)\text{sech}^3(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 244$$

$$\frac{\int \left(b\text{sech}^3(c+dx)+a\text{sech}(c+dx)\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\log(\cosh(c+dx))-\frac{1}{2}b\text{sech}^2(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]
```

output
```
(a*Log[Cosh[c + d*x]] - (b*Sech[c + d*x]^2)/2)/d
```

3.105.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.105.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
parts	$\frac{a\ln(\cosh(dx+c))}{d}+\frac{b\tanh(dx+c)^2}{2d}$	28
derivativedivides	$-\frac{b\,\mathrm{sech}(dx+c)^2}{2d}-\frac{a\ln(\mathrm{sech}(dx+c))}{d}$	29
default	$-\frac{b\,\mathrm{sech}(dx+c)^2}{2d}-\frac{a\ln(\mathrm{sech}(dx+c))}{d}$	29
risch	$-ax-\frac{2ac}{d}-\frac{2b\,\mathrm{e}^{2dx+2c}}{d(\mathrm{e}^{2dx+2c}+1)^2}+\frac{a\ln(\mathrm{e}^{2dx+2c}+1)}{d}$	58

input
```
int((a+b*sech(d*x+c)^2)*tanh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(cosh(d*x+c))/d+1/2*b*tanh(d*x+c)^2/d
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 12.38

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh(c+dx)\,dx = -\frac{adx\cosh\left(dx+c\right)^4+4\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+adx\sinh\left(dx+c\right)^4+adx+2\left(adx+b\right)\cosh(}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="fricas")
```

output
```
-(a*d*x*cosh(d*x + c)^4 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*x*si
nh(d*x + c)^4 + a*d*x + 2*(a*d*x + b)*cosh(d*x + c)^2 + 2*(3*a*d*x*cosh(d*
x + c)^2 + a*d*x + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x
+ c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*co
sh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c
))*sinh(d*x + c) + a)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
 + 4*(a*d*x*cosh(d*x + c)^3 + (a*d*x + b)*cosh(d*x + c))*sinh(d*x + c))/(d
*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 +
 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh(c+dx)\,dx = \begin{cases} ax - \frac{a\log\left(\tanh\left(c+dx\right)+1\right)}{d} - \frac{b\,\text{sech}^2\left(c+dx\right)}{2d} & \text{for}\ d \neq 0 \\ x\left(a+b\,\text{sech}^2\left(c\right)\right)\tanh\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c),x)
```

output
```
Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - b*sech(c + d*x)**2/(2*d), Ne
(d, 0)), (x*(a + b*sech(c)**2)*tanh(c), True))
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh(c+dx)\,dx=\frac{b\tanh\left(dx+c\right)^2}{2\,d}+\frac{a\log\left(\cosh\left(dx+c\right)\right)}{d}$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="maxima")
```

output
```
1/2*b*tanh(d*x + c)^2/d + a*log(cosh(d*x + c))/d
```

3.105.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(27) = 54.

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.86

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh(c+dx)\,dx$$
$$=-\frac{2\,(dx+c)a-2\,a\log\left(e^{(2\,dx+2\,c)}+1\right)+\frac{3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+3\,a}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a - 2*a*log(e^(2*d*x + 2*c) + 1) + (3*a*e^(4*d*x + 4*c)
+ 6*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + 3*a)/(e^(2*d*x + 2*c) + 1)^2
)/d
```

3.105.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.48

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh(c+dx)\,dx=\frac{2\,b}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-a\,x+\frac{a\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)}{d}$$

input
```
int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

output
```
(2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*b)/(d*(exp(2*c
+ 2*d*x) + 1)) - a*x + (a*log(exp(2*c)*exp(2*d*x) + 1))/d
```

3.105. $\int (a + b\text{sech}^2(c + dx)) \tanh(c + dx)\, dx$

3.106 $\int \left(a+b\text{sech}^2(c+dx)\right) dx$

3.106.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \left(a+b\text{sech}^2(c+dx)\right) dx = ax+\frac{b\tanh(c+dx)}{d}$$

output
```
a*x+b*tanh(d*x+c)/d
```

3.106.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\text{sech}^2(c+dx)\right) dx = ax+\frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[a + b*Sech[c + d*x]^2,x]
```

output
```
a*x + (b*Tanh[c + d*x])/d
```

3.106.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$ax+\frac{b\tanh(c+dx)}{d}$$

input
```
Int[a + b*Sech[c + d*x]^2,x]
```

output
```
a*x + (b*Tanh[c + d*x])/d
```

3.106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.106.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\tanh(dx+c)}{d}$	16
parts	$ax+\frac{b\tanh(dx+c)}{d}$	16
derivativedivides	$\frac{a(dx+c)+b\tanh(dx+c)}{d}$	21
risch	$ax-\frac{2b}{d\left(e^{2dx+2c}+1\right)}$	24
parallelrisch	$\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}+ax$	35

input
```
int(a+b*sech(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*tanh(d*x+c)/d
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.40

$$\int \left(a + b\text{sech}^2(c + dx)\right) \, dx = \frac{(adx - b)\cosh(dx + c) + b\sinh(dx + c)}{d\cosh(dx + c)}$$

input
```
integrate(a+b*sech(d*x+c)^2,x, algorithm="fricas")
```

output
```
((a*d*x - b)*cosh(d*x + c) + b*sinh(d*x + c))/(d*cosh(d*x + c))
```

3.106.6 Sympy [F]

$$\int \left(a + b\text{sech}^2(c + dx)\right) \, dx = \int \left(a + b\,\text{sech}^2(c + dx)\right) \, dx$$

input
```
integrate(a+b*sech(d*x+c)**2,x)
```

output
```
Integral(a + b*sech(c + d*x)**2, x)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \left(a + b\text{sech}^2(c + dx)\right) \, dx = ax + \frac{2\,b}{d(e^{(-2\,dx - 2\,c)} + 1)}$$

input
```
integrate(a+b*sech(d*x+c)^2,x, algorithm="maxima")
```

output
```
a*x + 2*b/(d*(e^(-2*d*x - 2*c) + 1))
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \left(a+b\text{sech}^2(c+dx)\right)\,dx = ax - \frac{2\,b}{d(e^{(2\,dx+2\,c)}+1)}$$

input
```
integrate(a+b*sech(d*x+c)^2,x, algorithm="giac")
```

output
```
a*x - 2*b/(d*(e^(2*d*x + 2*c) + 1))
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \left(a+b\text{sech}^2(c+dx)\right)\,dx = a\,x - \frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(a + b/cosh(c + d*x)^2,x)
```

output
```
a*x - (2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.107 $\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$

3.107.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=-\frac{b\log(\cosh(c+dx))}{d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

output
```
-b*ln(cosh(d*x+c))/d+(a+b)*ln(sinh(d*x+c))/d
```

3.107.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=-\frac{b(\log(\cosh(c+dx))-\log(\sinh(c+dx)))}{d}+\frac{a(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{d}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

output
```
-((b*(Log[Cosh[c + d*x]] - Log[Sinh[c + d*x]]))/d) + (a*(Log[Cosh[c + d*x]
] + Log[Tanh[c + d*x]]))/d
```

3.107.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 26, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\left(a+b\sec(ic+idx)^2\right)}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{b\sec(ic+idx)^2+a}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{ 4626}$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)\text{sech}(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)\text{sech}(c+dx)}{1-\cosh^2(c+dx)}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{\int \left(\frac{-a-b}{\cosh^2(c+dx)-1}+b\text{sech}(c+dx)\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\log\left(\cosh^2(c+dx)\right)-(a+b)\log\left(1-\cosh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/2*(b*Log[Cosh[c + d*x]^2] - (a + b)*Log[1 - Cosh[c + d*x]^2])/d
```

3.107.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.107.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{a\ln(\sinh(dx+c))+\ln(\tanh(dx+c))b}{d}$	24
default	$\frac{a\ln(\sinh(dx+c))+\ln(\tanh(dx+c))b}{d}$	24
risch	$-ax-\frac{2ac}{d}-\frac{b\ln(\mathrm{e}^{2dx+2c}+1)}{d}+\frac{\ln(\mathrm{e}^{2dx+2c}-1)a}{d}+\frac{\ln(\mathrm{e}^{2dx+2c}-1)b}{d}$	65

input
```
int(coth(d*x+c)*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sinh(d*x+c))+ln(tanh(d*x+c))*b)
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46

$$\int \coth(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx = -\frac{adx+b\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)-(a+b)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(a*d*x + b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - (a + b)
*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d
```

3.107.6 Sympy [F]

$$\int \coth(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx = \int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\coth\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*coth(c + d*x), x)
```

3.107.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.32

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}\right) + \frac{a\log\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c
) + 1)/d) + a*log(sinh(d*x + c))/d
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = -\frac{b\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-(a+b)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)}{2\,d}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(b*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (a + b)*log(e^(2*d*x
 + 2*c) + e^(-2*d*x - 2*c) - 2))/d
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.96

$$\int \coth(c+dx) \left(a+b\text{sech}^2(c+dx)\right) \, dx$$
$$= \frac{a \ln\left(4\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-4\,a^2-16\,b^2-16\,a\,b+16\,b^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+16\,a\,b\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}\right)}{2\,d}$$
$$-a\,x-\frac{\mathrm{atan}\left(\frac{a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2+4\,a\,b+4\,b^2}}+\frac{2\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2+4\,a\,b+4\,b^2}}\right)\,\sqrt{a^2+4\,a\,b+4\,b^2}}{\sqrt{-d^2}}$$

input
```
int(coth(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

output
```
(a*log(4*a^2*exp(4*c)*exp(4*d*x) - 4*a^2 - 16*b^2 - 16*a*b + 16*b^2*exp(4*
c)*exp(4*d*x) + 16*a*b*exp(4*c)*exp(4*d*x)))/(2*d) - a*x - (atan((a*exp(2*
c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(4*a*b + a^2 + 4*b^2)^(1/2)) + (2*b*exp(2*c
)*exp(2*d*x)*(-d^2)^(1/2))/(d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 +
 4*b^2)^(1/2))/(-d^2)^(1/2)
```

3.108 $\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$

3.108.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=ax-\frac{(a+b)\coth(c+dx)}{d}$$

output
```
a*x-(a+b)*coth(d*x+c)/d
```

3.108.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$
$$=-\frac{b\coth(c+dx)}{d}-\frac{a\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

output
```
-((b*Coth[c + d*x])/d) - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, 
Tanh[c + d*x]^2])/d
```

3.108.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 4629, 25, 2075, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{a+b\sec(ic+idx)^2}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{b\sec(ic+idx)^2+a}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \; 4629$$

$$-\frac{\int -\frac{\coth^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\coth^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\coth^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 359$$

$$-\frac{(a+b)\coth(c+dx)-a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{(a+b)\coth(c+dx)-a\text{arctanh}(\tanh(c+dx))}{d}$$

input
```
Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

output
```
-((-(a*ArcTanh[Tanh[c + d*x]]) + (a + b)*Coth[c + d*x])/d)
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.108.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

method	result	size
derivativedivides	$\frac{a(dx+c-\coth(dx+c))-b\coth(dx+c)}{d}$	30
default	$\frac{a(dx+c-\coth(dx+c))-b\coth(dx+c)}{d}$	30
risch	$ax-\frac{2a}{d(e^{2dx+2c}-1)}-\frac{2b}{d(e^{2dx+2c}-1)}$	43

input
```
int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(d*x+c-coth(d*x+c))-b*coth(d*x+c))
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = -\frac{(a+b)\cosh(dx+c)-(adx+a+b)\sinh(dx+c)}{d\sinh(dx+c)}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*cosh(d*x + c) - (a*d*x + a + b)*sinh(d*x + c))/(d*sinh(d*x + c))
```

3.108.6 Sympy [F]

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \int \left(a+b\,\text{sech}^2(c+dx)\right)\coth^2(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**2, x)
```

3.108.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(18) = 36$.

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.61

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = a\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+\frac{2\,b}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1)
)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{(dx+c)a-\frac{2\,(a+b)}{e^{(2\,dx+2\,c)}-1}}{d}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
((d*x + c)*a - 2*(a + b)/(e^(2*d*x + 2*c) - 1))/d
```

3.108.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = a\,x-\frac{2\,(a+b)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

output
```
a*x - (2*(a + b))/(d*(exp(2*c + 2*d*x) - 1))
```

3.109 $\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$

3.109.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=-\frac{(a+b)\text{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

output
```
-1/2*(a+b)*csch(d*x+c)^2/d+a*ln(sinh(d*x+c))/d
```

3.109.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$
$$=-\frac{b\text{csch}^2(c+dx)}{2d}-\frac{a\left(\coth^2(c+dx)-2\log(\cosh(c+dx))-2\log(\tanh(c+dx))\right)}{2d}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/2*(b*Csch[c + d*x]^2)/d - (a*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] -
2*Log[Tanh[c + d*x]]))/(2*d)
```

3.109.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a+b\sec(ic+idx)^2\right)}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{b\sec(ic+idx)^2+a}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{4626}$$

$$\frac{\int \frac{\cosh(c+dx)\left(a\cosh^2(c+dx)+b\right)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{a\cosh^2(c+dx)+b}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{a}{\cosh^2(c+dx)-1}+\frac{a+b}{\left(\cosh^2(c+dx)-1\right)^2}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a+b}{1-\cosh^2(c+dx)}+a\log\left(1-\cosh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + b)/(1 - Cosh[c + d*x]^2) + a*Log[1 - Cosh[c + d*x]^2])/(2*d)
```

3.109.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.109.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)-\frac{b}{2\sinh(dx+c)^2}}{d}$	37
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)-\frac{b}{2\sinh(dx+c)^2}}{d}$	37
risch	$-ax-\frac{2ac}{d}-\frac{2\,e^{2dx+2c}(a+b)}{d\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{2dx+2c}-1\right)a}{d}$	60

input
```
int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)-1/2*b/sinh(d*x+c)^2)
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 378, normalized size of antiderivative = 12.19

$$\int \coth^3(c+dx) \left(a+b\text{sech}^2(c+dx)\right) \, dx =$$
$$-\frac{adx\cosh\left(dx+c\right)^4+4\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+adx\sinh\left(dx+c\right)^4+adx-2\left(adx-a-b\right)\text{co}}{}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(a*d*x*cosh(d*x + c)^4 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*x*si
nh(d*x + c)^4 + a*d*x - 2*(a*d*x - a - b)*cosh(d*x + c)^2 + 2*(3*a*d*x*cos
h(d*x + c)^2 - a*d*x + a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*c
osh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2
*(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh
(d*x + c))*sinh(d*x + c) + a)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*
x + c))) + 4*(a*d*x*cosh(d*x + c)^3 - (a*d*x - a - b)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(
d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x +
c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

3.109.6 Sympy [F]

$$\int \coth^3(c+dx) \left(a+b\text{sech}^2(c+dx)\right) \, dx = \int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)\coth^3\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**3, x)
```

3.109.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(29) = 58$.

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.48

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$
$$= a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-\frac{2\,b}{d(e^{(dx+c)}-e^{(-dx-c)})^2}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d
*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*b/(d*(e^(d*
x + c) - e^(-d*x - c))^2)
```

3.109.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(29) = 58$.

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.71

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$
$$=-\frac{2\,(dx+c)a-2\,a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{3\,ae^{(4\,dx+4\,c)}-2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+3\,a}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a - 2*a*log(abs(e^(2*d*x + 2*c) - 1)) + (3*a*e^(4*d*x +
4*c) - 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + 3*a)/(e^(2*d*x + 2*c) -
 1)^2)/d
```

3.109.9 Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \coth^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx=\frac{a\ln\left(e^{2c}\,e^{2dx}-1\right)}{d}-a\,x-\frac{2\,(a+b)}{d\,\left(e^{2c+2dx}-1\right)}-\frac{2\,(a+b)}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}$$

input
```
int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

output
```
(a*log(exp(2*c)*exp(2*d*x) - 1))/d - a*x - (2*(a + b))/(d*(exp(2*c + 2*d*x
) - 1)) - (2*(a + b))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.110 $\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

3.110.1 Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=ax-\frac{a\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}$$

output
```
a*x-a*coth(d*x+c)/d-1/3*(a+b)*coth(d*x+c)^3/d
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$$
$$=-\frac{b\coth^3(c+dx)}{3d}-\frac{a\coth^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(c+dx)\right)}{3d}$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/3*(b*Coth[c + d*x]^3)/d - (a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1,
 -1/2, Tanh[c + d*x]^2])/(3*d)
```

3.110.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4629, 2075, 359, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(ic+idx)^2}{\tan(ic+idx)^4}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{359}$$

$$\frac{a\int \frac{\coth^2(c+dx)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{1}{3}(a+b)\coth^3(c+dx)}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{a\left(\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\coth(c+dx)\right)-\frac{1}{3}(a+b)\coth^3(c+dx)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{a(\text{arctanh}(\tanh(c+dx))-\coth(c+dx))-\frac{1}{3}(a+b)\coth^3(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

output
```
(a*(ArcTanh[Tanh[c + d*x]] - Coth[c + d*x]) - ((a + b)*Coth[c + d*x]^3)/3)
/d
```

3.110.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.110.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.88

method	result	size
risch	$ax-\frac{2\left(6a\,{\mathrm e}^{4dx+4c}+3b\,{\mathrm e}^{4dx+4c}-6\,{\mathrm e}^{2dx+2c}a+4a+b\right)}{3d\left({\mathrm e}^{2dx+2c}-1\right)^{3}}$	64
derivativedivides	$\frac{a\left(dx+c-\coth\left(dx+c\right)-\frac{\coth\left(dx+c\right)^{3}}{3}\right)+b\left(-\frac{\cosh\left(dx+c\right)}{2\sinh\left(dx+c\right)^{3}}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}\left(dx+c\right)^{2}}{3}\right)\coth\left(dx+c\right)}{2}\right)}{d}$	70
default	$\frac{a\left(dx+c-\coth\left(dx+c\right)-\frac{\coth\left(dx+c\right)^{3}}{3}\right)+b\left(-\frac{\cosh\left(dx+c\right)}{2\sinh\left(dx+c\right)^{3}}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}\left(dx+c\right)^{2}}{3}\right)\coth\left(dx+c\right)}{2}\right)}{d}$	70

input
```
int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
a*x-2/3*(6*a*exp(4*d*x+4*c)+3*b*exp(4*d*x+4*c)-6*exp(2*d*x+2*c)*a+4*a+b)/d
/(exp(2*d*x+2*c)-1)^3
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 4.12

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = -\frac{(4\,a+b)\cosh\left(dx+c\right)^{3}+3\,(4\,a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}-(3\,adx+4\,a+b)\sinh\left(dx+c\right)^{3}+3}{3\,\left(d\sinh\left(dx+c\right)^{3}+3\,\left(d\cosh\left(dx+c\right)\right.\right.}$$

input
```
integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/3*((4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^
2 - (3*a*d*x + 4*a + b)*sinh(d*x + c)^3 + 3*b*cosh(d*x + c) + 3*(3*a*d*x -
 (3*a*d*x + 4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c))/(d*sinh(d*x
 + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

3.110.6 Sympy [F]

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)\coth^4\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**4, x)
```

3.110.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(32) = 64$.

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.00

$$\begin{aligned}\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx\\ =\frac{1}{3}\,a\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)\\ +\frac{2}{3}\,b\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)\end{aligned}$$

input
```
integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3
*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 2/3*b*(
3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x
 - 6*c) - 1)) + 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) - 1)))
```

3.110.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{3\,(dx+c)a - \frac{2\left(6\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+4\,a+b\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 6*a*e^
(2*d*x + 2*c) + 4*a + b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.110.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 4.74

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = a\,x - \frac{\frac{2\,b}{3\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(2\,a+b)}{3\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1} - \frac{\frac{2\,(2\,a+b)}{3\,d}+\frac{4\,b\,e^{2\,c+2\,d\,x}}{3\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(2\,a+b)}{3\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1} - \frac{2\,(2\,a+b)}{3\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

output
```
a*x - ((2*b)/(3*d) + (2*exp(2*c + 2*d*x)*(2*a + b))/(3*d))/(exp(4*c + 4*d*
x) - 2*exp(2*c + 2*d*x) + 1) - ((2*(2*a + b))/(3*d) + (4*b*exp(2*c + 2*d*x
))/(3*d) + (2*exp(4*c + 4*d*x)*(2*a + b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*e
xp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (2*(2*a + b))/(3*d*(exp(2*c + 2*
d*x) - 1))
```

3.111 $\int \coth^5(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx$

3.111.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \coth^5(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx=-\frac{(2a+b)\mathrm{csch}^2(c+dx)}{2d}-\frac{(a+b)\mathrm{csch}^4(c+dx)}{4d}+\frac{a\log(\sinh(c+dx))}{d}$$

output
```
-1/2*(2*a+b)*csch(d*x+c)^2/d-1/4*(a+b)*csch(d*x+c)^4/d+a*ln(sinh(d*x+c))/d
```

3.111.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \coth^5(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx=-\frac{b\coth^4(c+dx)}{4d}-\frac{a\left(2\coth^2(c+dx)+\coth^4(c+dx)-4\log(\cosh(c+dx))-4\log(\tanh(c+dx))\right)}{4d}$$

input
```
Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/4*(b*Coth[c + d*x]^4)/d - (a*(2*Coth[c + d*x]^2 + Coth[c + d*x]^4 - 4*L
og[Cosh[c + d*x]] - 4*Log[Tanh[c + d*x]]))/(4*d)
```

3.111.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{i\left(a+b\sec(ic+idx)^2\right)}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \textbf{26}$$

$$i\int \frac{b\sec(ic+idx)^2+a}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \textbf{4626}$$

$$-\frac{\int \frac{\cosh^3(c+dx)\left(a\cosh^2(c+dx)+b\right)}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \textbf{354}$$

$$-\frac{\int \frac{\cosh^2(c+dx)\left(a\cosh^2(c+dx)+b\right)}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \textbf{86}$$

$$-\frac{\int \left(-\frac{a}{\cosh^2(c+dx)-1}+\frac{-2a-b}{\left(\cosh^2(c+dx)-1\right)^2}+\frac{-a-b}{\left(\cosh^2(c+dx)-1\right)^3}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\frac{a+b}{2\left(1-\cosh^2(c+dx)\right)^2}-\frac{2a+b}{1-\cosh^2(c+dx)}-a\log\left(1-\cosh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/2*((a + b)/(2*(1 - Cosh[c + d*x]^2)^2) - (2*a + b)/(1 - Cosh[c + d*x]^2
) - a*Log[1 - Cosh[c + d*x]^2])/d
```

3.111.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.111.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

method	result	size
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+b\left(-\frac{\cosh(dx+c)^2}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)}{d}$	67
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+b\left(-\frac{\cosh(dx+c)^2}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)}{d}$	67
risch	$-ax-\frac{2ac}{d}-\frac{2\,e^{2dx+2c}\left(2a\,e^{4dx+4c}+b\,e^{4dx+4c}-2\,e^{2dx+2c}a+2a+b\right)}{d\left(e^{2dx+2c}-1\right)^4}+\frac{\ln\left(e^{2dx+2c}-1\right)a}{d}$	97

input
```
int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+b*(-1/2/sinh(
d*x+c)^4*cosh(d*x+c)^2+1/4/sinh(d*x+c)^4))
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 1099, normalized size of antiderivative = 21.55

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(a*d*x*cosh(d*x + c)^8 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a*d*x*si
nh(d*x + c)^8 - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^6 + 2*(14*a*d*x*cosh(d
*x + c)^2 - 2*a*d*x + 2*a + b)*sinh(d*x + c)^6 + 4*(14*a*d*x*cosh(d*x + c)
^3 - 3*(2*a*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a*d*x - 2
*a)*cosh(d*x + c)^4 + 2*(35*a*d*x*cosh(d*x + c)^4 + 3*a*d*x - 15*(2*a*d*x
- 2*a - b)*cosh(d*x + c)^2 - 2*a)*sinh(d*x + c)^4 + 8*(7*a*d*x*cosh(d*x +
c)^5 - 5*(2*a*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*a*d*x - 2*a)*cosh(d*x +
c))*sinh(d*x + c)^3 + a*d*x - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(1
4*a*d*x*cosh(d*x + c)^6 - 15*(2*a*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*a*d*x
 + 6*(3*a*d*x - 2*a)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - (a*cosh(
d*x + c)^8 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 4*a*c
osh(d*x + c)^6 + 4*(7*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^6 + 8*(7*a*cosh
(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^5 + 6*a*cosh(d*x + c)^4 + 2
*(35*a*cosh(d*x + c)^4 - 30*a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c)^4 + 8*(
7*a*cosh(d*x + c)^5 - 10*a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c))*sinh(d*x +
 c)^3 - 4*a*cosh(d*x + c)^2 + 4*(7*a*cosh(d*x + c)^6 - 15*a*cosh(d*x + c)^
4 + 9*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - 3*a*
cosh(d*x + c)^5 + 3*a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d*x + c) + a
)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a*d*x*cosh(d
*x + c)^7 - 3*(2*a*d*x - 2*a - b)*cosh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*c...
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2),x)
```

output
```
Timed out
```

3.111.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(47) = 94$.

Time = 0.19 (sec) , antiderivative size = 251, normalized size of antiderivative = 4.92

$$\int \coth^5(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx$$
$$= a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx}\cdots}\right.$$
$$+2\,b\left(\frac{e^{(-2\,dx-2\,c)}}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}+\frac{e^{(-6\,dx-6}\cdots}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e\cdots}\right.$$

input
```
integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*
d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*b*(
e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4*e^(-2*d*x - 2*c)
 - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)))
```

3.111.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(47) = 94$.

Time = 0.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.35

$$\int \coth^5(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx =$$
$$-\frac{12\,(dx+c)a-12\,a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{25\,ae^{(8\,dx+8\,c)}-52\,ae^{(6\,dx+6\,c)}+24\,be^{(6\,dx+6\,c)}+102\,ae^{(4\,dx+4\,c)}-52\,ae^{(2\,dx+2\,c)}+2\cdots}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{12\,d}$$

input
```
integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/12*(12*(d*x + c)*a - 12*a*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a*e^(8*d*
x + 8*c) - 52*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x +
4*c) - 52*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c
) - 1)^4)/d
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.51

$$\int \coth^5(c+dx) \left(a+b\text{sech}^2(c+dx)\right) \, dx$$
$$= \frac{a \ln \left(e^{2c} e^{2dx}-1\right)}{d} - \frac{8\,(a+b)}{d\,\left(3\,e^{2c+2dx}-3\,e^{4c+4dx}+e^{6c+6dx}-1\right)} - \frac{2\,(2\,a+b)}{d\,\left(e^{2c+2dx}-1\right)}$$
$$- \frac{4\,(a+b)}{d\,\left(6\,e^{4c+4dx}-4\,e^{2c+2dx}-4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$- a\,x - \frac{2\,(4\,a+3\,b)}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}$$

input
```
int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2),x)
```

output
```
(a*log(exp(2*c)*exp(2*d*x) - 1))/d - (8*(a + b))/(d*(3*exp(2*c + 2*d*x) -
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (2*(2*a + b))/(d*(exp(2*c +
2*d*x) - 1)) - (4*(a + b))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4
*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - a*x - (2*(4*a + 3*b))/(d*(exp
(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.112 $\int (a+b\text{sech}^2(c+dx))^2 \tanh^4(c+dx)\,dx$

3.112.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int (a+b\text{sech}^2(c+dx))^2 \tanh^4(c+dx)\,dx = a^2x-\frac{a^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

output
```
a^2*x-a^2*tanh(d*x+c)/d-1/3*a^2*tanh(d*x+c)^3/d+1/5*b*(2*a+b)*tanh(d*x+c)^
5/d-1/7*b^2*tanh(d*x+c)^7/d
```

3.112.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 395 vs. 2(77) = 154.

Time = 2.32 (sec) , antiderivative size = 395, normalized size of antiderivative = 5.13

$$\int (a+b\text{sech}^2(c+dx))^2 \tanh^4(c+dx)\,dx = \frac{\text{sech}(c)\text{sech}^7(c+dx)\,(3675a^2dx\cosh(dx)+3675a^2dx\cosh(2c+dx)+2205a^2dx\cosh(2c+3dx)+2205a^2}{}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]
```

output
```
(Sech[c]*Sech[c + d*x]^7*(3675*a^2*d*x*Cosh[d*x] + 3675*a^2*d*x*Cosh[2*c +
 d*x] + 2205*a^2*d*x*Cosh[2*c + 3*d*x] + 2205*a^2*d*x*Cosh[4*c + 3*d*x] +
735*a^2*d*x*Cosh[4*c + 5*d*x] + 735*a^2*d*x*Cosh[6*c + 5*d*x] + 105*a^2*d*
x*Cosh[6*c + 7*d*x] + 105*a^2*d*x*Cosh[8*c + 7*d*x] - 5320*a^2*Sinh[d*x] +
 1680*a*b*Sinh[d*x] + 840*b^2*Sinh[d*x] + 4480*a^2*Sinh[2*c + d*x] - 1260*
a*b*Sinh[2*c + d*x] + 420*b^2*Sinh[2*c + d*x] - 3780*a^2*Sinh[2*c + 3*d*x]
 + 924*a*b*Sinh[2*c + 3*d*x] - 168*b^2*Sinh[2*c + 3*d*x] + 2100*a^2*Sinh[4
*c + 3*d*x] - 840*a*b*Sinh[4*c + 3*d*x] - 420*b^2*Sinh[4*c + 3*d*x] - 1540
*a^2*Sinh[4*c + 5*d*x] + 168*a*b*Sinh[4*c + 5*d*x] + 84*b^2*Sinh[4*c + 5*d
*x] + 420*a^2*Sinh[6*c + 5*d*x] - 420*a*b*Sinh[6*c + 5*d*x] - 280*a^2*Sinh
[6*c + 7*d*x] + 84*a*b*Sinh[6*c + 7*d*x] + 12*b^2*Sinh[6*c + 7*d*x]))/(134
40*d)
```

3.112.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(-b^2\tanh^6(c+dx)+b(2a+b)\tanh^4(c+dx)-a^2\tanh^2(c+dx)-a^2+\frac{a^2}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\text{arctanh}(\tanh(c+dx)) - \frac{1}{3}a^2\tanh^3(c+dx) - a^2\tanh(c+dx) + \frac{1}{5}b(2a+b)\tanh^5(c+dx) - \frac{1}{7}b^2\tanh^7(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]
```

output
```
(a^2*ArcTanh[Tanh[c + d*x]] - a^2*Tanh[c + d*x] - (a^2*Tanh[c + d*x]^3)/3
+ (b*(2*a + b)*Tanh[c + d*x]^5)/5 - (b^2*Tanh[c + d*x]^7)/7)/d
```

3.112.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.112.4 Maple [A] (verified)

Time = 31.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

method	result
parts	$\frac{a^2\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^7}{7}+\frac{\tanh(dx+c)^5}{5}\right)}{d}+\frac{2ab\,\mathrm{t}\cdots}{\cdots}$
derivativedivides	$\frac{a^2\left(dx+c-\tanh(dx+c)-\frac{\tanh(dx+c)^3}{3}\right)+2ab\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(\cdots}{8}\cdots\right.}{d}$
default	$\frac{a^2\left(dx+c-\tanh(dx+c)-\frac{\tanh(dx+c)^3}{3}\right)+2ab\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(\cdots}{8}\cdots\right.}{d}$
risch	$a^2x+\frac{4a^2e^{12dx+12c}-4ab\,e^{12dx+12c}+20a^2e^{10dx+10c}-8ab\,e^{10dx+10c}-4b^2e^{10dx+10c}+\frac{128a^2e^{8dx+8c}}{3}-12ab\,e^{8dx+8c}+4\cdots}{\cdots}$

input
```
int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(-1/3*tanh(d*x+c)^3-tanh(d*x+c)-1/2*ln(tanh(d*x+c)-1)+1/2*ln(tanh(d*
x+c)+1))+b^2/d*(-1/7*tanh(d*x+c)^7+1/5*tanh(d*x+c)^5)+2/5*a*b/d*tanh(d*x+c
)^5
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 721, normalized size of antiderivative = 9.36

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^2\tanh^4(c+dx)\,dx$$
$$=\frac{(105\,a^2dx+140\,a^2-42\,ab-6\,b^2)\cosh(dx+c)^7+7\,(105\,a^2dx+140\,a^2-42\,ab-6\,b^2)\cosh(dx+c)\sinh\cdots}{\cdots}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/105*((105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^7 + 7*(105*a
^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 - 2*(70*a
^2 - 21*a*b - 3*b^2)*sinh(d*x + c)^7 + 7*(105*a^2*d*x + 140*a^2 - 42*a*b -
 6*b^2)*cosh(d*x + c)^5 - 14*(3*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^2
+ 40*a^2 + 9*a*b - 3*b^2)*sinh(d*x + c)^5 + 35*((105*a^2*d*x + 140*a^2 - 4
2*a*b - 6*b^2)*cosh(d*x + c)^3 + (105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*
cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^
2)*cosh(d*x + c)^3 - 14*(5*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^4 + 10*
(40*a^2 + 9*a*b - 3*b^2)*cosh(d*x + c)^2 + 60*a^2 - 3*a*b + 21*b^2)*sinh(d
*x + c)^3 + 7*(3*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^5
+ 10*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^3 + 9*(105*a^2
*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*
a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c) - 14*((70*a^2 - 21*a*b -
 3*b^2)*cosh(d*x + c)^6 + 5*(40*a^2 + 9*a*b - 3*b^2)*cosh(d*x + c)^4 + 9*(
20*a^2 - a*b + 7*b^2)*cosh(d*x + c)^2 + 30*a^2 - 15*a*b - 45*b^2)*sinh(d*x
 + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d
*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*
d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*co
sh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))
```

3.112.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^4(c+dx)\,dx=\int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)^2\tanh^4\left(c+dx\right)dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**4,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**4, x)
```

3.112.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(71)=142$.

Time = 0.19 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.43

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^4(c+dx)\,dx$$
$$=\frac{2\,ab\tanh\left(dx+c\right)^5}{5\,d}+\frac{1}{3}\,a^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+\frac{4}{35}\,b^2\left(\frac{7\,e^{(-2\,dx-2\,c)}}{d(7\,e^{(-2\,dx-2\,c)}+21\,e^{(-4\,dx-4\,c)}+35\,e^{(-6\,dx-6\,c)}+35\,e^{(-8\,dx-8\,c)}+21\,e^{(-10\,dx-10\,c)}+7\,e^{(-12\,dx-12\,c)}\right.$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="maxima")
```

output
```
2/5*a*b*tanh(d*x + c)^5/d + 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c)
 + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^
(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^
(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x
 - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*
c) + e^(-14*d*x - 14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c)
 + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^
(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^
(-8*d*x - 8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x
 - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*
c) + e^(-14*d*x - 14*c) + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*
c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*
e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(
d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(
-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x
- 14*c) + 1)))
```

3.112.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(71) = 142.

Time = 0.35 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.61

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^4(c+dx)\,dx$$
$$=\frac{105\,(dx+c)a^2+\frac{4\left(105\,a^2e^{(12\,dx+12\,c)}-105\,abe^{(12\,dx+12\,c)}+525\,a^2e^{(10\,dx+10\,c)}-210\,abe^{(10\,dx+10\,c)}-105\,b^2e^{(10\,dx+10\,c)}+1120\,a^2e^{(8\,dx+8}\right.}{}}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="giac")
```

output
```
1/105*(105*(d*x + c)*a^2 + 4*(105*a^2*e^(12*d*x + 12*c) - 105*a*b*e^(12*d*
x + 12*c) + 525*a^2*e^(10*d*x + 10*c) - 210*a*b*e^(10*d*x + 10*c) - 105*b^
2*e^(10*d*x + 10*c) + 1120*a^2*e^(8*d*x + 8*c) - 315*a*b*e^(8*d*x + 8*c) +
 105*b^2*e^(8*d*x + 8*c) + 1330*a^2*e^(6*d*x + 6*c) - 420*a*b*e^(6*d*x + 6
*c) - 210*b^2*e^(6*d*x + 6*c) + 945*a^2*e^(4*d*x + 4*c) - 231*a*b*e^(4*d*x
 + 4*c) + 42*b^2*e^(4*d*x + 4*c) + 385*a^2*e^(2*d*x + 2*c) - 42*a*b*e^(2*d
*x + 2*c) - 21*b^2*e^(2*d*x + 2*c) + 70*a^2 - 21*a*b - 3*b^2)/(e^(2*d*x +
2*c) + 1)^7)/d
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1022, normalized size of antiderivative = 13.27

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^2\tanh^4(c+dx)\,dx$$
$$=\frac{\frac{4\,(7\,a^2+a\,b+8\,b^2)}{105\,d}-\frac{4\,\mathrm{e}^{8\,c+8\,d\,x}\,(a\,b-a^2)}{7\,d}-\frac{16\,\mathrm{e}^{2\,c+2\,d\,x}\,(-2\,a^2+a\,b+3\,b^2)}{35\,d}+\frac{16\,\mathrm{e}^{6\,c+6\,d\,x}\,(2\,a^2+a\,b-b^2)}{21\,d}+\frac{8\,\mathrm{e}^{4\,c+4\,d\,x}\,(7\,a^2+a\,b+8\,b^2)}{35\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1}$$
$$-\frac{\frac{4\,(-2\,a^2+a\,b+3\,b^2)}{35\,d}+\frac{4\,\mathrm{e}^{6\,c+6\,d\,x}\,(a\,b-a^2)}{7\,d}-\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(2\,a^2+a\,b-b^2)}{7\,d}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(7\,a^2+a\,b+8\,b^2)}{35\,d}}{4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}$$
$$+a^2\,x+\frac{\frac{4\,(7\,a^2+a\,b+8\,b^2)}{105\,d}-\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(a\,b-a^2)}{7\,d}+\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a^2+a\,b-b^2)}{21\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}$$
$$+\frac{\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a^2+a\,b-b^2)}{7\,d}-\frac{4\,\mathrm{e}^{12\,c+12\,d\,x}\,(a\,b-a^2)}{7\,d}-\frac{4\,(a\,b-a^2)}{7\,d}-\frac{16\,\mathrm{e}^{6\,c+6\,d\,x}\,(-2\,a^2+a\,b+3\,b^2)}{7\,d}+\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(7\,a^2+a\,b+8\,b^2)}{7\,d}+8\,\mathrm{e}\cdots}{7\,\mathrm{e}^{2\,c+2\,d\,x}+21\,\mathrm{e}^{4\,c+4\,d\,x}+35\,\mathrm{e}^{6\,c+6\,d\,x}+35\,\mathrm{e}^{8\,c+8\,d\,x}+21\,\mathrm{e}^{10\,c+10\,d\,x}+7\,\mathrm{e}^{12\,c+12\,d\,x}\cdots}$$
$$+\frac{\frac{4\,(2\,a^2+a\,b-b^2)}{21\,d}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a\,b-a^2)}{7\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}$$
$$+\frac{\frac{4\,(2\,a^2+a\,b-b^2)}{21\,d}-\frac{4\,\mathrm{e}^{10\,c+10\,d\,x}\,(a\,b-a^2)}{7\,d}-\frac{8\,\mathrm{e}^{4\,c+4\,d\,x}\,(-2\,a^2+a\,b+3\,b^2)}{7\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(7\,a^2+a\,b+8\,b^2)}{21\,d}+\frac{20\,\mathrm{e}^{8\,c+8\,d\,x}\,(2\,a^2+a\,b-b^2)}{21\,d}+\cdots}{6\,\mathrm{e}^{2\,c+2\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}+6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1}$$
$$-\frac{4\,(a\,b-a^2)}{7\,d\,(\mathrm{e}^{2\,c+2\,d\,x}+1)}$$

input
```
int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
((4*(a*b + 7*a^2 + 8*b^2))/(105*d) - (4*exp(8*c + 8*d*x)*(a*b - a^2))/(7*d
) - (16*exp(2*c + 2*d*x)*(a*b - 2*a^2 + 3*b^2))/(35*d) + (16*exp(6*c + 6*d
*x)*(a*b + 2*a^2 - b^2))/(21*d) + (8*exp(4*c + 4*d*x)*(a*b + 7*a^2 + 8*b^2
))/(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x)
 + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((4*(a*b - 2*a^2 + 3*b^2
))/(35*d) + (4*exp(6*c + 6*d*x)*(a*b - a^2))/(7*d) - (4*exp(4*c + 4*d*x)*(
a*b + 2*a^2 - b^2))/(7*d) - (4*exp(2*c + 2*d*x)*(a*b + 7*a^2 + 8*b^2))/(35
*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1) + a^2*x + ((4*(a*b + 7*a^2 + 8*b^2))/(105*d) - (4*exp(4*c
 + 4*d*x)*(a*b - a^2))/(7*d) + (8*exp(2*c + 2*d*x)*(a*b + 2*a^2 - b^2))/(2
1*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (
(8*exp(2*c + 2*d*x)*(a*b + 2*a^2 - b^2))/(7*d) - (4*exp(12*c + 12*d*x)*(a*
b - a^2))/(7*d) - (4*(a*b - a^2))/(7*d) - (16*exp(6*c + 6*d*x)*(a*b - 2*a^
2 + 3*b^2))/(7*d) + (4*exp(4*c + 4*d*x)*(a*b + 7*a^2 + 8*b^2))/(7*d) + (8*
exp(10*c + 10*d*x)*(a*b + 2*a^2 - b^2))/(7*d) + (4*exp(8*c + 8*d*x)*(a*b +
 7*a^2 + 8*b^2))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp
(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c +
 12*d*x) + exp(14*c + 14*d*x) + 1) + ((4*(a*b + 2*a^2 - b^2))/(21*d) - (4*
exp(2*c + 2*d*x)*(a*b - a^2))/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x
) + 1) + ((4*(a*b + 2*a^2 - b^2))/(21*d) - (4*exp(10*c + 10*d*x)*(a*b -...
```

3.112. $\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \tanh^4(c+dx)\,dx$

3.113 $\int (a+b\text{sech}^2(c+dx))^2 \tanh^3(c+dx)\,dx$

3.113.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int (a+b\text{sech}^2(c+dx))^2 \tanh^3(c+dx)\,dx = \frac{a^2\log(\cosh(c+dx))}{d} + \frac{a(a-2b)\text{sech}^2(c+dx)}{2d} + \frac{(2a-b)b\text{sech}^4(c+dx)}{4d} + \frac{b^2\text{sech}^6(c+dx)}{6d}$$

output
```
a^2*ln(cosh(d*x+c))/d+1/2*a*(a-2*b)*sech(d*x+c)^2/d+1/4*(2*a-b)*b*sech(d*x
+c)^4/d+1/6*b^2*sech(d*x+c)^6/d
```

3.113.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int (a+b\text{sech}^2(c+dx))^2 \tanh^3(c+dx)\,dx = \frac{\cosh^4(c+dx)\,(a+b\text{sech}^2(c+dx))^2\,(12a^2\log(\cosh(c+dx))+6a(a-2b)\text{sech}^2(c+dx)+3(2a-b)b\text{sech}}{3d(a+2b+a\cosh(2c+2dx))^2}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]
```

output
```
(Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2*(12*a^2*Log[Cosh[c + d*x]] + 6*
a*(a - 2*b)*Sech[c + d*x]^2 + 3*(2*a - b)*b*Sech[c + d*x]^4 + 2*b^2*Sech[c
 + d*x]^6))/(3*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^2)
```

3.113.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \left(b\sec(ic+idx)^2+a\right)^2\tan(ic+idx)^3dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2\text{sech}^7(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2\text{sech}^4(c+dx)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{85}$$

$$-\frac{\int \left(b^2\text{sech}^4(c+dx)+(2a-b)b\text{sech}^3(c+dx)+a(a-2b)\text{sech}^2(c+dx)-a^2\text{sech}(c+dx)\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^2\log\left(\cosh^2(c+dx)\right)-\frac{1}{2}b(2a-b)\text{sech}^2(c+dx)-a(a-2b)\text{sech}(c+dx)-\frac{1}{3}b^2\text{sech}^3(c+dx)}{2d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]
```

output
```
-1/2*(-(a^2*Log[Cosh[c + d*x]^2]) - a*(a - 2*b)*Sech[c + d*x] - ((2*a - b)
*b*Sech[c + d*x]^2)/2 - (b^2*Sech[c + d*x]^3)/3)/d
```

3.113.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.113. $\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh^3(c+dx)\,dx$

3.113.4 Maple [A] (verified)

Time = 20.83 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\operatorname{sech}(dx+c)^6 b^2}{6}+\frac{\operatorname{sech}(dx+c)^4 ab}{2}-\frac{\operatorname{sech}(dx+c)^4 b^2}{4}+\frac{\operatorname{sech}(dx+c)^2 a^2}{2}-\operatorname{sech}(dx+c)^2 ab-a^2\ln(\operatorname{sech}(dx+c))}{d}$
default	$\frac{\frac{\operatorname{sech}(dx+c)^6 b^2}{6}+\frac{\operatorname{sech}(dx+c)^4 ab}{2}-\frac{\operatorname{sech}(dx+c)^4 b^2}{4}+\frac{\operatorname{sech}(dx+c)^2 a^2}{2}-\operatorname{sech}(dx+c)^2 ab-a^2\ln(\operatorname{sech}(dx+c))}{d}$
parts	$\frac{a^2\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^6}{6}+\frac{\tanh(dx+c)^4}{4}\right)}{d}+\frac{ab\tanh(dx+c)^4}{2d}$
risch	$-a^2x-\frac{2a^2c}{d}+\frac{2\,\mathrm{e}^{2dx+2c}\left(3a^2\,\mathrm{e}^{8dx+8c}-6ab\,\mathrm{e}^{8dx+8c}+12a^2\,\mathrm{e}^{6dx+6c}-12ab\,\mathrm{e}^{6dx+6c}-6b^2\,\mathrm{e}^{6dx+6c}+18a^2\,\mathrm{e}^{4dx+4c}-12a\right.}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^6}$

input
```
int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/6*sech(d*x+c)^6*b^2+1/2*sech(d*x+c)^4*a*b-1/4*sech(d*x+c)^4*b^2+1/2
*sech(d*x+c)^2*a^2-sech(d*x+c)^2*a*b-a^2*ln(sech(d*x+c)))
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2591 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 2591, normalized size of antiderivative = 33.65

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\tanh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^
11 + 3*a^2*d*x*sinh(d*x + c)^12 + 6*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c
)^10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 + 3*a^2*d*x - a^2 + 2*a*b)*sinh(d*x +
 c)^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 + (3*a^2*d*x - a^2 + 2*a*b)*cosh(d
*x + c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x
 + c)^8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x + 90*(3*a^2*d*x - a^
2 + 2*a*b)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^8 + 24*(
99*a^2*d*x*cosh(d*x + c)^5 + 30*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^3
+ (15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*
(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh
(d*x + c)^6 + 315*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^4 + 15*a^2*d*x +
 21*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 - 9*a^2 + 6*a*b -
 2*b^2)*sinh(d*x + c)^6 + 24*(99*a^2*d*x*cosh(d*x + c)^7 + 63*(3*a^2*d*x -
 a^2 + 2*a*b)*cosh(d*x + c)^5 + 7*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cos
h(d*x + c)^3 + (15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(495
*a^2*d*x*cosh(d*x + c)^8 + 420*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^6 +
 70*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 15*a^2*d*x + 20
*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*
b^2)*sinh(d*x + c)^4 + 3*a^2*d*x + 4*(165*a^2*d*x*cosh(d*x + c)^9 + 180...
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^3(c+dx)\,dx$$
$$=\begin{cases} a^2x-\frac{a^2\log\left(\tanh\left(c+dx\right)+1\right)}{d}-\frac{a^2\tanh^2\left(c+dx\right)}{2d}-\frac{ab\tanh^2\left(c+dx\right)\operatorname{sech}^2\left(c+dx\right)}{2d}-\frac{ab\operatorname{sech}^2\left(c+dx\right)}{2d}-\frac{b^2\tanh^2\left(c+dx\right)\operatorname{sech}^4\left(c+dx\right)}{6d} \\ x\left(a+b\operatorname{sech}^2\left(c\right)\right)^2\tanh^3\left(c\right) \end{cases}$$

input
```
integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**3,x)
```

output
```
Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/
(2*d) - a*b*tanh(c + d*x)**2*sech(c + d*x)**2/(2*d) - a*b*sech(c + d*x)**2
/(2*d) - b**2*tanh(c + d*x)**2*sech(c + d*x)**4/(6*d) - b**2*sech(c + d*x)
**4/(12*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c)**3, True))
```

3.113.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.32

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^3(c+dx)\,dx$$
$$= \frac{ab\tanh\left(dx+c\right)^4}{2\,d}+a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$-\frac{4}{3}b^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right.$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*a*b*tanh(d*x + c)^4/d + a^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2
*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4/3*b
^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e
^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x
- 12*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x -
 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) +
 e^(-12*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) + 15
*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d
*x - 10*c) + e^(-12*d*x - 12*c) + 1)))
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(71) = 142$.

Time = 0.34 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.17

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^3(c+dx)\,dx =$$
$$-\frac{60\,(dx+c)a^2-60\,a^2\log\left(e^{(2\,dx+2\,c)}+1\right)+\frac{147\,a^2e^{(12\,dx+12\,c)}+762\,a^2e^{(10\,dx+10\,c)}+240\,abe^{(10\,dx+10\,c)}+1725\,a^2e^{(8\,dx+8\,c)}+4}{}}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/60*(60*(d*x + c)*a^2 - 60*a^2*log(e^(2*d*x + 2*c) + 1) + (147*a^2*e^(12
*d*x + 12*c) + 762*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c) + 172
5*a^2*e^(8*d*x + 8*c) + 480*a*b*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c)
+ 2220*a^2*e^(6*d*x + 6*c) + 480*a*b*e^(6*d*x + 6*c) - 160*b^2*e^(6*d*x +
6*c) + 1725*a^2*e^(4*d*x + 4*c) + 480*a*b*e^(4*d*x + 4*c) + 240*b^2*e^(4*d
*x + 4*c) + 762*a^2*e^(2*d*x + 2*c) + 240*a*b*e^(2*d*x + 2*c) + 147*a^2)/(
e^(2*d*x + 2*c) + 1)^6)/d
```

3.113.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 349, normalized size of antiderivative = 4.53

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^3(c+dx)\,dx$$
$$= \frac{4\,(2\,a\,b-9\,b^2)}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)} - \frac{32\,b^2}{3\,d\,\left(6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)} - \frac{2\,(a^2-6\,a\,b+2\,b^2)}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} - \frac{2\,(2\,a\,b-a^2)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{8\,(6\,a\,b-7\,b^2)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)} - a^2\,x + \frac{a^2\,\ln\left(e^{2\,c}\,e^{2\,d\,x}+1\right)}{d} + \frac{32\,b^2}{d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$

input
```
int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(4*(2*a*b - 9*b^2))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*
c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2)/(3*d*(6*exp(2*c + 2*d*x) +
15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10
*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (2*(a^2 - 6*a*b + 2*b^2))/(d*(2*
exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*(2*a*b - a^2))/(d*(exp(2*c
+ 2*d*x) + 1)) - (8*(6*a*b - 7*b^2))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c
+ 4*d*x) + exp(6*c + 6*d*x) + 1)) - a^2*x + (a^2*log(exp(2*c)*exp(2*d*x) +
 1))/d + (32*b^2)/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*
c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1))
```

3.114 $\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh^2(c+dx)\,dx$

3.114.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh^2(c+dx)\,dx = a^2x-\frac{a^2\tanh(c+dx)}{d}+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
a^2*x-a^2*tanh(d*x+c)/d+1/3*b*(2*a+b)*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^
5/d
```

3.114.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(59) = 118.

Time = 1.84 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.76

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh^2(c+dx)\,dx$$
$$=\frac{\text{sech}(c)\text{sech}^5(c+dx)\left(150a^2dx\cosh(dx)+150a^2dx\cosh(2c+dx)+75a^2dx\cosh(2c+3dx)+75a^2dx\cosh\right.}{}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]
```

output
```
(Sech[c]*Sech[c + d*x]^5*(150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c + d
*x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] + 75*a^2*d*x*Cosh[4*c + 3*d*x] + 15*a^2
*d*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] - 180*a^2*Sinh[d*x]
+ 80*a*b*Sinh[d*x] - 20*b^2*Sinh[d*x] + 120*a^2*Sinh[2*c + d*x] - 120*a*b*
Sinh[2*c + d*x] - 60*b^2*Sinh[2*c + d*x] - 120*a^2*Sinh[2*c + 3*d*x] + 40*
a*b*Sinh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] + 30*a^2*Sinh[4*c + 3*d*x
] - 60*a*b*Sinh[4*c + 3*d*x] - 30*a^2*Sinh[4*c + 5*d*x] + 20*a*b*Sinh[4*c
+ 5*d*x] + 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)
```

3.114.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 4629, 25, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ic+idx)^2\left(-\left(a+b\sec(ic+idx)^2\right)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(b\sec(ic+idx)^2+a\right)^2\tan(ic+idx)^2dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\tanh^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(-b^2\tanh^4(c+dx)+b(2a+b)\tanh^2(c+dx)-a^2+\frac{a^2}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^2\text{arctanh}(\tanh(c+dx))+a^2\tanh(c+dx)-\frac{1}{3}b(2a+b)\tanh^3(c+dx)+\frac{1}{5}b^2\tanh^5(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]
```

output
```
-((-(a^2*ArcTanh[Tanh[c + d*x]]) + a^2*Tanh[c + d*x] - (b*(2*a + b)*Tanh[c
 + d*x]^3)/3 + (b^2*Tanh[c + d*x]^5)/5)/d)
```

3.114.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.114.4 Maple [A] (verified)

Time = 14.88 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

method	result
parts	$\frac{a^2\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^5}{5}+\frac{\tanh(dx+c)^3}{3}\right)}{d}+\frac{2ab\tanh(dx+c)^3}{3d}$
derivativedivides	$\frac{a^2(dx+c-\tanh(dx+c))+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)}{5}\right.}{}\right.}{d}$
default	$\frac{a^2(dx+c-\tanh(dx+c))+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)}{5}\right.}{}\right.}{d}$
risch	$a^2x+\frac{2a^2e^{8dx+8c}-4ab\,e^{8dx+8c}+8a^2e^{6dx+6c}-8ab\,e^{6dx+6c}-4b^2e^{6dx+6c}+12a^2e^{4dx+4c}-\frac{16ab\,e^{4dx+4c}}{3}+\frac{4\,e^{4dx+4c}b^2}{3}+}{d\left(e^{2dx+2c}+1\right)^5}$

input
```
int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(-tanh(d*x+c)-1/2*ln(tanh(d*x+c)-1)+1/2*ln(tanh(d*x+c)+1))+b^2/d*(-1
/5*tanh(d*x+c)^5+1/3*tanh(d*x+c)^3)+2/3*a*b*tanh(d*x+c)^3/d
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(55) = 110$.

Time = 0.27 (sec) , antiderivative size = 435, normalized size of antiderivative = 7.37

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\tanh^2(c+dx)\,dx$$
$$=\frac{(15\,a^2dx+15\,a^2-10\,ab-2\,b^2)\cosh(dx+c)^5+5\,(15\,a^2dx+15\,a^2-10\,ab-2\,b^2)\cosh(dx+c)\sinh(dx}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/15*((15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(15*a^2*d
*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2 - 10
*a*b - 2*b^2)*sinh(d*x + c)^5 + 5*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*c
osh(d*x + c)^3 - 5*(2*(15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^2 + 9*a^2 +
2*a*b - 2*b^2)*sinh(d*x + c)^3 + 5*(2*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^
2)*cosh(d*x + c)^3 + 3*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c
))*sinh(d*x + c)^2 + 10*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x +
c) - 5*((15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^4 + 3*(9*a^2 + 2*a*b - 2*b
^2)*cosh(d*x + c)^2 + 6*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x +
c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*co
sh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

3.114.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^2(c+dx)\,dx=\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2\tanh^2\left(c+dx\right)dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**2, x)
```

3.114.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(55) = 110$.

Time = 0.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 5.51

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^2(c+dx)\,dx$$
$$=\frac{2\,ab\tanh\left(dx+c\right)^3}{3\,d}+a^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$
$$+\frac{4}{15}b^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{}{d(5\,e^{(-2\,dx-2\,c}}\right.$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="maxima")
```

output
```
2/3*a*b*tanh(d*x + c)^3/d + a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) +
 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5
*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x
- 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c)
+ 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c)
 + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10
*d*x - 10*c) + 1)))
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(55) = 110$.

Time = 0.33 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.32

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\tanh^2(c+dx)\,dx$$
$$=\frac{15\,(dx+c)a^2+\frac{2\left(15\,a^2e^{(8\,dx+8\,c)}-30\,abe^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}-60\,abe^{(6\,dx+6\,c)}-30\,b^2e^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}-40\,abe^{(4\,dx+4\,c)}+\cdots\right.}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)*a^2 + 2*(15*a^2*e^(8*d*x + 8*c) - 30*a*b*e^(8*d*x + 8*c
) + 60*a^2*e^(6*d*x + 6*c) - 60*a*b*e^(6*d*x + 6*c) - 30*b^2*e^(6*d*x + 6*
c) + 90*a^2*e^(4*d*x + 4*c) - 40*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4
*c) + 60*a^2*e^(2*d*x + 2*c) - 20*a*b*e^(2*d*x + 2*c) - 10*b^2*e^(2*d*x +
2*c) + 15*a^2 - 10*a*b - 2*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.114.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 513, normalized size of antiderivative = 8.69

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\tanh^2(c+dx)\,dx = \frac{\frac{8\,e^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,\left(2\,a\,b-a^2\right)}{5\,d}+\frac{8\,e^{6\,c+6\,d\,x}\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,e^{8\,c+8\,d\,x}\,\left(2\,a\,b-a^2\right)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,\left(3\,a^2+2\,a\,b+4\,b^2\right)}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}+\frac{\frac{2\,\left(3\,a^2+2\,a\,b+4\,b^2\right)}{15\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,e^{4\,c+4\,d\,x}\,\left(2\,a\,b-a^2\right)}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}+\frac{\frac{2\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,\left(2\,a\,b-a^2\right)}{5\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}+\frac{\frac{2\,\left(a^2-b^2\right)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,e^{6\,c+6\,d\,x}\,\left(2\,a\,b-a^2\right)}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,\left(3\,a^2+2\,a\,b+4\,b^2\right)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}+a^2\,x-\frac{2\,\left(2\,a\,b-a^2\right)}{5\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
((8*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d) - (2*(2*a*b - a^2))/(5*d) + (8*exp
(6*c + 6*d*x)*(a^2 - b^2))/(5*d) - (2*exp(8*c + 8*d*x)*(2*a*b - a^2))/(5*d
) + (4*exp(4*c + 4*d*x)*(2*a*b + 3*a^2 + 4*b^2))/(5*d))/(5*exp(2*c + 2*d*x
) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(1
0*c + 10*d*x) + 1) + ((2*(2*a*b + 3*a^2 + 4*b^2))/(15*d) + (4*exp(2*c + 2*
d*x)*(a^2 - b^2))/(5*d) - (2*exp(4*c + 4*d*x)*(2*a*b - a^2))/(5*d))/(3*exp
(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((2*(a^2 - b^
2))/(5*d) - (2*exp(2*c + 2*d*x)*(2*a*b - a^2))/(5*d))/(2*exp(2*c + 2*d*x)
+ exp(4*c + 4*d*x) + 1) + ((2*(a^2 - b^2))/(5*d) + (6*exp(4*c + 4*d*x)*(a^
2 - b^2))/(5*d) - (2*exp(6*c + 6*d*x)*(2*a*b - a^2))/(5*d) + (2*exp(2*c +
2*d*x)*(2*a*b + 3*a^2 + 4*b^2))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4
*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + a^2*x - (2*(2*a*b - a
^2))/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.115 $\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx=\frac{a^2\log(\cosh(c+dx))}{d}-\frac{ab\text{sech}^2(c+dx)}{d}-\frac{b^2\text{sech}^4(c+dx)}{4d}$$

output
```
a^2*ln(cosh(d*x+c))/d-a*b*sech(d*x+c)^2/d-1/4*b^2*sech(d*x+c)^4/d
```

3.115.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.71

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx =\frac{\cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\left(4a^2\log(\cosh(c+dx))-4ab\text{sech}^2(c+dx)-b^2\text{sech}^4(c+dx)\right)}{d(a+2b+a\cosh(2c+2dx))^2}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x],x]
```

output
```
(Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2*(4*a^2*Log[Cosh[c + d*x]] - 4*a
*b*Sech[c + d*x]^2 - b^2*Sech[c + d*x]^4))/(d*(a + 2*b + a*Cosh[2*c + 2*d*
x])^2)
```

3.115.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\tan(ic+idx)\left(a+b\sec(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \ 26$$

$$-i\int \left(b\sec(ic+idx)^2+a\right)^2\tan(ic+idx)dx$$

$$\Big\downarrow \ 4626$$

$$\frac{\int \left(a\cosh^2(c+dx)+b\right)^2\text{sech}^5(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 243$$

$$\frac{\int \left(a\cosh^2(c+dx)+b\right)^2\text{sech}^3(c+dx)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(b^2\text{sech}^3(c+dx)+2ab\text{sech}^2(c+dx)+a^2\text{sech}(c+dx)\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2\log\left(\cosh^2(c+dx)\right)-2ab\text{sech}(c+dx)-\frac{1}{2}b^2\text{sech}^2(c+dx)}{2d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x],x]
```

output
```
(a^2*Log[Cosh[c + d*x]^2] - 2*a*b*Sech[c + d*x] - (b^2*Sech[c + d*x]^2)/2)
/(2*d)
```

3.115.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.115.4 Maple [A] (verified)

Time = 9.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\frac{\operatorname{sech}(dx+c)^4 b^2}{4}+\operatorname{sech}(dx+c)^2 ab+a^2\ln(\operatorname{sech}(dx+c))}{d}$	42
default	$-\frac{\frac{\operatorname{sech}(dx+c)^4 b^2}{4}+\operatorname{sech}(dx+c)^2 ab+a^2\ln(\operatorname{sech}(dx+c))}{d}$	42
parts	$\frac{a^2\ln(\cosh(dx+c))}{d}-\frac{b^2\operatorname{sech}(dx+c)^4}{4d}+\frac{ab\tanh(dx+c)^2}{d}$	46
risch	$-a^2x-\frac{2a^2c}{d}-\frac{4b\,\mathrm{e}^{2dx+2c}\left(a\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+b\,\mathrm{e}^{2dx+2c}+a\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^4}+\frac{a^2\ln\left(\mathrm{e}^{2dx+2c}+1\right)}{d}$	100

input
```
int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/4*sech(d*x+c)^4*b^2+sech(d*x+c)^2*a*b+a^2*ln(sech(d*x+c)))
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 1180, normalized size of antiderivative = 24.58

$$\int \left(a + b\text{sech}^2(c + dx)\right)^2 \tanh(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="fricas")
```

output
```
-(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*
d*x*sinh(d*x + c)^8 + 4*(a^2*d*x + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d*x*cos
h(d*x + c)^2 + a^2*d*x + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)
^3 + 3*(a^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2*d*x + 4*a
*b + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x +
30*(a^2*d*x + a*b)*cosh(d*x + c)^2 + 4*a*b + 2*b^2)*sinh(d*x + c)^4 + a^2*
d*x + 8*(7*a^2*d*x*cosh(d*x + c)^5 + 10*(a^2*d*x + a*b)*cosh(d*x + c)^3 +
(3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2*d*x +
a*b)*cosh(d*x + c)^2 + 4*(7*a^2*d*x*cosh(d*x + c)^6 + 15*(a^2*d*x + a*b)*c
osh(d*x + c)^4 + a^2*d*x + 3*(3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x + c)^2 +
 a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*
x + c)^7 + a^2*sinh(d*x + c)^8 + 4*a^2*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x
 + c)^2 + a^2)*sinh(d*x + c)^6 + 6*a^2*cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x
 + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4
 + 30*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^2
 + 8*(7*a^2*cosh(d*x + c)^5 + 10*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*(7*a^2*cosh(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 9*
a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7
+ 3*a^2*cosh(d*x + c)^5 + 3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(
d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2...
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx = \begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} - \frac{ab\,\text{sech}^2(c+dx)}{d} - \frac{b^2\,\text{sech}^4(c+dx)}{4d} & \text{for } d\neq 0 \\ x\left(a+b\,\text{sech}^2(c)\right)^2\tanh(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c),x)
```

output
```
Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a*b*sech(c + d*x)**2/d
 - b**2*sech(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c
), True))
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx = \frac{ab\tanh(dx+c)^2}{d} + \frac{a^2\log(\cosh(dx+c))}{d} - \frac{4\,b^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="maxima")
```

output
```
a*b*tanh(d*x + c)^2/d + a^2*log(cosh(d*x + c))/d - 4*b^2/(d*(e^(d*x + c) +
 e^(-d*x - c))^4)
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(46) = 92.

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.38

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx = -\frac{12\,(dx+c)a^2 - 12\,a^2\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{25\,a^2e^{(8\,dx+8\,c)}+100\,a^2e^{(6\,dx+6\,c)}+48\,abe^{(6\,dx+6\,c)}+150\,a^2e^{(4\,dx+4\,c)}+96\,abe^{(4\,dx}}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{12\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="giac")
```

output
```
-1/12*(12*(d*x + c)*a^2 - 12*a^2*log(e^(2*d*x + 2*c) + 1) + (25*a^2*e^(8*d
*x + 8*c) + 100*a^2*e^(6*d*x + 6*c) + 48*a*b*e^(6*d*x + 6*c) + 150*a^2*e^(
4*d*x + 4*c) + 96*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 100*a^2*e
^(2*d*x + 2*c) + 48*a*b*e^(2*d*x + 2*c) + 25*a^2)/(e^(2*d*x + 2*c) + 1)^4)
/d
```

3.115.9 Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.79

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\tanh(c+dx)\,dx$$
$$= \frac{4\,(a\,b-b^2)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-a^2\,x+\frac{8\,b^2}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}$$
$$-\frac{4\,b^2}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$
$$+\frac{a^2\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)}{d}-\frac{4\,a\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(4*(a*b - b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - a^2*x +
(8*b^2)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1
)) - (4*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1)) + (a^2*log(exp(2*c)*exp(2*d*x) + 1))/d - (4*a
*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.116 $\int \left(a+b\text{sech}^2(c+dx)\right)^2 dx$

3.116.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 dx = a^2x+\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
a^2*x+b*(2*a+b)*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

3.116.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 dx = a^2x+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
a^2*x + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d - (b^2*Tanh[c + d*
x]^3)/(3*d)
```

3.116.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\left(-b\tanh^2(c+dx)+a+b\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{a^2}{1-\tanh^2(c+dx)}-b^2\tanh^2(c+dx)+b(2a+b)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\text{arctanh}(\tanh(c+dx))+b(2a+b)\tanh(c+dx)-\frac{1}{3}b^2\tanh^3(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(a^2*ArcTanh[Tanh[c + d*x]] + b*(2*a + b)*Tanh[c + d*x] - (b^2*Tanh[c + d*
x]^3)/3)/d
```

3.116.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.116.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

method	result
parts	$a^{2}x+\frac{b^{2}\left(\frac{2}{3}+\frac{\operatorname{sech}\left(dx+c\right)^{2}}{3}\right)\tanh\left(dx+c\right)}{d}+\frac{2ab\tanh\left(dx+c\right)}{d}$
derivativedivides	$\frac{a^{2}\left(dx+c\right)+2ab\tanh\left(dx+c\right)+b^{2}\left(\frac{2}{3}+\frac{\operatorname{sech}\left(dx+c\right)^{2}}{3}\right)\tanh\left(dx+c\right)}{d}$
default	$\frac{a^{2}\left(dx+c\right)+2ab\tanh\left(dx+c\right)+b^{2}\left(\frac{2}{3}+\frac{\operatorname{sech}\left(dx+c\right)^{2}}{3}\right)\tanh\left(dx+c\right)}{d}$
risch	$a^{2}x-\frac{4b\left(3a\,{\mathrm e}^{4dx+4c}+6\,{\mathrm e}^{2dx+2c}a+3b\,{\mathrm e}^{2dx+2c}+3a+b\right)}{3d\left({\mathrm e}^{2dx+2c}+1\right)^{3}}$
parallelrisch	$\frac{3x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a^{2}d+6\left(2ab+b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+9x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a^{2}d+4\left(6ab+b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+9x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{3d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)^{3}}$

input
```
int((a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2/d*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*tanh(d*x+c)/d
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 4.40

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{(3\,a^2dx-6\,ab-2\,b^2)\cosh{(dx+c)}^3+3\,(3\,a^2dx-6\,ab-2\,b^2)\cosh{(dx+c)}\sinh{(dx+c)}^2+2\,(3\,ab+b^2)}{3\,\left(d\cosh{(dx+c)}^3+3\,d\cosh{(dx-}\right.}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*((3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2*d*x - 6*a*b -
2*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + 3
*(3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c) + 6*((3*a*b + b^2)*cosh(d*x + c
)^2 + a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sin
h(d*x + c)^2 + 3*d*cosh(d*x + c))
```

3.116.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\,dx=\int \left(a+b\,\text{sech}^2\,(c+dx)\right)^2\,dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2, x)
```

3.116.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(38) = 76$.

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.00

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\,dx=a^2x$$
$$+\frac{4}{3}\,b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+\frac{4\,ab}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) + 1))
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int \left(a + b\text{sech}^2(c + dx)\right)^2 \, dx = \frac{3\,(dx + c)a^2 - \frac{4\,(3\,abe^{(4\,dx+4\,c)} + 6\,abe^{(2\,dx+2\,c)} + 3\,b^2e^{(2\,dx+2\,c)} + 3\,ab + b^2)}{(e^{(2\,dx+2\,c)} + 1)^3}}{3\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^2 - 4*(3*a*b*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) +
3*b^2*e^(2*d*x + 2*c) + 3*a*b + b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.116.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.08

$$\int \left(a + b\text{sech}^2(c + dx)\right)^2 \, dx = a^2\,x - \frac{\frac{4\,(b^2 + a\,b)}{3\,d} + \frac{4\,a\,b\,e^{2\,c+2\,d\,x}}{3\,d}}{2\,e^{2\,c+2\,d\,x} + e^{4\,c+4\,d\,x} + 1} - \frac{\frac{8\,e^{2\,c+2\,d\,x}\,(b^2 + a\,b)}{3\,d} + \frac{4\,a\,b}{3\,d} + \frac{4\,a\,b\,e^{4\,c+4\,d\,x}}{3\,d}}{3\,e^{2\,c+2\,d\,x} + 3\,e^{4\,c+4\,d\,x} + e^{6\,c+6\,d\,x} + 1} - \frac{4\,a\,b}{3\,d\,(e^{2\,c+2\,d\,x} + 1)}$$

input
```
int((a + b/cosh(c + d*x)^2)^2,x)
```

output
```
a^2*x - ((4*(a*b + b^2))/(3*d) + (4*a*b*exp(2*c + 2*d*x))/(3*d))/(2*exp(2*
c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*
d) + (4*a*b)/(3*d) + (4*a*b*exp(4*c + 4*d*x))/(3*d))/(3*exp(2*c + 2*d*x) +
 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (4*a*b)/(3*d*(exp(2*c + 2*d*
x) + 1))
```

3.117 $\int \coth(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

3.117.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=-\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{(a+b)^2\log(\sinh(c+dx))}{d}+\frac{b^2\text{sech}^2(c+dx)}{2d}$$

output
```
-b*(2*a+b)*ln(cosh(d*x+c))/d+(a+b)^2*ln(sinh(d*x+c))/d+1/2*b^2*sech(d*x+c)
^2/d
```

3.117.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{2\left(b^2+2\cosh^2(c+dx)\left(-b(2a+b)\log(\cosh(c+dx))+(a+b)^2\log(\sinh(c+dx))\right)\right)(a\cosh(c+dx)+b\text{se}}{d(a+2b+a\cosh(2(c+dx)))^2}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(2*(b^2 + 2*Cosh[c + d*x]^2*(-(b*(2*a + b)*Log[Cosh[c + d*x]]) + (a + b)^2
*Log[Sinh[c + d*x]]))*(a*Cosh[c + d*x] + b*Sech[c + d*x])^2)/(d*(a + 2*b +
 a*Cosh[2*(c + d*x)])^2)
```

3.117.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\left(a+b\sec(ic+idx)^2\right)^2}{\tan(ic+idx)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\left(b\sec(ic+idx)^2+a\right)^2}{\tan(ic+idx)}dx$$

$$\Big\downarrow \; 4626$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^2\text{sech}^3(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 354$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^2\text{sech}^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \; 99$$

$$-\frac{\int \left(-\frac{(a+b)^2}{\cosh^2(c+dx)-1}+b^2\text{sech}^2(c+dx)+b(2a+b)\text{sech}(c+dx)\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{b(2a+b)\log\left(\cosh^2(c+dx)\right)-(a+b)^2\log\left(1-\cosh^2(c+dx)\right)+b^2(-\text{sech}(c+dx))}{2d}$$

input
```
Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-1/2*(b*(2*a + b)*Log[Cosh[c + d*x]^2] - (a + b)^2*Log[1 - Cosh[c + d*x]^2
] - b^2*Sech[c + d*x])/d
```

3.117. $\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

3.117.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.117.4 Maple [A] (verified)

Time = 6.52 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^2\ln(\sinh(dx+c))+2a\ln(\tanh(dx+c))b+b^2\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)}{d}$
default	$\frac{a^2\ln(\sinh(dx+c))+2a\ln(\tanh(dx+c))b+b^2\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)}{d}$
risch	$-a^2x-\frac{2a^2c}{d}+\frac{2b^2\mathrm{e}^{2dx+2c}}{d\left(\mathrm{e}^{2dx+2c}+1\right)^2}-\frac{2ab\ln\left(\mathrm{e}^{2dx+2c}+1\right)}{d}-\frac{b^2\ln\left(\mathrm{e}^{2dx+2c}+1\right)}{d}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-1\right)a^2}{d}+\frac{2\ln(\mathrm{e}^{2dx+}}{d}$

input
```
int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sinh(d*x+c))+2*a*ln(tanh(d*x+c))*b+b^2*(1/2/cosh(d*x+c)^2+ln(t
anh(d*x+c))))
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(51) = 102$.

Time = 0.27 (sec) , antiderivative size = 665, normalized size of antiderivative = 12.55

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx =$$

$$-\frac{a^2dx\cosh\left(dx+c\right)^4+4\,a^2dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+a^2dx\sinh\left(dx+c\right)^4+a^2dx+2\left(a^2dx-b^2\right)}{}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*
d*x*sinh(d*x + c)^4 + a^2*d*x + 2*(a^2*d*x - b^2)*cosh(d*x + c)^2 + 2*(3*a
^2*d*x*cosh(d*x + c)^2 + a^2*d*x - b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*c
osh(d*x + c)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b +
b^2)*sinh(d*x + c)^4 + 2*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2
)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b
 + b^2)*cosh(d*x + c)^3 + (2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(
2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 + 2*a*b + b^2)*co
sh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 +
2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c
)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 +
2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x +
 c) - sinh(d*x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 + (a^2*d*x - b^2)*cosh(
d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
 c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 +
 d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c
) + d)
```

3.117.6 Sympy [F]

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=\int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)^2\coth\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x), x)
```

3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(51) = 102.

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.04

$$\begin{aligned}&\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx\\&=b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)\\&\quad+2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}\right)\\&\quad+\frac{a^2\log\left(\sinh\left(dx+c\right)\right)}{d}\end{aligned}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2
*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)
+ 1))) + 2*a*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^
(-2*d*x - 2*c) + 1)/d) + a^2*log(sinh(d*x + c))/d
```

3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(51) = 102$.

Time = 0.33 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.83

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx =$$
$$-\frac{(2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-(a^2+2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{2\,ab(e^{(2\,d}}{}}{2\,d}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((2*a*b + b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (a^2 + 2
*a*b + b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (2*a*b*(e^(2*d*x
 + 2*c) + e^(-2*d*x - 2*c)) + b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4
*a*b + 6*b^2)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2))/d
```

3.117.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 308, normalized size of antiderivative = 5.81

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{2\,b^2}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-a^2\,x+\frac{a^2\,\ln\left(e^{4\,c+4\,d\,x}-1\right)}{2\,d}-\frac{2\,b^2}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$
$$-\frac{\text{atan}\left(\frac{e^{2\,c}\,e^{2\,d\,x}\,\left(a^4\,\sqrt{-d^2}+4\,b^4\,\sqrt{-d^2}+16\,a\,b^3\,\sqrt{-d^2}+8\,a^3\,b\,\sqrt{-d^2}+20\,a^2\,b^2\,\sqrt{-d^2}\right)}{a^2\,d\,\sqrt{a^4+8\,a^3\,b+20\,a^2\,b^2+16\,a\,b^3+4\,b^4}+2\,b^2\,d\,\sqrt{a^4+8\,a^3\,b+20\,a^2\,b^2+16\,a\,b^3+4\,b^4}+4\,a\,b\,d\,\sqrt{a^4+8\,a^3\,b+20\,a^2\,b^2+16\,a\,b^3+4\,b^4}}\right)\,\sqrt{}}{\sqrt{-d^2}}$$

input
```
int(coth(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) - a^2*x + (a^2*log(exp(4*c + 4*d*x) - 1
))/(2*d) - (2*b^2)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (atan
((exp(2*c)*exp(2*d*x)*(a^4*(-d^2)^(1/2) + 4*b^4*(-d^2)^(1/2) + 16*a*b^3*(-
d^2)^(1/2) + 8*a^3*b*(-d^2)^(1/2) + 20*a^2*b^2*(-d^2)^(1/2)))/(a^2*d*(16*a
*b^3 + 8*a^3*b + a^4 + 4*b^4 + 20*a^2*b^2)^(1/2) + 2*b^2*d*(16*a*b^3 + 8*a
^3*b + a^4 + 4*b^4 + 20*a^2*b^2)^(1/2) + 4*a*b*d*(16*a*b^3 + 8*a^3*b + a^4
 + 4*b^4 + 20*a^2*b^2)^(1/2)))*(16*a*b^3 + 8*a^3*b + a^4 + 4*b^4 + 20*a^2*
b^2)^(1/2))/(-d^2)^(1/2)
```

3.118 $\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

3.118.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

output
```
a^2*x-(a+b)^2*coth(d*x+c)/d-b^2*tanh(d*x+c)/d
```

3.118.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. $2(36)=72$.

Time = 3.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.28

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{4\left(b+a\cosh^2(c+dx)\right)^2\text{sech}(c+dx)\left(a^2dx\cosh(c+dx)+\left((a+b)^2\coth(c+dx)\text{csch}(c)-b^2\text{sech}(c)\right)\sinh\right.}{d(a+2b+a\cosh(2(c+dx)))^2}$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]*(a^2*d*x*Cosh[c + d*x] + ((a +
b)^2*Coth[c + d*x]*Csch[c] - b^2*Sech[c])*Sinh[d*x]))/(d*(a + 2*b + a*Cosh
[2*(c + d*x)])^2)
```

3.118.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 4629, 25, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\left(a+b\sec(ic+idx)^2\right)^2}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\left(b\sec(ic+idx)^2+a\right)^2}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4629}$$

$$-\frac{\int -\frac{\coth^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2075}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 364}$$

$$\frac{\int \left(-\frac{a^2}{\tanh^2(c+dx)-1}-b^2+(a+b)^2\coth^2(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2(-\text{arctanh}(\tanh(c+dx)))+(a+b)^2\coth(c+dx)+b^2\tanh(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

3.118. $\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$

output
```
-((-(a^2*ArcTanh[Tanh[c + d*x]]) + (a + b)^2*Coth[c + d*x] + b^2*Tanh[c +
d*x])/d)
```

3.118.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.118. $\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

3.118.4 Maple [A] (verified)

Time = 7.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

method	result	size
derivativedivides	$\frac{a^{2}(dx+c-\coth(dx+c))-2\coth(dx+c)ab+b^{2}\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	64
default	$\frac{a^{2}(dx+c-\coth(dx+c))-2\coth(dx+c)ab+b^{2}\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	64
risch	$a^{2}x-\frac{2(a^{2}e^{2dx+2c}+2ab\,e^{2dx+2c}+a^{2}+2ab+2b^{2})}{d(e^{2dx+2c}+1)(e^{2dx+2c}-1)}$	77

input
```
int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(d*x+c-coth(d*x+c))-2*coth(d*x+c)*a*b+b^2*(-1/sinh(d*x+c)/cosh(d*
x+c)-2*tanh(d*x+c)))
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.94

$$\int \coth^{2}(c+dx)\left(a+b\operatorname{sech}^{2}(c+dx)\right)^{2}\,dx =$$
$$-\frac{(a^{2}+2\,ab+2\,b^{2})\cosh{(dx+c)}^{2}-2\,(a^{2}dx+a^{2}+2\,ab+2\,b^{2})\cosh{(dx+c)}\sinh{(dx+c)}+(a^{2}+2\,ab+}{2\,d\cosh{(dx+c)}\sinh{(dx+c)}}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((a^2 + 2*a*b + 2*b^2)*cosh(d*x + c)^2 - 2*(a^2*d*x + a^2 + 2*a*b + 2
*b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + 2*b^2)*sinh(d*x + c)^2
+ a^2 + 2*a*b)/(d*cosh(d*x + c)*sinh(d*x + c))
```

3.118.6 Sympy [F]

$$\int \coth^2(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx=\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^2\coth^2\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x)**2, x)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \coth^2(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx=a^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+\frac{4\,ab}{d(e^{(-2\,dx-2\,c)}-1)}+\frac{4\,b^2}{d(e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c)
- 1)) + 4*b^2/(d*(e^(-4*d*x - 4*c) - 1))
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.89

$$\int \coth^2(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx=\frac{(dx+c)a^2-\frac{2\left(a^2e^{(2\,dx+2\,c)}+2\,abe^{(2\,dx+2\,c)}+a^2+2\,ab+2\,b^2\right)}{e^{(4\,dx+4\,c)}-1}}{d}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
((d*x + c)*a^2 - 2*(a^2*e^(2*d*x + 2*c) + 2*a*b*e^(2*d*x + 2*c) + a^2 + 2*
a*b + 2*b^2)/(e^(4*d*x + 4*c) - 1))/d
```

3.118.9 Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \coth^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx=a^2\,x-\frac{\frac{2\,\left(a^2+2\,a\,b+2\,b^2\right)}{d}+\frac{2\,a\,e^{2\,c+2\,d\,x}\,\left(a+2\,b\right)}{d}}{e^{4\,c+4\,d\,x}-1}$$

input
```
int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
a^2*x - ((2*(2*a*b + a^2 + 2*b^2))/d + (2*a*exp(2*c + 2*d*x)*(a + 2*b))/d)
/(exp(4*c + 4*d*x) - 1)
```

3.119 $\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx$

3.119.1 Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx=-\frac{(a+b)^2\text{csch}^2(c+dx)}{2d}+\frac{b^2\log(\cosh(c+dx))}{d}+\frac{(a^2-b^2)\log(\sinh(c+dx))}{d}$$

output
```
-1/2*(a+b)^2*csch(d*x+c)^2/d+b^2*ln(cosh(d*x+c))/d+(a^2-b^2)*ln(sinh(d*x+c
))/d
```

3.119.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx = -\frac{2\left(b+a\cosh^2(c+dx)\right)^2\left((a+b)^2\text{csch}^2(c+dx)-2(b^2\log(\cosh(c+dx))+(a^2-b^2)\log(\sinh(c+dx)))\right)}{d(a+2b+a\cosh(2(c+dx)))^2}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(-2*(b + a*Cosh[c + d*x]^2)^2*((a + b)^2*Csch[c + d*x]^2 - 2*(b^2*Log[Cosh
[c + d*x]] + (a^2 - b^2)*Log[Sinh[c + d*x]])))/(d*(a + 2*b + a*Cosh[2*(c +
 d*x)])^2)
```

3.119.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\left(a+b\sec(ic+idx)^2\right)^2}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\left(b\sec(ic+idx)^2+a\right)^2}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{ 4626}$$

$$\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^2\text{sech}(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^2\text{sech}(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(\text{sech}(c+dx)b^2+\frac{a^2-b^2}{\cosh^2(c+dx)-1}+\frac{(a+b)^2}{\left(\cosh^2(c+dx)-1\right)^2}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a^2-b^2)\log\left(1-\cosh^2(c+dx)\right)+\frac{(a+b)^2}{1-\cosh^2(c+dx)}+b^2\log\left(\cosh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + b)^2/(1 - Cosh[c + d*x]^2) + b^2*Log[Cosh[c + d*x]^2] + (a^2 - b^2)*
Log[1 - Cosh[c + d*x]^2])/(2*d)
```

3.119.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.119.4 Maple [A] (verified)

Time = 10.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{a^{2}\left(\ln\left(\sinh\left(dx+c\right)\right)-\frac{\coth\left(dx+c\right)^{2}}{2}\right)-\frac{ab}{\sinh\left(dx+c\right)^{2}}+b^{2}\left(-\frac{1}{2\sinh\left(dx+c\right)^{2}}-\ln\left(\tanh\left(dx+c\right)\right)\right)}{d}$	64
default	$\frac{a^{2}\left(\ln\left(\sinh\left(dx+c\right)\right)-\frac{\coth\left(dx+c\right)^{2}}{2}\right)-\frac{ab}{\sinh\left(dx+c\right)^{2}}+b^{2}\left(-\frac{1}{2\sinh\left(dx+c\right)^{2}}-\ln\left(\tanh\left(dx+c\right)\right)\right)}{d}$	64
risch	$-a^{2}x-\frac{2a^{2}c}{d}-\frac{2\,{\mathrm e}^{2dx+2c}\left(a^{2}+2ab+b^{2}\right)}{d\left({\mathrm e}^{2dx+2c}-1\right)^{2}}+\frac{\ln\left({\mathrm e}^{2dx+2c}-1\right)a^{2}}{d}-\frac{\ln\left({\mathrm e}^{2dx+2c}-1\right)b^{2}}{d}+\frac{b^{2}\ln\left({\mathrm e}^{2dx+2c}+1\right)}{d}$	113

input
```
int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)-a*b/sinh(d*x+c)^2+b^2*(-1/2/s
inh(d*x+c)^2-ln(tanh(d*x+c))))
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(53) = 106$.

Time = 0.27 (sec) , antiderivative size = 637, normalized size of antiderivative = 11.58

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx =$$

$$-\frac{a^2dx\cosh\left(dx+c\right)^4+4\,a^2dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+a^2dx\sinh\left(dx+c\right)^4+a^2dx-2\left(a^2dx-a^2\right.}{}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*
d*x*sinh(d*x + c)^4 + a^2*d*x - 2*(a^2*d*x - a^2 - 2*a*b - b^2)*cosh(d*x +
 c)^2 + 2*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + 2*a*b + b^2)*sinh(d
*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b
^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^
2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sin
h(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 -
 b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 - b^2)*sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)
*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)
*cosh(d*x + c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*
x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 - (a^
2*d*x - a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^
4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x +
 c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3
 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

3.119.6 Sympy [F]

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=\int\left(a+b\,\text{sech}^2\left(c+dx\right)\right)^2\coth^3\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x)**3, x)
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.75

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$= a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-\frac{4\,ab}{d(e^{(dx+c)}-e^{(-dx-c)})^2}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2
*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b^2*(log(e^
(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d
- 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 4*
a*b/(d*(e^(d*x + c) - e^(-d*x - c))^2)
```

3.119.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(53) = 106$.

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.69

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{b^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+(a^2-b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{a^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)-b^2(}{e^{(2\,dx+2\,c}}}{2\,d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + (a^2 - b^2)*log(e^(
2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^2*(e^(2*d*x + 2*c) + e^(-2*d*x -
 2*c)) - b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a^2 + 8*a*b + 6*b^2)
/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2))/d
```

3.119.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.36

$$\int \coth^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{\ln\left(e^{4c+4dx}-1\right)\left(d\left(a^2-b^2\right)+b^2 d\right)}{2d^2} - a^2 x - \frac{2\left(a^2+2ab+b^2\right)}{d\left(e^{2c+2dx}-1\right)}$$
$$- \frac{\operatorname{atan}\left(\frac{e^{2c}e^{2dx}\left(a^4\sqrt{-d^2}+4b^4\sqrt{-d^2}-4a^2b^2\sqrt{-d^2}\right)}{a^2 d\sqrt{a^4-4a^2b^2+4b^4}-2b^2 d\sqrt{a^4-4a^2b^2+4b^4}}\right)\sqrt{a^4-4a^2b^2+4b^4}}{\sqrt{-d^2}}$$
$$- \frac{2\left(a^2+2ab+b^2\right)}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}$$

input

```
int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

output

```
(log(exp(4*c + 4*d*x) - 1)*(d*(a^2 - b^2) + b^2*d))/(2*d^2) - a^2*x - (2*(
2*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) - (atan((exp(2*c)*exp(2*d*x
)*(a^4*(-d^2)^(1/2) + 4*b^4*(-d^2)^(1/2) - 4*a^2*b^2*(-d^2)^(1/2)))/(a^2*d
*(a^4 + 4*b^4 - 4*a^2*b^2)^(1/2) - 2*b^2*d*(a^4 + 4*b^4 - 4*a^2*b^2)^(1/2)
))*(a^4 + 4*b^4 - 4*a^2*b^2)^(1/2))/(-d^2)^(1/2) - (2*(2*a*b + a^2 + b^2))
/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.120 $\int \coth^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^2 dx$

3.120.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \coth^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^2 dx = a^2x - \frac{(a^2-b^2)\coth(c+dx)}{d} - \frac{(a+b)^2\coth^3(c+dx)}{3d}$$

output
```
a^2*x-(a^2-b^2)*coth(d*x+c)/d-1/3*(a+b)^2*coth(d*x+c)^3/d
```

3.120.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(46) = 92.

Time = 1.44 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.48

$$\int \coth^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^2 dx = \frac{\text{csch}(c)\text{csch}^3(c+dx)\,(9a^2dx\cosh(dx) - 9a^2dx\cosh(2c+dx) - 3a^2dx\cosh(2c+3dx) + 3a^2dx\cosh(4c+3}{}$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(Csch[c]*Csch[c + d*x]^3*(9*a^2*d*x*Cosh[d*x] - 9*a^2*d*x*Cosh[2*c + d*x]
- 3*a^2*d*x*Cosh[2*c + 3*d*x] + 3*a^2*d*x*Cosh[4*c + 3*d*x] - 12*a^2*Sinh[
d*x] + 12*b^2*Sinh[d*x] - 12*a^2*Sinh[2*c + d*x] - 12*a*b*Sinh[2*c + d*x]
+ 8*a^2*Sinh[2*c + 3*d*x] + 4*a*b*Sinh[2*c + 3*d*x] - 4*b^2*Sinh[2*c + 3*d
*x]))/(24*d)
```

3.120.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^2}{\tan(ic+idx)^4}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left((a+b)^2\coth^4(c+dx)+\left(a^2-b^2\right)\coth^2(c+dx)-\frac{a^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\text{arctanh}(\tanh(c+dx))-\left(a^2-b^2\right)\coth(c+dx)-\frac{1}{3}(a+b)^2\coth^3(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

3.120. $\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

output
```
(a^2*ArcTanh[Tanh[c + d*x]] - (a^2 - b^2)*Coth[c + d*x] - ((a + b)^2*Coth[
c + d*x]^3)/3)/d
```

3.120.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.120.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(44) = 88$.

Time = 15.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

method	result
risch	$a^2x-\frac{4\left(3a^2 e^{4dx+4c}+3ab\, e^{4dx+4c}-3a^2 e^{2dx+2c}+3\, e^{2dx+2c}b^2+2a^2+ab-b^2\right)}{3d\left(e^{2dx+2c}-1\right)^3}$
derivativedivides	$\frac{a^2\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}\right)+2ab\left(-\frac{\cosh(dx+c)}{2\sinh(dx+c)^3}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)}{2}\right)+b^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\operatorname{co}}{d}$
default	$\frac{a^2\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}\right)+2ab\left(-\frac{\cosh(dx+c)}{2\sinh(dx+c)^3}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)}{2}\right)+b^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\operatorname{co}}{d}$

input
```
int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-4/3*(3*a^2*exp(4*d*x+4*c)+3*a*b*exp(4*d*x+4*c)-3*a^2*exp(2*d*x+2*c)+
3*exp(2*d*x+2*c)*b^2+2*a^2+a*b-b^2)/d/(exp(2*d*x+2*c)-1)^3
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.37

$$\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx =$$
$$-\frac{2\left(2a^2+ab-b^2\right)\cosh(dx+c)^3+6\left(2a^2+ab-b^2\right)\cosh(dx+c)\sinh(dx+c)^2-\left(3a^2dx+4a^2+2a\right.}{3\left(d\sinh(\right.}$$

input
```
integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*(2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + 6*(2*a^2 + a*b - b^2)*cosh(d
*x + c)*sinh(d*x + c)^2 - (3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c
)^3 + 6*(a*b + b^2)*cosh(d*x + c) + 3*(3*a^2*d*x - (3*a^2*d*x + 4*a^2 + 2*
a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c))/(d*si
nh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \coth^4(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.120.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(44) = 88$.

Time = 0.19 (sec) , antiderivative size = 268, normalized size of antiderivative = 5.83

$$\begin{aligned}&\int \coth^4(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx\\&=\frac{1}{3}a^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)\\&\quad+\frac{4}{3}b^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)\\&\quad+\frac{4}{3}ab\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}+\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)\end{aligned}$$

input
```
integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*
(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4/3*b
^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) - 1))) + 4/3*a*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) - 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) + 1/(d*(3*e^(-2*d*x - 2*c) - 3*
e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

3.120.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(44) = 88.

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.17

$$\int \coth^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^2 \, dx = \frac{3\,(dx+c)a^2 - \frac{4\,\left(3\,a^2e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}-3\,a^2e^{(2\,dx+2\,c)}+3\,b^2e^{(2\,dx+2\,c)}+2\,a^2+ab-b^2\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^2 - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) -
3*a^2*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 2*a^2 + a*b - b^2)/(e^(2*d
*x + 2*c) - 1)^3)/d
```

3.120.9 Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.98

$$\int \coth^4(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^2 \, dx = a^2\,x - \frac{\frac{4\,\left(a^2+b\,a\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+a\,b\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1} - \frac{\frac{4\,\left(b^2+a\,b\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1} - \frac{4\,\left(a^2+b\,a\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
a^2*x - ((4*(a*b + a^2))/(3*d) + (4*exp(4*c + 4*d*x)*(a*b + a^2))/(3*d) +
(8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) - 1) - ((4*(a*b + b^2))/(3*d) + (4*exp(2*c + 2*d
*x)*(a*b + a^2))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (4*(
a*b + a^2))/(3*d*(exp(2*c + 2*d*x) - 1))
```

3.121 $\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

3.121.1 Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=-\frac{a(a+b)\text{csch}^2(c+dx)}{d}-\frac{(a+b)^2\text{csch}^4(c+dx)}{4d}+\frac{a^2\log(\sinh(c+dx))}{d}$$

output
```
-a*(a+b)*csch(d*x+c)^2/d-1/4*(a+b)^2*csch(d*x+c)^4/d+a^2*ln(sinh(d*x+c))/d
```

3.121.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=-\frac{\left(b+a\cosh^2(c+dx)\right)^2\left(4a(a+b)\text{csch}^2(c+dx)+(a+b)^2\text{csch}^4(c+dx)-4a^2\log(\sinh(c+dx))\right)}{d(a+2b+a\cosh(2(c+dx)))^2}$$

input
```
Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-(((b + a*Cosh[c + d*x]^2)^2*(4*a*(a + b)*Csch[c + d*x]^2 + (a + b)^2*Csch
[c + d*x]^4 - 4*a^2*Log[Sinh[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)]
)^2))
```

3.121.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sec(ic+idx)^2\right)^2}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sec(ic+idx)^2+a\right)^2}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cosh(c+dx)\left(a\cosh^2(c+dx)+b\right)^2}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^2}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(-\frac{a^2}{\cosh^2(c+dx)-1}-\frac{2(a+b)a}{\left(\cosh^2(c+dx)-1\right)^2}-\frac{(a+b)^2}{\left(\cosh^2(c+dx)-1\right)^3}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(-\log\left(1-\cosh^2(c+dx)\right)\right)-\frac{2a(a+b)}{1-\cosh^2(c+dx)}+\frac{(a+b)^2}{2\left(1-\cosh^2(c+dx)\right)^2}}{2d}$$

input
```
Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-1/2*((a + b)^2/(2*(1 - Cosh[c + d*x]^2)^2) - (2*a*(a + b))/(1 - Cosh[c +
d*x]^2) - a^2*Log[1 - Cosh[c + d*x]^2])/d
```

3.121.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.121.4 Maple [A] (verified)

Time = 23.42 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.62

method	result	size
derivativedivides	$\frac{a^{2}\left(\ln\left(\sinh\left(dx+c\right)\right)-\frac{\coth\left(dx+c\right)^{2}}{2}-\frac{\coth\left(dx+c\right)^{4}}{4}\right)+2ab\left(-\frac{\cosh\left(dx+c\right)^{2}}{2\sinh\left(dx+c\right)^{4}}+\frac{1}{4\sinh\left(dx+c\right)^{4}}\right)-\frac{b^{2}}{4\sinh\left(dx+c\right)^{4}}}{d}$	84
default	$\frac{a^{2}\left(\ln\left(\sinh\left(dx+c\right)\right)-\frac{\coth\left(dx+c\right)^{2}}{2}-\frac{\coth\left(dx+c\right)^{4}}{4}\right)+2ab\left(-\frac{\cosh\left(dx+c\right)^{2}}{2\sinh\left(dx+c\right)^{4}}+\frac{1}{4\sinh\left(dx+c\right)^{4}}\right)-\frac{b^{2}}{4\sinh\left(dx+c\right)^{4}}}{d}$	84
risch	$-a^{2}x-\frac{2a^{2}c}{d}-\frac{4\,\mathrm{e}^{2dx+2c}\left(a^{2}\mathrm{e}^{4dx+4c}+ab\,\mathrm{e}^{4dx+4c}-a^{2}\mathrm{e}^{2dx+2c}+\mathrm{e}^{2dx+2c}b^{2}+a^{2}+ab\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)^{4}}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-1\right)a^{2}}{d}$	122

input
```
int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+2*a*b*(-1/2
/sinh(d*x+c)^4*cosh(d*x+c)^2+1/4/sinh(d*x+c)^4)-1/4*b^2/sinh(d*x+c)^4)
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1252 vs. $2(50) = 100$.

Time = 0.26 (sec) , antiderivative size = 1252, normalized size of antiderivative = 24.08

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*
d*x*sinh(d*x + c)^8 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d
*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*c
osh(d*x + c)^3 - 3*(a^2*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 +
2*(3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c
)^4 + 3*a^2*d*x - 30*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 + 2*b^2
)*sinh(d*x + c)^4 + a^2*d*x + 8*(7*a^2*d*x*cosh(d*x + c)^5 - 10*(a^2*d*x -
 a^2 - a*b)*cosh(d*x + c)^3 + (3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*d*x*co
sh(d*x + c)^6 - 15*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^4 - a^2*d*x + 3*(3*
a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 - (a
^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x +
c)^8 - 4*a^2*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x +
c)^6 + 6*a^2*cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 - 3*a^2*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 - 30*a^2*cosh(d*x + c)^2
+ 3*a^2)*sinh(d*x + c)^4 - 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cosh(d*x + c)^
5 - 10*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a
^2*cosh(d*x + c)^6 - 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)^2 - a^2)
*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 - 3*a^2*cosh(d*x + c)^5 +
3*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*...
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.121.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(50) = 100$.

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 5.42

$$\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx$$
$$= a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8}}\right.$$
$$+4\,ab\left(\frac{e^{(-2\,dx-2\,c)}}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}+\frac{e^{(-6\,dx-}}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4}\right.$$
$$-\frac{4\,b^2}{d\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4}$$

input
```
integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c)
 - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 4*a
*b*(e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6
*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4*e^(-2*d*x -
2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) -
 4*b^2/(d*(e^(d*x + c) - e^(-d*x - c))^4)
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(50) = 100.

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.88

$$\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{12\,(dx+c)a^2-12\,a^2\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{25\,a^2e^{(8\,dx+8\,c)}-52\,a^2e^{(6\,dx+6\,c)}+48\,abe^{(6\,dx+6\,c)}+102\,a^2e^{(4\,dx+4\,c)}+48\,b^2e^{(4\,d}}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{12\,d}$$

input
```
integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/12*(12*(d*x + c)*a^2 - 12*a^2*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a^2*e
^(8*d*x + 8*c) - 52*a^2*e^(6*d*x + 6*c) + 48*a*b*e^(6*d*x + 6*c) + 102*a^2
*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 52*a^2*e^(2*d*x + 2*c) + 48*a*
b*e^(2*d*x + 2*c) + 25*a^2)/(e^(2*d*x + 2*c) - 1)^4)/d
```

3.121.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.98

$$\begin{aligned}
&\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx \\
&= \frac{a^2\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)}{d}-\frac{4\,(a^2+2\,a\,b+b^2)}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)} \\
&\quad -\frac{4\,(2\,a^2+3\,a\,b+b^2)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-a^2\,x-\frac{4\,(a^2+b\,a)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} \\
&\quad -\frac{8\,(a^2+2\,a\,b+b^2)}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}
\end{aligned}$$

input
```
int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(a^2*log(exp(2*c)*exp(2*d*x) - 1))/d - (4*(2*a*b + a^2 + b^2))/(d*(6*exp(4
*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) +
 1)) - (4*(3*a*b + 2*a^2 + b^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
 + 1)) - a^2*x - (4*(a*b + a^2))/(d*(exp(2*c + 2*d*x) - 1)) - (8*(2*a*b +
a^2 + b^2))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x)
 - 1))
```

3.122 $\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

3.122.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = a^2x-\frac{a^2\coth(c+dx)}{d}-\frac{(a^2-b^2)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\coth^5(c+dx)}{5d}$$

output
```
a^2*x-a^2*coth(d*x+c)/d-1/3*(a^2-b^2)*coth(d*x+c)^3/d-1/5*(a+b)^2*coth(d*x
+c)^5/d
```

3.122.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 256 vs. 2(64) = 128.

Time = 4.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.00

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{\text{csch}(c)\text{csch}^5(c+dx)\left(-150a^2dx\cosh(dx)+150a^2dx\cosh(2c+dx)+75a^2dx\cosh(2c+3dx)-75a^2dx\,\text{co}\right.}{}$$

input
```
Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(Csch[c]*Csch[c + d*x]^5*(-150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c +
d*x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] - 75*a^2*d*x*Cosh[4*c + 3*d*x] - 15*a^
2*d*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] + 280*a^2*Sinh[d*x]
 + 120*a*b*Sinh[d*x] + 20*b^2*Sinh[d*x] + 180*a^2*Sinh[2*c + d*x] - 60*b^2
*Sinh[2*c + d*x] - 140*a^2*Sinh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] -
90*a^2*Sinh[4*c + 3*d*x] - 60*a*b*Sinh[4*c + 3*d*x] + 46*a^2*Sinh[4*c + 5*
d*x] + 12*a*b*Sinh[4*c + 5*d*x] - 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)
```

3.122.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4629, 25, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+b\sec(ic+idx)^2\right)^2}{\tan(ic+idx)^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sec(ic+idx)^2+a\right)^2}{\tan(ic+idx)^6}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\coth^6(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^6(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^6(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left((a+b)^2\coth^6(c+dx)+\left(a^2-b^2\right)\coth^4(c+dx)+a^2\coth^2(c+dx)-\frac{a^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^2\text{arctanh}(\tanh(c+dx))+\frac{1}{3}\left(a^2-b^2\right)\coth^3(c+dx)+a^2\coth(c+dx)+\frac{1}{5}(a+b)^2\coth^5(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-((-(a^2*ArcTanh[Tanh[c + d*x]]) + a^2*Coth[c + d*x] + ((a^2 - b^2)*Coth[c
 + d*x]^3)/3 + ((a + b)^2*Coth[c + d*x]^5)/5)/d)
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(60) = 120$.

Time = 34.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.55

method	result
derivativedivides	$\frac{a^2\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}-\frac{\coth(dx+c)^5}{5}\right)+2ab\left(-\frac{\cosh(dx+c)^3}{2\sinh(dx+c)^5}+\frac{3\cosh(dx+c)}{8\sinh(dx+c)^5}+\frac{3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}}{}\right.}{8}\right.}{d}$
default	$\frac{a^2\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}-\frac{\coth(dx+c)^5}{5}\right)+2ab\left(-\frac{\cosh(dx+c)^3}{2\sinh(dx+c)^5}+\frac{3\cosh(dx+c)}{8\sinh(dx+c)^5}+\frac{3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}}{}\right.}{8}\right.}{d}$
risch	$a^2x-\frac{2\left(45a^2e^{8dx+8c}+30ab\,e^{8dx+8c}-90a^2e^{6dx+6c}+30b^2e^{6dx+6c}+140a^2e^{4dx+4c}+60ab\,e^{4dx+4c}+10\,e^{4dx+4c}b^2-70a^2\right.}{15d\left(e^{2dx+2c}-1\right)^5}$

input
```
int(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+2*a*b*(-1
/2/sinh(d*x+c)^5*cosh(d*x+c)^3+3/8/sinh(d*x+c)^5*cosh(d*x+c)+3/8*(-8/15-1/
5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))+b^2*(-1/4/sinh(d*x+c)^5*c
osh(d*x+c)-1/4*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)))
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(60) = 120$.

Time = 0.24 (sec) , antiderivative size = 425, normalized size of antiderivative = 6.64

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx =$$
$$-\frac{(23\,a^2+6\,ab-2\,b^2)\cosh(dx+c)^5+5\,(23\,a^2+6\,ab-2\,b^2)\cosh(dx+c)\sinh(dx+c)^4-(15\,a^2dx+2}{}$$

input
```
integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*((23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 6*a*b - 2*b^
2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*s
inh(d*x + c)^5 - 5*(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 5*(15*a^2*d*x
 - 2*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 + 23*a^2 + 6*a*
b - 2*b^2)*sinh(d*x + c)^3 + 5*(2*(23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^3
 - 3*(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 +
6*a*b + 4*b^2)*cosh(d*x + c) - 5*((15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*co
sh(d*x + c)^4 + 30*a^2*d*x - 3*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(
d*x + c)^2 + 46*a^2 + 12*a*b - 4*b^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 +
5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*c
osh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(60) = 120$.

Time = 0.21 (sec) , antiderivative size = 613, normalized size of antiderivative = 9.58

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{15}a^2\left(15x + \frac{15c}{d} - \frac{2\left(70e^{(-2dx-2c)} - 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} - 45e^{(-8dx-8c)} - 23\right)}{d(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1)}\right)$$
$$+ \frac{4}{15}b^2\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1)} + \frac{}{d(5e^{(-2dx-2c}}\right.$$
$$+ \frac{4}{5}ab\left(\frac{10e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1)} + \frac{}{d(5e^{(-2dx-2c)}}\right.$$

input
```
integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/15*a^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) - 1))) + 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10
*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) - 1)) + 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x -
 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)
) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5
*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*
x - 8*c) + e^(-10*d*x - 10*c) - 1))) + 4/5*a*b*(10*e^(-4*d*x - 4*c)/(d*(5*
e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x
 - 8*c) + e^(-10*d*x - 10*c) - 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x -
2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) - 1)) + 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)))
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(60) = 120$.

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.66

$$\int \coth^6(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$
$$= \frac{15\,(dx+c)a^2 - \frac{2\,(45\,a^2e^{(8\,dx+8\,c)}+30\,abe^{(8\,dx+8\,c)}-90\,a^2e^{(6\,dx+6\,c)}+30\,b^2e^{(6\,dx+6\,c)}+140\,a^2e^{(4\,dx+4\,c)}+60\,abe^{(4\,dx+4\,c)}+10\,b^2e^{(4\,dx+4\,c)}}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

input
```
integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)*a^2 - 2*(45*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c
) - 90*a^2*e^(6*d*x + 6*c) + 30*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4
*c) + 60*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x +
2*c) + 10*b^2*e^(2*d*x + 2*c) + 23*a^2 + 6*a*b - 2*b^2)/(e^(2*d*x + 2*c) -
 1)^5)/d
```

3.122.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 511, normalized size of antiderivative = 7.98

$$\int \coth^6(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx = a^2x - \frac{\frac{2\,(5\,a^2+6\,a\,b+4\,b^2)}{15\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(3\,a^2+2\,b\,a)}{5\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1} - \frac{\frac{2\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(3\,a^2+2\,b\,a)}{5\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}$$
$$- \frac{\frac{2\,(b^2+2\,a\,b)}{5\,d}+\frac{6\,\mathrm{e}^{4\,c+4\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,\mathrm{e}^{6\,c+6\,d\,x}\,(3\,a^2+2\,b\,a)}{5\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(5\,a^2+6\,a\,b+4\,b^2)}{5\,d}}{6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}$$
$$- \frac{\frac{2\,(3\,a^2+2\,b\,a)}{5\,d}+\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{8\,\mathrm{e}^{6\,c+6\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,\mathrm{e}^{8\,c+8\,d\,x}\,(3\,a^2+2\,b\,a)}{5\,d}+\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(5\,a^2+6\,a\,b+4\,b^2)}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1}$$
$$- \frac{2\,(3\,a^2+2\,b\,a)}{5\,d\,(\mathrm{e}^{2\,c+2\,d\,x}-1)}$$

input
```
int(coth(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
a^2*x - ((2*(6*a*b + 5*a^2 + 4*b^2))/(15*d) + (4*exp(2*c + 2*d*x)*(2*a*b +
 b^2))/(5*d) + (2*exp(4*c + 4*d*x)*(2*a*b + 3*a^2))/(5*d))/(3*exp(2*c + 2*
d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*(2*a*b + b^2))/(5*
d) + (2*exp(2*c + 2*d*x)*(2*a*b + 3*a^2))/(5*d))/(exp(4*c + 4*d*x) - 2*exp
(2*c + 2*d*x) + 1) - ((2*(2*a*b + b^2))/(5*d) + (6*exp(4*c + 4*d*x)*(2*a*b
 + b^2))/(5*d) + (2*exp(6*c + 6*d*x)*(2*a*b + 3*a^2))/(5*d) + (2*exp(2*c +
 2*d*x)*(6*a*b + 5*a^2 + 4*b^2))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c +
2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(2*a*b + 3*a^2))
/(5*d) + (8*exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(2
*a*b + b^2))/(5*d) + (2*exp(8*c + 8*d*x)*(2*a*b + 3*a^2))/(5*d) + (4*exp(4
*c + 4*d*x)*(6*a*b + 5*a^2 + 4*b^2))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4
*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x
) - 1) - (2*(2*a*b + 3*a^2))/(5*d*(exp(2*c + 2*d*x) - 1))
```

3.123 $\int \coth^7(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

3.123.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \coth^7(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx=-\frac{a(a+b)\mathrm{csch}^2(c+dx)}{d}-\frac{(a+b)^2\mathrm{csch}^4(c+dx)}{4d}-\frac{\left(b+a\cosh^2(c+dx)\right)^3\mathrm{csch}^6(c+dx)}{6(a+b)d}+\frac{a^2\log(\sinh(c+dx))}{d}$$

output
```
-a*(a+b)*csch(d*x+c)^2/d-1/4*(a+b)^2*csch(d*x+c)^4/d-1/6*(b+a*cosh(d*x+c)^
2)^3*csch(d*x+c)^6/(a+b)/d+a^2*ln(sinh(d*x+c))/d
```

3.123.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.24

$$\int \coth^7(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx=$$
$$-\frac{\left(b+a\cosh^2(c+dx)\right)^2\left(6a(3a+2b)\mathrm{csch}^2(c+dx)+3\left(3a^2+4ab+b^2\right)\mathrm{csch}^4(c+dx)+2(a+b)^2\mathrm{csch}^6(c\right.}{3d(a+2b+a\cosh(2(c+dx)))^2}$$

input
```
Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-1/3*((b + a*Cosh[c + d*x]^2)^2*(6*a*(3*a + 2*b)*Csch[c + d*x]^2 + 3*(3*a^
2 + 4*a*b + b^2)*Csch[c + d*x]^4 + 2*(a + b)^2*Csch[c + d*x]^6 - 12*a^2*Lo
g[Sinh[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

3.123.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4626, 354, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a+b\sec(ic+idx)^2\right)^2}{\tan(ic+idx)^7}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(b\sec(ic+idx)^2+a\right)^2}{\tan(ic+idx)^7}dx$$

$$\Big\downarrow \text{4626}$$

$$\frac{\int \frac{\cosh^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2}{\left(1-\cosh^2(c+dx)\right)^4}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cosh^2(c+dx)\left(a\cosh^2(c+dx)+b\right)^2}{\left(1-\cosh^2(c+dx)\right)^4}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{\left(a\cosh^2(c+dx)+b\right)^3}{3(a+b)\left(1-\cosh^2(c+dx)\right)^3}-\int \frac{\left(a\cosh^2(c+dx)+b\right)^2}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\frac{\left(a\cosh^2(c+dx)+b\right)^3}{3(a+b)\left(1-\cosh^2(c+dx)\right)^3}-\int \left(-\frac{a^2}{\cosh^2(c+dx)-1}-\frac{2(a+b)a}{\left(\cosh^2(c+dx)-1\right)^2}-\frac{(a+b)^2}{\left(\cosh^2(c+dx)-1\right)^3}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\log\left(1-\cosh^2(c+dx)\right)+\frac{\left(a\cosh^2(c+dx)+b\right)^3}{3(a+b)\left(1-\cosh^2(c+dx)\right)^3}+\frac{2a(a+b)}{1-\cosh^2(c+dx)}-\frac{(a+b)^2}{2\left(1-\cosh^2(c+dx)\right)^2}}{2d}$$

input
```
Int[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(-1/2*(a + b)^2/(1 - Cosh[c + d*x]^2)^2 + (2*a*(a + b))/(1 - Cosh[c + d*x]
^2) + (b + a*Cosh[c + d*x]^2)^3/(3*(a + b)*(1 - Cosh[c + d*x]^2)^3) + a^2*
Log[1 - Cosh[c + d*x]^2])/(2*d)
```

3.123.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.123. $\int \coth^7(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.123.4 Maple [A] (verified)

Time = 49.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}-\frac{\coth(dx+c)^6}{6}\right)+2ab\left(-\frac{\cosh(dx+c)^4}{2\sinh(dx+c)^6}+\frac{\cosh(dx+c)^2}{2\sinh(dx+c)^6}-\frac{1}{6\sinh(dx+c)^6}\right)+b}{d}$
default	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}-\frac{\coth(dx+c)^6}{6}\right)+2ab\left(-\frac{\cosh(dx+c)^4}{2\sinh(dx+c)^6}+\frac{\cosh(dx+c)^2}{2\sinh(dx+c)^6}-\frac{1}{6\sinh(dx+c)^6}\right)+b}{d}$
risch	$-a^2x-\frac{2a^2c}{d}-\frac{2\,e^{2dx+2c}\left(9a^2e^{8dx+8c}+6ab\,e^{8dx+8c}-18a^2e^{6dx+6c}+6b^2e^{6dx+6c}+34a^2e^{4dx+4c}+20ab\,e^{4dx+4c}+4\,e^4\right.}{3d\left(e^{2dx+2c}-1\right)^6}$

input
```
int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4-1/6*coth(d*x
+c)^6)+2*a*b*(-1/2/sinh(d*x+c)^6*cosh(d*x+c)^4+1/2/sinh(d*x+c)^6*cosh(d*x+
c)^2-1/6/sinh(d*x+c)^6)+b^2*(-1/4/sinh(d*x+c)^6*cosh(d*x+c)^2+1/12/sinh(d*
x+c)^6))
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2548 vs. $2(82) = 164$.

Time = 0.28 (sec) , antiderivative size = 2548, normalized size of antiderivative = 29.63

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^
11 + 3*a^2*d*x*sinh(d*x + c)^12 - 6*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x +
 c)^10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x + 3*a^2 + 2*a*b)*sinh(d
*x + c)^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x - 3*a^2 - 2*a*b)*
cosh(d*x + c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x
+ c)^8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x - 90*(3*a^2*d*x - 3*a
^2 - 2*a*b)*cosh(d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^8 + 24*(99*a^2
*d*x*cosh(d*x + c)^5 - 30*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^3 + (1
5*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^2*d*x
 - 17*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh(d*x + c)
^6 - 315*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 15*a^2*d*x + 21*(15
*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^2 + 17*a^2 + 10*a*b + 2*b^2)*sinh
(d*x + c)^6 + 24*(99*a^2*d*x*cosh(d*x + c)^7 - 63*(3*a^2*d*x - 3*a^2 - 2*a
*b)*cosh(d*x + c)^5 + 7*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^3 - (1
5*a^2*d*x - 17*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(1
5*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^4 + 3*(495*a^2*d*x*cosh(d*x + c)
^8 - 420*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^6 + 70*(15*a^2*d*x - 12
*a^2 + 4*b^2)*cosh(d*x + c)^4 + 15*a^2*d*x - 20*(15*a^2*d*x - 17*a^2 - 10*
a*b - 2*b^2)*cosh(d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^4 + 3*a^2*d*x
 + 4*(165*a^2*d*x*cosh(d*x + c)^9 - 180*(3*a^2*d*x - 3*a^2 - 2*a*b)*cos...
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \coth^7(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.123.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(82) = 164$.

Time = 0.20 (sec) , antiderivative size = 696, normalized size of antiderivative = 8.09

$$\int \coth^7(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$$

$$= \frac{1}{3}a^2\left(3x + \frac{3c}{d} + \frac{3\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(9e^{(-2dx-2c)} - 18e^{(-4dx-4c)} + \cdots\right.}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx\cdots}}\right.$$

$$+\frac{4}{3}ab\left(\frac{3e^{(-2dx-2c)}}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)}\cdots\right.$$

$$+\frac{4}{3}b^2\left(\frac{3e^{(-4dx-4c)}}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)}\cdots\right.$$

input
```
integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)
/d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 1
8*e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(
-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x -
 10*c) - e^(-12*d*x - 12*c) - 1))) + 4/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(
-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x -
 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 10*e^(-6*d*x - 6
*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 1
5*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e
^(-10*d*x - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*
d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*
c) - 1))) + 4/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10
*c) - e^(-12*d*x - 12*c) - 1)) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c)
 - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(
-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-
2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x -
8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)))
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(82) = 164.

Time = 0.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.55

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx =$$

$$-\frac{60\,(dx+c)a^2-60\,a^2\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{147\,a^2e^{(12\,dx+12\,c)}-522\,a^2e^{(10\,dx+10\,c)}+240\,abe^{(10\,dx+10\,c)}+1485\,a^2e^{(8\,dx+8\,c)}}{}}{}$$

input
```
integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/60*(60*(d*x + c)*a^2 - 60*a^2*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^2*
e^(12*d*x + 12*c) - 522*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c)
+ 1485*a^2*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c) - 1580*a^2*e^(6*d*x +
 6*c) + 800*a*b*e^(6*d*x + 6*c) + 160*b^2*e^(6*d*x + 6*c) + 1485*a^2*e^(4*
d*x + 4*c) + 240*b^2*e^(4*d*x + 4*c) - 522*a^2*e^(2*d*x + 2*c) + 240*a*b*e
^(2*d*x + 2*c) + 147*a^2)/(e^(2*d*x + 2*c) - 1)^6)/d
```

3.123.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.38

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$$
$$=\frac{a^2\,\ln\left(e^{2\,c}\,e^{2\,d\,x}-1\right)}{d}$$
$$-\frac{32\,(a^2+2\,a\,b+b^2)}{d\,\left(5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1\right)}$$
$$-\frac{2\,(3\,a^2+2\,b\,a)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}$$
$$-\frac{32\,(a^2+2\,a\,b+b^2)}{3\,d\,\left(15\,e^{4\,c+4\,d\,x}-6\,e^{2\,c+2\,d\,x}-20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}-6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}$$
$$-\frac{2\,(9\,a^2+10\,a\,b+2\,b^2)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,(13\,a^2+20\,a\,b+7\,b^2)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}$$
$$-\frac{4\,(11\,a^2+20\,a\,b+9\,b^2)}{d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}-a^2\,x$$

input
```
int(coth(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
(a^2*log(exp(2*c)*exp(2*d*x) - 1))/d - (32*(2*a*b + a^2 + b^2))/(d*(5*exp(
2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d
*x) + exp(10*c + 10*d*x) - 1)) - (2*(2*a*b + 3*a^2))/(d*(exp(2*c + 2*d*x)
- 1)) - (32*(2*a*b + a^2 + b^2))/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2
*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) +
 exp(12*c + 12*d*x) + 1)) - (2*(10*a*b + 9*a^2 + 2*b^2))/(d*(exp(4*c + 4*d
*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*(20*a*b + 13*a^2 + 7*b^2))/(3*d*(3*exp
(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(20*a*b +
 11*a^2 + 9*b^2))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c
+ 6*d*x) + exp(8*c + 8*d*x) + 1)) - a^2*x
```

3.123. $\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

3.124 $\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx$

3.124.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx = a^3x-\frac{a^3\tanh(c+dx)}{d}-\frac{a^3\tanh^3(c+dx)}{3d}+\frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d}-\frac{b^2(3a+2b)\tanh^7(c+dx)}{7d}+\frac{b^3\tanh^9(c+dx)}{9d}$$

output
```
a^3*x-a^3*tanh(d*x+c)/d-1/3*a^3*tanh(d*x+c)^3/d+1/5*b*(3*a^2+3*a*b+b^2)*ta
nh(d*x+c)^5/d-1/7*b^2*(3*a+2*b)*tanh(d*x+c)^7/d+1/9*b^3*tanh(d*x+c)^9/d
```

3.124.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(110) = 220.

Time = 8.13 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.74

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx$$
$$= \frac{8\left(b+a\cosh^2(c+dx)\right)^3 \text{sech}^9(c+dx)\left(315a^3dx\cosh^9(c+dx)+35b^3\text{sech}(c)\sinh(dx)+5(27a-10b)b^2\,\text{co}\ldots\right.}{}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]
```

output
```
(8*(b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^9*(315*a^3*d*x*Cosh[c + d*x]^9
+ 35*b^3*Sech[c]*Sinh[d*x] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^2*Sech[c]*S
inh[d*x] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^4*Sech[c]*Sinh[d*x] +
 (105*a^3 - 378*a^2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^6*Sech[c]*Sinh[d*x
] - (420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*Cosh[c + d*x]^8*Sech[c]*Sinh[
d*x] + 35*b^3*Cosh[c + d*x]*Tanh[c] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^3*
Tanh[c] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^5*Tanh[c] + (105*a^3 -
 378*a^2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^7*Tanh[c]))/(315*d*(a + 2*b +
 a*Cosh[2*(c + d*x)])^3)
```

3.124.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(b^3\tanh^8(c+dx)-b^2(3a+2b)\tanh^6(c+dx)+b\left(3a^2+3ba+b^2\right)\tanh^4(c+dx)-a^3\tanh^2(c+dx)-a^3+\frac{}{1-}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\text{arctanh}(\tanh(c+dx)) - \frac{1}{3}a^3\tanh^3(c+dx) - a^3\tanh(c+dx) + \frac{1}{5}b(3a^2+3ab+b^2)\tanh^5(c+dx) - \frac{1}{7}b^2(3a+}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]
```

output
```
(a^3*ArcTanh[Tanh[c + d*x]] - a^3*Tanh[c + d*x] - (a^3*Tanh[c + d*x]^3)/3
+ (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^5)/5 - (b^2*(3*a + 2*b)*Tanh[c +
d*x]^7)/7 + (b^3*Tanh[c + d*x]^9)/9)/d
```

3.124.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.124.4 Maple [A] (verified)

Time = 149.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23

method	result
parts	$\frac{a^3\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^3\left(\frac{\tanh(dx+c)^9}{9}-\frac{2\tanh(dx+c)^7}{7}+\frac{\tanh(dx+c}{5}\right.}{d}$
derivativedivides	$\frac{a^3\left(dx+c-\tanh(dx+c)-\frac{\tanh(dx+c)^3}{3}\right)+3a^2b\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh}{8}\right.}{}$
default	$\frac{a^3\left(dx+c-\tanh(dx+c)-\frac{\tanh(dx+c)^3}{3}\right)+3a^2b\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh}{8}\right.}{}$
risch	$a^3x+\frac{156a^3\mathrm{e}^{10dx+10c}+16b^3\mathrm{e}^{10dx+10c}+180a^3\mathrm{e}^{8dx+8c}-48a^2b\,\mathrm{e}^{12dx+12c}+28a^3\mathrm{e}^{14dx+14c}-\frac{12a\,b^2}{35}-6a^2b\,\mathrm{e}^{16dx+16c}-2}{}$

input
```
int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(-1/3*tanh(d*x+c)^3-tanh(d*x+c)-1/2*ln(tanh(d*x+c)-1)+1/2*ln(tanh(d*
x+c)+1))+b^3/d*(1/9*tanh(d*x+c)^9-2/7*tanh(d*x+c)^7+1/5*tanh(d*x+c)^5)+3/5
*a^2*b/d*tanh(d*x+c)^5+3*a*b^2/d*(-1/7*tanh(d*x+c)^7+1/5*tanh(d*x+c)^5)
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. 2(102) = 204.

Time = 0.26 (sec) , antiderivative size = 1323, normalized size of antiderivative = 12.03

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\tanh^4(c+dx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/315*((315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c
)^9 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x +
c)*sinh(d*x + c)^8 - (420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*sinh(d*x + c
)^9 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x +
c)^7 - 9*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3 + 4*(420*a^3 - 189*a^2*b -
 54*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(315*a^3*d*x +
 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(315*a^3*d*x
+ 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 +
 36*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^5
 - 9*(14*(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 700*a^
3 + 84*a^2*b + 204*a*b^2 - 32*b^3 + 21*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(315*a^3*d*x + 420*a^3 - 189
*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 35*(315*a^3*d*x + 420*a^3 - 1
89*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 20*(315*a^3*d*x + 420*a^3 -
 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(315*a^
3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 3*(28*(4
20*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 105*(280*a^3 + 21
*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 2660*a^3 - 252*a^2*b - 252*a*
b^2 + 896*b^3 + 120*(175*a^3 + 21*a^2*b + 51*a*b^2 - 8*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 9*(4*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2...
```

3.124.6 Sympy [F]

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\tanh^4(c+dx)\,dx=\int \left(a+b\operatorname{sech}^2{(c+dx)}\right)^3\tanh^4{(c+dx)}\,dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**4,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**4, x)
```

3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1453 vs. $2(102) = 204$.

Time = 0.20 (sec) , antiderivative size = 1453, normalized size of antiderivative = 13.21

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\tanh^4(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="maxima")
```

output
```
3/5*a^2*b*tanh(d*x + c)^5/d + 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1))) + 16/315*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x -
2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) +
126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9
*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9
*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*
d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*
x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 126*e^(-6*d*
x - 6*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c
) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c)
+ 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))
+ 441*e^(-8*d*x - 8*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e
^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-1
2*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x
- 18*c) + 1)) - 315*e^(-10*d*x - 10*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d
*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x -
10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*
c) + e^(-18*d*x - 18*c) + 1)) + 210*e^(-12*d*x - 12*c)/(d*(9*e^(-2*d*x - 2
*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) ...
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(102) = 204$.

Time = 0.38 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.32

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\tanh^4(c+dx)\,dx$$
$$= \frac{315\,(dx+c)a^3 + \frac{2\,(630\,a^3e^{(16\,dx+16\,c)}-945\,a^2be^{(16\,dx+16\,c)}+4410\,a^3e^{(14\,dx+14\,c)}-3780\,a^2be^{(14\,dx+14\,c)}-1890\,ab^2e^{(14\,dx+14\,c)}+13650\,a^3}{}}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="giac")
```

output
```
1/315*(315*(d*x + c)*a^3 + 2*(630*a^3*e^(16*d*x + 16*c) - 945*a^2*b*e^(16*
d*x + 16*c) + 4410*a^3*e^(14*d*x + 14*c) - 3780*a^2*b*e^(14*d*x + 14*c) -
1890*a*b^2*e^(14*d*x + 14*c) + 13650*a^3*e^(12*d*x + 12*c) - 7560*a^2*b*e^
(12*d*x + 12*c) - 1890*a*b^2*e^(12*d*x + 12*c) - 1680*b^3*e^(12*d*x + 12*c
) + 24570*a^3*e^(10*d*x + 10*c) - 11340*a^2*b*e^(10*d*x + 10*c) - 1890*a*b
^2*e^(10*d*x + 10*c) + 2520*b^3*e^(10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8
*c) - 12474*a^2*b*e^(8*d*x + 8*c) - 4914*a*b^2*e^(8*d*x + 8*c) - 3528*b^3*
e^(8*d*x + 8*c) + 21630*a^3*e^(6*d*x + 6*c) - 8316*a^2*b*e^(6*d*x + 6*c) -
 2646*a*b^2*e^(6*d*x + 6*c) + 1008*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*
x + 4*c) - 3024*a^2*b*e^(4*d*x + 4*c) - 54*a*b^2*e^(4*d*x + 4*c) - 288*b^3
*e^(4*d*x + 4*c) + 3150*a^3*e^(2*d*x + 2*c) - 756*a^2*b*e^(2*d*x + 2*c) -
486*a*b^2*e^(2*d*x + 2*c) - 72*b^3*e^(2*d*x + 2*c) + 420*a^3 - 189*a^2*b -
 54*a*b^2 - 8*b^3)/(e^(2*d*x + 2*c) + 1)^9)/d
```

3.124.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 1834, normalized size of antiderivative = 16.67

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx = \text{Too large to display}$$

input
```
int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
((3*a*b^2 + 13*a^3 + 16*b^3)/(63*d) + (10*exp(4*c + 4*d*x)*(3*a*b^2 + 13*a
^3 + 16*b^3))/(63*d) + (20*exp(6*c + 6*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4
*b^3))/(63*d) - (2*exp(2*c + 2*d*x)*(8*a*b^2 + 3*a^2*b - 10*a^3 + 16*b^3))
/(21*d) - (5*exp(8*c + 8*d*x)*(a*b^2 - a^3))/(3*d) - (2*exp(10*c + 10*d*x)
*(3*a^2*b - 2*a^3))/(9*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*
exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c +
 12*d*x) + 1) - ((2*exp(2*c + 2*d*x)*(a*b^2 - a^3))/(3*d) - (2*(6*a*b^2 +
3*a^2*b + 8*a^3 - 4*b^3))/(63*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b - 2*a^3))/
(9*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) +
((3*a*b^2 + 13*a^3 + 16*b^3)/(63*d) + (2*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2
*b + 8*a^3 - 4*b^3))/(21*d) - (exp(4*c + 4*d*x)*(a*b^2 - a^3))/d - (2*exp(
6*c + 6*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4
*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((a*b^2 - a^3)/(3*d)
+ (2*exp(2*c + 2*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(2*exp(2*c + 2*d*x) + exp(
4*c + 4*d*x) + 1) + a^3*x + ((2*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(63*d
) + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(21*d) + (20*exp(6*c
+ 6*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(63*d) + (10*exp(8*c + 8*d*x)*(6*a*b
^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(21*d) - (2*exp(4*c + 4*d*x)*(8*a*b^2 + 3*a
^2*b - 10*a^3 + 16*b^3))/(7*d) - (2*exp(10*c + 10*d*x)*(a*b^2 - a^3))/d -
(2*exp(12*c + 12*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(7*exp(2*c + 2*d*x) + 2...
```

3.124. $\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx$

3.125 $\int \left(a+b\mathbf{sech}^2(c+dx)\right)^3 \tanh^3(c+dx)\,dx$

3.125.1 Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^3(c+dx)\,dx = \frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\text{sech}^2(c+dx)}{2d}-\frac{3ab^2\text{sech}^4(c+dx)}{4d}-\frac{b^3\text{sech}^6(c+dx)}{6d}+\frac{\left(b+a\cosh^2(c+dx)\right)^4\text{sech}^8(c+dx)}{8bd}$$

output
```
a^3*ln(cosh(d*x+c))/d-3/2*a^2*b*sech(d*x+c)^2/d-3/4*a*b^2*sech(d*x+c)^4/d-
1/6*b^3*sech(d*x+c)^6/d+1/8*(b+a*cosh(d*x+c)^2)^4*sech(d*x+c)^8/b/d
```

3.125.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.24

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^3(c+dx)\,dx = \frac{\cosh^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\left(24a^3\log(\cosh(c+dx))+12a^2(a-3b)\text{sech}^2(c+dx)+18a(a-b)b\text{s}\right.}{3d(a+2b+a\cosh(2c+2dx))^3}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]
```

output
```
(Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(24*a^3*Log[Cosh[c + d*x]] + 12
*a^2*(a - 3*b)*Sech[c + d*x]^2 + 18*a*(a - b)*b*Sech[c + d*x]^4 + 4*(3*a -
 b)*b^2*Sech[c + d*x]^6 + 3*b^3*Sech[c + d*x]^8))/(3*d*(a + 2*b + a*Cosh[2
*c + 2*d*x])^3)
```

3.125.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4626, 354, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$i\int \left(b\sec(ic+idx)^2+a\right)^3\tan(ic+idx)^3dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^3\text{sech}^9(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^3\text{sech}^5(c+dx)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{87}$$

$$-\frac{-\int \left(a\cosh^2(c+dx)+b\right)^3\text{sech}^4(c+dx)d\cosh^2(c+dx)-\frac{\text{sech}^4(c+dx)\left(a\cosh^2(c+dx)+b\right)^4}{4b}}{2d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{-\int \left(b^3\text{sech}^4(c+dx)+3ab^2\text{sech}^3(c+dx)+3a^2b\text{sech}^2(c+dx)+a^3\text{sech}(c+dx)\right)d\cosh^2(c+dx)-\frac{\text{sech}^4(c+dx)(}{}}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^3\log\left(\cosh^2(c+dx)\right)+3a^2b\operatorname{sech}(c+dx)+\frac{3}{2}ab^2\operatorname{sech}^2(c+dx)-\frac{\operatorname{sech}^4(c+dx)\left(a\cosh^2(c+dx)+b\right)^4}{4b}+\frac{1}{3}b^3\operatorname{sech}^3(c+}{2d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]
```

output
```
-1/2*(-(a^3*Log[Cosh[c + d*x]^2]) + 3*a^2*b*Sech[c + d*x] + (3*a*b^2*Sech[
c + d*x]^2)/2 + (b^3*Sech[c + d*x]^3)/3 - ((b + a*Cosh[c + d*x]^2)^4*Sech[
c + d*x]^4)/(4*b))/d
```

3.125.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.125.4 Maple [A] (verified)

Time = 111.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{b^3 \operatorname{sech}(dx+c)^8}{8}+\frac{a\,b^2 \operatorname{sech}(dx+c)^6}{2}-\frac{\operatorname{sech}(dx+c)^6 b^3}{6}+\frac{3a^2 b \operatorname{sech}(dx+c)^4}{4}-\frac{3 \operatorname{sech}(dx+c)^4 a\,b^2}{4}+\frac{a^3 \operatorname{sech}(dx+c)^2}{2}-\frac{3 \operatorname{sech}(dx+c)^2 a^2 b}{2}}{d}$
default	$\frac{\frac{b^3 \operatorname{sech}(dx+c)^8}{8}+\frac{a\,b^2 \operatorname{sech}(dx+c)^6}{2}-\frac{\operatorname{sech}(dx+c)^6 b^3}{6}+\frac{3a^2 b \operatorname{sech}(dx+c)^4}{4}-\frac{3 \operatorname{sech}(dx+c)^4 a\,b^2}{4}+\frac{a^3 \operatorname{sech}(dx+c)^2}{2}-\frac{3 \operatorname{sech}(dx+c)^2 a^2 b}{2}}{d}$
parts	$\frac{a^3\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^3\left(\frac{\operatorname{sech}(dx+c)^8}{8}-\frac{\operatorname{sech}(dx+c)^6}{6}\right)}{d}+\frac{3a^2 b \tanh(dx+c)^4}{4d}+$
risch	$-a^3 x-\frac{2a^3 c}{d}+\frac{2\,\mathrm{e}^{2dx+2c}(3a^3\mathrm{e}^{12dx+12c}-9a^2 b\,\mathrm{e}^{12dx+12c}+18a^3\mathrm{e}^{10dx+10c}-36a^2 b\,\mathrm{e}^{10dx+10c}-18a\,b^2\mathrm{e}^{10dx+10c}+45a}{}$

input
```
int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*b^3*sech(d*x+c)^8+1/2*a*b^2*sech(d*x+c)^6-1/6*sech(d*x+c)^6*b^3+3
/4*a^2*b*sech(d*x+c)^4-3/4*sech(d*x+c)^4*a*b^2+1/2*a^3*sech(d*x+c)^2-3/2*s
ech(d*x+c)^2*a^2*b-a^3*ln(sech(d*x+c)))
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4658 vs. 2(95) = 190.

Time = 0.30 (sec) , antiderivative size = 4658, normalized size of antiderivative = 45.22

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\tanh^3(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/3*(3*a^3*d*x*cosh(d*x + c)^16 + 48*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^
15 + 3*a^3*d*x*sinh(d*x + c)^16 + 6*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x +
 c)^14 + 6*(60*a^3*d*x*cosh(d*x + c)^2 + 4*a^3*d*x - a^3 + 3*a^2*b)*sinh(d
*x + c)^14 + 84*(20*a^3*d*x*cosh(d*x + c)^3 + (4*a^3*d*x - a^3 + 3*a^2*b)*
cosh(d*x + c))*sinh(d*x + c)^13 + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^
2)*cosh(d*x + c)^12 + 6*(910*a^3*d*x*cosh(d*x + c)^4 + 14*a^3*d*x - 6*a^3
+ 12*a^2*b + 6*a*b^2 + 91*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^2)*sin
h(d*x + c)^12 + 24*(546*a^3*d*x*cosh(d*x + c)^5 + 91*(4*a^3*d*x - a^3 + 3*
a^2*b)*cosh(d*x + c)^3 + 6*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*
x + c))*sinh(d*x + c)^11 + 2*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 +
16*b^3)*cosh(d*x + c)^10 + 2*(12012*a^3*d*x*cosh(d*x + c)^6 + 84*a^3*d*x +
 3003*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^4 - 45*a^3 + 63*a^2*b + 24
*a*b^2 + 16*b^3 + 396*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c
)^2)*sinh(d*x + c)^10 + 4*(8580*a^3*d*x*cosh(d*x + c)^7 + 3003*(4*a^3*d*x
- a^3 + 3*a^2*b)*cosh(d*x + c)^5 + 660*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*
b^2)*cosh(d*x + c)^3 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b
^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(105*a^3*d*x - 60*a^3 + 72*a^2*b +
12*a*b^2 - 16*b^3)*cosh(d*x + c)^8 + 2*(19305*a^3*d*x*cosh(d*x + c)^8 + 90
09*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^6 + 105*a^3*d*x + 2970*(7*a^3
*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 - 60*a^3 + 72*a^2*b +...
```

3.125.6 Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.73

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\tanh^3(c+dx)\,dx$$
$$=\begin{cases} a^3x-\frac{a^3\log(\tanh(c+dx)+1)}{d}-\frac{a^3\tanh^2(c+dx)}{2d}-\frac{3a^2b\tanh^2(c+dx)\operatorname{sech}^2(c+dx)}{4d}-\frac{3a^2b\operatorname{sech}^2(c+dx)}{4d}-\frac{ab^2\tanh^2(c+dx)\operatorname{sech}^4(\ }{2d} \\ x\left(a+b\operatorname{sech}^2(c)\right)^3\tanh^3(c) \end{cases}$$

input
```
integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**3,x)
```

output
```
Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/
(2*d) - 3*a**2*b*tanh(c + d*x)**2*sech(c + d*x)**2/(4*d) - 3*a**2*b*sech(c
 + d*x)**2/(4*d) - a*b**2*tanh(c + d*x)**2*sech(c + d*x)**4/(2*d) - a*b**2
*sech(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2*sech(c + d*x)**6/(8*d) - b
**3*sech(c + d*x)**6/(24*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c)**
3, True))
```

3.125.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(95) = 190$.

Time = 0.29 (sec) , antiderivative size = 652, normalized size of antiderivative = 6.33

$$\int \left(a + b\text{sech}^2(c + dx)\right)^3 \tanh^3(c + dx)\,dx = \frac{3\,a^2 b \tanh\left(dx + c\right)^4}{4\,d}$$
$$+ a^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx - 2\,c)} + 1\right)}{d} + \frac{2\,e^{(-2\,dx - 2\,c)}}{d(2\,e^{(-2\,dx - 2\,c)} + e^{(-4\,dx - 4\,c)} + 1)}\right)$$
$$- 4\,ab^2\left(\frac{3\,e^{(-4\,dx - 4\,c)}}{d(6\,e^{(-2\,dx - 2\,c)} + 15\,e^{(-4\,dx - 4\,c)} + 20\,e^{(-6\,dx - 6\,c)} + 15\,e^{(-8\,dx - 8\,c)} + 6\,e^{(-10\,dx - 10\,c)} + e^{(-12\,dx - 12\,c)} + 1)}\right.$$
$$- \frac{32}{3}\,b^3\left(\frac{e^{(-6\,dx - 6\,c)}}{d(8\,e^{(-2\,dx - 2\,c)} + 28\,e^{(-4\,dx - 4\,c)} + 56\,e^{(-6\,dx - 6\,c)} + 70\,e^{(-8\,dx - 8\,c)} + 56\,e^{(-10\,dx - 10\,c)} + 28\,e^{(-12\,dx - 12\,c)}}\right.$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="maxima")
```

output
```
3/4*a^2*b*tanh(d*x + c)^4/d + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d +
 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a
*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20
*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*
x - 12*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x
 - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 + e^(-12*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) +
15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10
*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 32/3*b^3*(e^(-6*d*x - 6*c)/(d*(
8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*
d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x
- 14*c) + e^(-16*d*x - 16*c) + 1)) - e^(-8*d*x - 8*c)/(d*(8*e^(-2*d*x - 2*
c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*
e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16
*d*x - 16*c) + 1)) + e^(-10*d*x - 10*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*
d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 1
0*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) +
 1)))
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(95) = 190$.

Time = 0.38 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.76

$$\int \left(a + b\text{sech}^2(c + dx)\right)^3 \tanh^3(c + dx)\, dx =$$

$$-\frac{840\,(dx+c)a^3 - 840\,a^3\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{2283\,a^3e^{(16\,dx+16\,c)}+16584\,a^3e^{(14\,dx+14\,c)}+5040\,a^2be^{(14\,dx+14\,c)}+53844\,a^3e^{(1}}{}}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/840*(840*(d*x + c)*a^3 - 840*a^3*log(e^(2*d*x + 2*c) + 1) + (2283*a^3*e
^(16*d*x + 16*c) + 16584*a^3*e^(14*d*x + 14*c) + 5040*a^2*b*e^(14*d*x + 14
*c) + 53844*a^3*e^(12*d*x + 12*c) + 20160*a^2*b*e^(12*d*x + 12*c) + 10080*
a*b^2*e^(12*d*x + 12*c) + 102648*a^3*e^(10*d*x + 10*c) + 35280*a^2*b*e^(10
*d*x + 10*c) + 13440*a*b^2*e^(10*d*x + 10*c) + 8960*b^3*e^(10*d*x + 10*c)
+ 126210*a^3*e^(8*d*x + 8*c) + 40320*a^2*b*e^(8*d*x + 8*c) + 6720*a*b^2*e^
(8*d*x + 8*c) - 8960*b^3*e^(8*d*x + 8*c) + 102648*a^3*e^(6*d*x + 6*c) + 35
280*a^2*b*e^(6*d*x + 6*c) + 13440*a*b^2*e^(6*d*x + 6*c) + 8960*b^3*e^(6*d*
x + 6*c) + 53844*a^3*e^(4*d*x + 4*c) + 20160*a^2*b*e^(4*d*x + 4*c) + 10080
*a*b^2*e^(4*d*x + 4*c) + 16584*a^3*e^(2*d*x + 2*c) + 5040*a^2*b*e^(2*d*x +
 2*c) + 2283*a^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.125.9 Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 573, normalized size of antiderivative = 5.56

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^3\tanh^3(c+dx)\,dx$$
$$=\frac{32\,\left(3\,a\,b^2-5\,b^3\right)}{d\,\left(5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1\right)}-a^3\,x$$
$$-\frac{128\,b^3}{d\,\left(7\,{\mathrm{e}}^{2\,c+2\,d\,x}+21\,{\mathrm{e}}^{4\,c+4\,d\,x}+35\,{\mathrm{e}}^{6\,c+6\,d\,x}+35\,{\mathrm{e}}^{8\,c+8\,d\,x}+21\,{\mathrm{e}}^{10\,c+10\,d\,x}+7\,{\mathrm{e}}^{12\,c+12\,d\,x}+{\mathrm{e}}^{14\,c+14\,d\,x}+1\right)}$$
$$-\frac{32\,\left(3\,a\,b^2-19\,b^3\right)}{3\,d\,\left(6\,{\mathrm{e}}^{2\,c+2\,d\,x}+15\,{\mathrm{e}}^{4\,c+4\,d\,x}+20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}+6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d\,x}+1\right)}$$
$$+\frac{32\,b^3}{d\,\left(8\,{\mathrm{e}}^{2\,c+2\,d\,x}+28\,{\mathrm{e}}^{4\,c+4\,d\,x}+56\,{\mathrm{e}}^{6\,c+6\,d\,x}+70\,{\mathrm{e}}^{8\,c+8\,d\,x}+56\,{\mathrm{e}}^{10\,c+10\,d\,x}+28\,{\mathrm{e}}^{12\,c+12\,d\,x}+8\,{\mathrm{e}}^{14\,c+14\,d\,x}+{\mathrm{e}}^{16\,c}\cdots}$$
$$-\frac{8\,\left(9\,a^2\,b-21\,a\,b^2+4\,b^3\right)}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}$$
$$+\frac{4\,\left(3\,a^2\,b-27\,a\,b^2+16\,b^3\right)}{d\,\left(4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}$$
$$-\frac{2\,\left(3\,a^2\,b-a^3\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{a^3\,\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+1\right)}{d}-\frac{2\,\left(a^3-9\,a^2\,b+6\,a\,b^2\right)}{d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}$$

input
```
int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(32*(3*a*b^2 - 5*b^3))/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - a^3*x -
(128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*
x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) +
exp(14*c + 14*d*x) + 1)) - (32*(3*a*b^2 - 19*b^3))/(3*d*(6*exp(2*c + 2*d*x
) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*ex
p(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (32*b^3)/(d*(8*exp(2*c + 2*d
*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56
*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(1
6*c + 16*d*x) + 1)) - (8*(9*a^2*b - 21*a*b^2 + 4*b^3))/(3*d*(3*exp(2*c + 2
*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*(3*a^2*b - 27*a*b
^2 + 16*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*
d*x) + exp(8*c + 8*d*x) + 1)) - (2*(3*a^2*b - a^3))/(d*(exp(2*c + 2*d*x) +
 1)) + (a^3*log(exp(2*c)*exp(2*d*x) + 1))/d - (2*(6*a*b^2 - 9*a^2*b + a^3)
)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.126 $\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^2(c+dx)\,dx$

3.126.1 Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^2(c+dx)\,dx = a^3x-\frac{a^3\tanh(c+dx)}{d}+\frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+2b)\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

output
```
a^3*x-a^3*tanh(d*x+c)/d+1/3*b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)^3/d-1/5*b^2*(3
*a+2*b)*tanh(d*x+c)^5/d+1/7*b^3*tanh(d*x+c)^7/d
```

3.126.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 479 vs. 2(92) = 184.

Time = 3.78 (sec) , antiderivative size = 479, normalized size of antiderivative = 5.21

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^2(c+dx)\,dx = \frac{\text{sech}(c)\text{sech}^7(c+dx)\left(3675a^3dx\cosh(dx)+3675a^3dx\cosh(2c+dx)+2205a^3dx\cosh(2c+3dx)+2205a^3\right.}{}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]
```

output
```
(Sech[c]*Sech[c + d*x]^7*(3675*a^3*d*x*Cosh[d*x] + 3675*a^3*d*x*Cosh[2*c +
 d*x] + 2205*a^3*d*x*Cosh[2*c + 3*d*x] + 2205*a^3*d*x*Cosh[4*c + 3*d*x] +
735*a^3*d*x*Cosh[4*c + 5*d*x] + 735*a^3*d*x*Cosh[6*c + 5*d*x] + 105*a^3*d*
x*Cosh[6*c + 7*d*x] + 105*a^3*d*x*Cosh[8*c + 7*d*x] - 4200*a^3*Sinh[d*x] +
 3360*a^2*b*Sinh[d*x] + 840*a*b^2*Sinh[d*x] - 560*b^3*Sinh[d*x] + 3150*a^3
*Sinh[2*c + d*x] - 3990*a^2*b*Sinh[2*c + d*x] - 2100*a*b^2*Sinh[2*c + d*x]
 - 1120*b^3*Sinh[2*c + d*x] - 3150*a^3*Sinh[2*c + 3*d*x] + 1890*a^2*b*Sinh
[2*c + 3*d*x] + 504*a*b^2*Sinh[2*c + 3*d*x] + 336*b^3*Sinh[2*c + 3*d*x] +
1260*a^3*Sinh[4*c + 3*d*x] - 2520*a^2*b*Sinh[4*c + 3*d*x] - 1260*a*b^2*Sin
h[4*c + 3*d*x] - 1260*a^3*Sinh[4*c + 5*d*x] + 840*a^2*b*Sinh[4*c + 5*d*x]
+ 588*a*b^2*Sinh[4*c + 5*d*x] + 112*b^3*Sinh[4*c + 5*d*x] + 210*a^3*Sinh[6
*c + 5*d*x] - 630*a^2*b*Sinh[6*c + 5*d*x] - 210*a^3*Sinh[6*c + 7*d*x] + 21
0*a^2*b*Sinh[6*c + 7*d*x] + 84*a*b^2*Sinh[6*c + 7*d*x] + 16*b^3*Sinh[6*c +
 7*d*x]))/(13440*d)
```

3.126.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 4629, 25, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ic+idx)^2 \left(-\left(a+b\sec(ic+idx)^2\right)^3\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(b\sec(ic+idx)^2+a\right)^3\tan(ic+idx)^2dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\tanh^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^2(c+dx)(-b\tanh^2(c+dx)+a+b)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(b^3\tanh^6(c+dx)-b^2(3a+2b)\tanh^4(c+dx)+b(3a^2+3ba+b^2)\tanh^2(c+dx)-a^3+\frac{a^3}{1-\tanh^2(c+dx)}\right)d\tanh(}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^3\text{arctanh}(\tanh(c+dx))+a^3\tanh(c+dx)-\frac{1}{3}b(3a^2+3ab+b^2)\tanh^3(c+dx)+\frac{1}{5}b^2(3a+2b)\tanh^5(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]
```

output
```
-((-(a^3*ArcTanh[Tanh[c + d*x]]) + a^3*Tanh[c + d*x] - (b*(3*a^2 + 3*a*b +
 b^2)*Tanh[c + d*x]^3)/3 + (b^2*(3*a + 2*b)*Tanh[c + d*x]^5)/5 - (b^3*Tanh
[c + d*x]^7)/7)/d)
```

3.126.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.126. $\int \left(a+b\text{sech}^2(c+dx)\right)^3\tanh^2(c+dx)\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.126.4 Maple [A] (verified)

Time = 84.91 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.35

method	result
parts	$\frac{a^3\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^3\left(\frac{\tanh(dx+c)^7}{7}-\frac{2\tanh(dx+c)^5}{5}+\frac{\tanh(dx+c)^3}{3}\right)}{d}+\frac{a^2b\,\mathrm{ta}}{}$ …
derivativedivides	$a^3(dx+c-\tanh(dx+c))+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3a\,b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\mathrm{sech}(dx}{5}\right.}{}\right.$ …
default	$a^3(dx+c-\tanh(dx+c))+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3a\,b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\mathrm{sech}(dx}{5}\right.}{}\right.$ …
risch	$a^3x+\frac{12a^3e^{10dx+10c}+30a^3e^{8dx+8c}-6a^2b\,e^{12dx+12c}-\frac{4a\,b^2}{5}+2a^3-\frac{16\,e^{4dx+4c}b^3}{5}-24a^2b\,e^{10dx+10c}-2a^2b-\frac{28\,e^{2dx+2c}}{5}}{}$ …

input
```
int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(-tanh(d*x+c)-1/2*ln(tanh(d*x+c)-1)+1/2*ln(tanh(d*x+c)+1))+b^3/d*(1/
7*tanh(d*x+c)^7-2/5*tanh(d*x+c)^5+1/3*tanh(d*x+c)^3)+a^2*b*tanh(d*x+c)^3/d
+3*a*b^2/d*(-1/5*tanh(d*x+c)^5+1/3*tanh(d*x+c)^3)
```

3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(86) = 172$.

Time = 0.26 (sec) , antiderivative size = 881, normalized size of antiderivative = 9.58

$$\int \left(a + b\text{sech}^2(c+dx)\right)^3 \tanh^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/105*((105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c
)^7 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x +
c)*sinh(d*x + c)^6 - (105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*sinh(d*x + c
)^7 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x +
c)^5 - 7*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3 + 3*(105*a^3 - 105*a^2*b -
42*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((105*a^3*d*x + 10
5*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + (105*a^3*d*x + 105
*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(
105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 7*
(5*(105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 135*a^3 + 45
*a^2*b + 54*a*b^2 - 24*b^3 + 10*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(105*a^3*d*x + 105*a^3 - 105*a^2*b -
42*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 10*(105*a^3*d*x + 105*a^3 - 105*a^2*b
- 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 9*(105*a^3*d*x + 105*a^3 - 105*a^2*b
 - 42*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3*d*x + 10
5*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c) - 7*((105*a^3 - 105*a^
2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 5*(75*a^3 - 15*a^2*b - 42*a*b^2
- 8*b^3)*cosh(d*x + c)^4 + 75*a^3 + 45*a^2*b + 90*a*b^2 + 120*b^3 + 9*(45*
a^3 + 15*a^2*b + 18*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh
(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 +...
```

3.126.6 Sympy [F]

$$\int \left(a + b\text{sech}^2(c+dx)\right)^3 \tanh^2(c+dx)\,dx = \int \left(a + b\,\text{sech}^2\left(c+dx\right)\right)^3 \tanh^2\left(c+dx\right)dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**2, x)
```

3.126.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(86) = 172$.

Time = 0.20 (sec) , antiderivative size = 788, normalized size of antiderivative = 8.57

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\tanh^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="maxima")
```

output
```
a^2*b*tanh(d*x + c)^3/d + a^3*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 1
6/105*b^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e
^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
 - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c
) + 1)) - 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 70*e^(-8*d*x - 8*c)/(d*(7*e
^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
 - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c
) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x -
6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
+ e^(-14*d*x - 14*c) + 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*
x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
+ e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6...
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(86) = 172$.

Time = 0.35 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.90

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\tanh^2(c+dx)\,dx$$
$$= \frac{105\,(dx+c)a^3 + \frac{2\left(105\,a^3e^{(12\,dx+12\,c)} - 315\,a^2be^{(12\,dx+12\,c)} + 630\,a^3e^{(10\,dx+10\,c)} - 1260\,a^2be^{(10\,dx+10\,c)} - 630\,ab^2e^{(10\,dx+10\,c)} + 1575\,a^3e^{(8\ldots}{\ldots}}{\ldots}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="giac")
```

output
```
1/105*(105*(d*x + c)*a^3 + 2*(105*a^3*e^(12*d*x + 12*c) - 315*a^2*b*e^(12*
d*x + 12*c) + 630*a^3*e^(10*d*x + 10*c) - 1260*a^2*b*e^(10*d*x + 10*c) - 6
30*a*b^2*e^(10*d*x + 10*c) + 1575*a^3*e^(8*d*x + 8*c) - 1995*a^2*b*e^(8*d*
x + 8*c) - 1050*a*b^2*e^(8*d*x + 8*c) - 560*b^3*e^(8*d*x + 8*c) + 2100*a^3
*e^(6*d*x + 6*c) - 1680*a^2*b*e^(6*d*x + 6*c) - 420*a*b^2*e^(6*d*x + 6*c)
+ 280*b^3*e^(6*d*x + 6*c) + 1575*a^3*e^(4*d*x + 4*c) - 945*a^2*b*e^(4*d*x
+ 4*c) - 252*a*b^2*e^(4*d*x + 4*c) - 168*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(
2*d*x + 2*c) - 420*a^2*b*e^(2*d*x + 2*c) - 294*a*b^2*e^(2*d*x + 2*c) - 56*
b^3*e^(2*d*x + 2*c) + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)/(e^(2*d*x +
2*c) + 1)^7)/d
```

3.126.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1133, normalized size of antiderivative = 12.32

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^3\tanh^2(c+dx)\,dx = \text{Too large to display}$$

input
```
int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
a^3*x - ((2*(2*a*b^2 + a^2*b - a^3))/(7*d) - (2*exp(2*c + 2*d*x)*(3*a^2*b
+ 15*a^3 - 16*b^3))/(21*d) - (4*exp(6*c + 6*d*x)*(3*a^2*b + 15*a^3 - 16*b^
3))/(21*d) + (10*exp(8*c + 8*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) - (4*exp(
4*c + 4*d*x)*(6*a*b^2 + 3*a^2*b + 5*a^3 + 8*b^3))/(7*d) + (2*exp(10*c + 10
*d*x)*(3*a^2*b - a^3))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) +
20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*
c + 12*d*x) + 1) - ((2*(3*a^2*b - a^3))/(7*d) - (2*exp(4*c + 4*d*x)*(3*a^2
*b + 15*a^3 - 16*b^3))/(7*d) - (2*exp(8*c + 8*d*x)*(3*a^2*b + 15*a^3 - 16*
b^3))/(7*d) + (12*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) + (12*ex
p(10*c + 10*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) - (8*exp(6*c + 6*d*x)*(6*a
*b^2 + 3*a^2*b + 5*a^3 + 8*b^3))/(7*d) + (2*exp(12*c + 12*d*x)*(3*a^2*b -
a^3))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*
x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) +
exp(14*c + 14*d*x) + 1) - ((4*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b - a^3))/(7
*d) - (2*(3*a^2*b + 15*a^3 - 16*b^3))/(105*d) + (2*exp(4*c + 4*d*x)*(3*a^2
*b - a^3))/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d
*x) + 1) + ((2*(6*a*b^2 + 3*a^2*b + 5*a^3 + 8*b^3))/(35*d) + (2*exp(2*c +
2*d*x)*(3*a^2*b + 15*a^3 - 16*b^3))/(35*d) - (6*exp(4*c + 4*d*x)*(2*a*b^2
+ a^2*b - a^3))/(7*d) - (2*exp(6*c + 6*d*x)*(3*a^2*b - a^3))/(7*d))/(4*exp
(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d...
```

3.126. $\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\tanh^2(c+dx)\,dx$

3.127 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx$

3.127.1 Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx=\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\operatorname{sech}^2(c+dx)}{2d}-\frac{3ab^2\operatorname{sech}^4(c+dx)}{4d}-\frac{b^3\operatorname{sech}^6(c+dx)}{6d}$$

output
```
a^3*ln(cosh(d*x+c))/d-3/2*a^2*b*sech(d*x+c)^2/d-3/4*a*b^2*sech(d*x+c)^4/d-
1/6*b^3*sech(d*x+c)^6/d
```

3.127.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx$$
$$=\frac{\cosh^6(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\left(8a^3\log(\cosh(c+dx))-12a^2b\operatorname{sech}^2(c+dx)-6ab^2\operatorname{sech}^4(c+dx)-\right.}{d(a+2b+a\cosh(2c+2dx))^3}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x],x]
```

output
```
(Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(8*a^3*Log[Cosh[c + d*x]] - 12*
a^2*b*Sech[c + d*x]^2 - 6*a*b^2*Sech[c + d*x]^4 - (4*b^3*Sech[c + d*x]^6)/
3))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

3.127.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ic+idx)\left(a+b\sec(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \left(b\sec(ic+idx)^2+a\right)^3\tan(ic+idx)dx$$

$$\Big\downarrow \text{4626}$$

$$\frac{\int \left(a\cosh^2(c+dx)+b\right)^3\text{sech}^7(c+dx)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \left(a\cosh^2(c+dx)+b\right)^3\text{sech}^4(c+dx)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(b^3\text{sech}^4(c+dx)+3ab^2\text{sech}^3(c+dx)+3a^2b\text{sech}^2(c+dx)+a^3\text{sech}(c+dx)\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\log\left(\cosh^2(c+dx)\right)-3a^2b\text{sech}(c+dx)-\frac{3}{2}ab^2\text{sech}^2(c+dx)-\frac{1}{3}b^3\text{sech}^3(c+dx)}{2d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x],x]
```

output
```
(a^3*Log[Cosh[c + d*x]^2] - 3*a^2*b*Sech[c + d*x] - (3*a*b^2*Sech[c + d*x]
^2)/2 - (b^3*Sech[c + d*x]^3)/3)/(2*d)
```

3.127.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.127.4 Maple [A] (verified)

Time = 60.58 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{\frac{\operatorname{sech}(dx+c)^6 b^3}{6}+\frac{3\operatorname{sech}(dx+c)^4 a\,b^2}{4}+\frac{3\operatorname{sech}(dx+c)^2 a^2 b}{2}+a^3\ln(\operatorname{sech}(dx+c))}{d}$
default	$-\frac{\frac{\operatorname{sech}(dx+c)^6 b^3}{6}+\frac{3\operatorname{sech}(dx+c)^4 a\,b^2}{4}+\frac{3\operatorname{sech}(dx+c)^2 a^2 b}{2}+a^3\ln(\operatorname{sech}(dx+c))}{d}$
parts	$\frac{a^3\ln(\cosh(dx+c))}{d}-\frac{b^3\operatorname{sech}(dx+c)^6}{6d}+\frac{3a^2b\tanh(dx+c)^2}{2d}-\frac{3a\,b^2\operatorname{sech}(dx+c)^4}{4d}$
risch	$-a^3x-\frac{2a^3c}{d}-\frac{2b\,e^{2dx+2c}(9a^2e^{8dx+8c}+36a^2e^{6dx+6c}+18ab\,e^{6dx+6c}+54a^2e^{4dx+4c}+36ab\,e^{4dx+4c}+16\,e^{4dx+4c}b^2+}{3d(e^{2dx+2c}+1)^6}$

input
```
int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/6*sech(d*x+c)^6*b^3+3/4*sech(d*x+c)^4*a*b^2+3/2*sech(d*x+c)^2*a^2*
b+a^3*ln(sech(d*x+c)))
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2519 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 2519, normalized size of antiderivative = 35.48

$$\int \left(a + b\text{sech}^2(c + dx)\right)^3 \tanh(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="fricas")
```

output
```
-1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^
11 + 3*a^3*d*x*sinh(d*x + c)^12 + 18*(a^3*d*x + a^2*b)*cosh(d*x + c)^10 +
18*(11*a^3*d*x*cosh(d*x + c)^2 + a^3*d*x + a^2*b)*sinh(d*x + c)^10 + 60*(1
1*a^3*d*x*cosh(d*x + c)^3 + 3*(a^3*d*x + a^2*b)*cosh(d*x + c))*sinh(d*x +
c)^9 + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^8 + 9*(165*a^3*d*x*
cosh(d*x + c)^4 + 5*a^3*d*x + 8*a^2*b + 4*a*b^2 + 90*(a^3*d*x + a^2*b)*cos
h(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*x + c)^5 + 30*(a^3*d
*x + a^2*b)*cosh(d*x + c)^3 + (5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c
))*sinh(d*x + c)^7 + 4*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x
 + c)^6 + 4*(693*a^3*d*x*cosh(d*x + c)^6 + 15*a^3*d*x + 945*(a^3*d*x + a^2
*b)*cosh(d*x + c)^4 + 27*a^2*b + 18*a*b^2 + 8*b^3 + 63*(5*a^3*d*x + 8*a^2*
b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*a^3*d*x*cosh(d*x +
c)^7 + 189*(a^3*d*x + a^2*b)*cosh(d*x + c)^5 + 21*(5*a^3*d*x + 8*a^2*b + 4
*a*b^2)*cosh(d*x + c)^3 + (15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(
d*x + c))*sinh(d*x + c)^5 + 3*a^3*d*x + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*
cosh(d*x + c)^4 + 3*(495*a^3*d*x*cosh(d*x + c)^8 + 1260*(a^3*d*x + a^2*b)*
cosh(d*x + c)^6 + 15*a^3*d*x + 210*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*
x + c)^4 + 24*a^2*b + 12*a*b^2 + 20*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 + 5
40*(a^3*d*x + a^2*b)*cosh(d*x + c)^7 + 126*(5*a^3*d*x + 8*a^2*b + 4*a*b...
```

3.127.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx = \begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\,\text{sech}^2(c+dx)}{2d} - \frac{3ab^2\,\text{sech}^4(c+dx)}{4d} - \frac{b^3\,\text{sech}^6(c+dx)}{6d} & \text{for } d\neq 0 \\ x\left(a+b\,\text{sech}^2(c)\right)^3\tanh(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c),x)
```

output
```
Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - 3*a**2*b*sech(c + d*x)
**2/(2*d) - 3*a*b**2*sech(c + d*x)**4/(4*d) - b**3*sech(c + d*x)**6/(6*d),
 Ne(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c), True))
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx = \frac{3\,a^2b\tanh(dx+c)^2}{2\,d} + \frac{a^3\log(\cosh(dx+c))}{d} - \frac{12\,ab^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4} - \frac{32\,b^3}{3\,d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^6}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="maxima")
```

output
```
3/2*a^2*b*tanh(d*x + c)^2/d + a^3*log(cosh(d*x + c))/d - 12*a*b^2/(d*(e^(d
*x + c) + e^(-d*x - c))^4) - 32/3*b^3/(d*(e^(d*x + c) + e^(-d*x - c))^6)
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(65) = 130$.

Time = 0.32 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.82

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx =$$

$$-\frac{60\,(dx+c)a^3 - 60\,a^3\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{147\,a^3e^{(12\,dx+12\,c)}+882\,a^3e^{(10\,dx+10\,c)}+360\,a^2be^{(10\,dx+10\,c)}+2205\,a^3e^{(8\,dx+8\,c)}+}{}}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="giac")
```

output
```
-1/60*(60*(d*x + c)*a^3 - 60*a^3*log(e^(2*d*x + 2*c) + 1) + (147*a^3*e^(12
*d*x + 12*c) + 882*a^3*e^(10*d*x + 10*c) + 360*a^2*b*e^(10*d*x + 10*c) + 2
205*a^3*e^(8*d*x + 8*c) + 1440*a^2*b*e^(8*d*x + 8*c) + 720*a*b^2*e^(8*d*x
+ 8*c) + 2940*a^3*e^(6*d*x + 6*c) + 2160*a^2*b*e^(6*d*x + 6*c) + 1440*a*b^
2*e^(6*d*x + 6*c) + 640*b^3*e^(6*d*x + 6*c) + 2205*a^3*e^(4*d*x + 4*c) + 1
440*a^2*b*e^(4*d*x + 4*c) + 720*a*b^2*e^(4*d*x + 4*c) + 882*a^3*e^(2*d*x +
 2*c) + 360*a^2*b*e^(2*d*x + 2*c) + 147*a^3)/(e^(2*d*x + 2*c) + 1)^6)/d
```

3.127.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 347, normalized size of antiderivative = 4.89

$$\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx$$
$$= \frac{32\,b^3}{3\,d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}+6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)} - \frac{4\,\left(3\,a\,b^2-8\,b^3\right)}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)} - a^3\,x - \frac{6\,\left(2\,a\,b^2-a^2\,b\right)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} + \frac{a^3\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)}{d} + \frac{8\,\left(9\,a\,b^2-4\,b^3\right)}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)} - \frac{32\,b^3}{d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)} - \frac{6\,a^2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(32*b^3)/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d
*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)
) - (4*(3*a*b^2 - 8*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*
exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - a^3*x - (6*(2*a*b^2 - a^2*b))/
(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (a^3*log(exp(2*c)*exp(2*
d*x) + 1))/d + (8*(9*a*b^2 - 4*b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c
+ 4*d*x) + exp(6*c + 6*d*x) + 1)) - (32*b^3)/(d*(5*exp(2*c + 2*d*x) + 10*e
xp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10
*d*x) + 1)) - (6*a^2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.128 $\int (a + b\text{sech}^2(c + dx))^3\, dx$

3.128.1 Optimal result

Integrand size = 14, antiderivative size = 73

$$\int (a + b\text{sech}^2(c + dx))^3\, dx = a^3x + \frac{b(3a^2 + 3ab + b^2)\tanh(c + dx)}{d} - \frac{b^2(3a + 2b)\tanh^3(c + dx)}{3d} + \frac{b^3\tanh^5(c + dx)}{5d}$$

output
```
a^3*x+b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(3*a+2*b)*tanh(d*x+c)^3/d+
1/5*b^3*tanh(d*x+c)^5/d
```

3.128.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int (a + b\text{sech}^2(c + dx))^3\, dx = a^3x + \frac{3a^2b\tanh(c + dx)}{d} + \frac{3ab^2\tanh(c + dx)}{d} + \frac{b^3\tanh(c + dx)}{d} - \frac{ab^2\tanh^3(c + dx)}{d} - \frac{2b^3\tanh^3(c + dx)}{3d} + \frac{b^3\tanh^5(c + dx)}{5d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
a^3*x + (3*a^2*b*Tanh[c + d*x])/d + (3*a*b^2*Tanh[c + d*x])/d + (b^3*Tanh[
c + d*x])/d - (a*b^2*Tanh[c + d*x]^3)/d - (2*b^3*Tanh[c + d*x]^3)/(3*d) +
(b^3*Tanh[c + d*x]^5)/(5*d)
```

3.128.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a+b\sec(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{ 4616}$$

$$\frac{\int \frac{\left(-b\tanh^2(c+dx)+a+b\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(b^3\tanh^4(c+dx)-b^2(3a+2b)\tanh^2(c+dx)+b\left(3a^2+3ba+b^2\right)+\frac{a^3}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3\text{arctanh}(\tanh(c+dx))+b\left(3a^2+3ab+b^2\right)\tanh(c+dx)-\frac{1}{3}b^2(3a+2b)\tanh^3(c+dx)+\frac{1}{5}b^3\tanh^5(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(a^3*ArcTanh[Tanh[c + d*x]] + b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x] - (b^2
*(3*a + 2*b)*Tanh[c + d*x]^3)/3 + (b^3*Tanh[c + d*x]^5)/5)/d
```

3.128.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.128.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a^{3}(dx+c)+3a^{2}b\tanh(dx+c)+3a\,b^{2}\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^{2}}{3}\right)\tanh(dx+c)+b^{3}\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^{4}}{5}+\frac{4\operatorname{sech}(dx+c)^{2}}{15}\right)\tanh(dx+c)}{d}$
default	$\frac{a^{3}(dx+c)+3a^{2}b\tanh(dx+c)+3a\,b^{2}\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^{2}}{3}\right)\tanh(dx+c)+b^{3}\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^{4}}{5}+\frac{4\operatorname{sech}(dx+c)^{2}}{15}\right)\tanh(dx+c)}{d}$
parts	$a^{3}x+\frac{b^{3}\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^{4}}{5}+\frac{4\operatorname{sech}(dx+c)^{2}}{15}\right)\tanh(dx+c)}{d}+\frac{3a^{2}b\tanh(dx+c)}{d}+\frac{3a\,b^{2}\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^{2}}{3}\right)\tanh(dx+}{d}$
risch	$a^{3}x-\frac{2b(45a^{2}e^{8dx+8c}+180a^{2}e^{6dx+6c}+90ab\,e^{6dx+6c}+270a^{2}e^{4dx+4c}+210ab\,e^{4dx+4c}+80\,e^{4dx+4c}b^{2}+180a^{2}e^{2dx+2c}+}{15d\left(e^{2dx+2c}+1\right)^{5}}$
parallelrisch	$15x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}a^{3}d+30(3a^{2}b+3a\,b^{2}+b^{3})\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+75x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a^{3}d+360\left(a+\frac{b}{3}\right)^{2}b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+1$

input
```
int((a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)+3*a^2*b*tanh(d*x+c)+3*a*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(
d*x+c)+b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(69) = 138$.

Time = 0.25 (sec) , antiderivative size = 470, normalized size of antiderivative = 6.44

$$\int \left(a + b\text{sech}^2(c + dx)\right)^3 dx$$
$$= \frac{(15\,a^3dx - 45\,a^2b - 30\,ab^2 - 8\,b^3)\cosh{(dx + c)}^5 + 5\,(15\,a^3dx - 45\,a^2b - 30\,ab^2 - 8\,b^3)\cosh{(dx + c)}\sinh}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*((15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 5*(15*a
^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (45*
a^2*b + 30*a*b^2 + 8*b^3)*sinh(d*x + c)^5 + 5*(15*a^3*d*x - 45*a^2*b - 30*
a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 5*(27*a^2*b + 30*a*b^2 + 8*b^3 + 2*(45*a^
2*b + 30*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(15*a^3*d*
x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(15*a^3*d*x - 45*a^2*
b - 30*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a^3*d*x - 45
*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c) + 5*((45*a^2*b + 30*a*b^2 + 8*b^3
)*cosh(d*x + c)^4 + 18*a^2*b + 24*a*b^2 + 16*b^3 + 3*(27*a^2*b + 30*a*b^2
+ 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x
 + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d
*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

3.128.6 Sympy [F]

$$\int \left(a + b\text{sech}^2(c + dx)\right)^3 dx = \int \left(a + b\,\text{sech}^2\left(c + dx\right)\right)^3 dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3, x)
```

3.128.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(69) = 138$.

Time = 0.19 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.55

$$\int \left(a + b\mathrm{sech}^2(c + dx)\right)^3 \, dx = a^3x + \frac{16}{15}b^3\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)} + \frac{}{d(5\,e^{(-2\,dx-2\,c}}\right. + 4\,ab^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)} + \frac{1}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right. + \frac{6\,a^2b}{d(e^{(-2\,dx-2\,c)} + 1)}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*x + 16/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*
x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d
*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8
*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3
*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 6*a^2*b
/(d*(e^(-2*d*x - 2*c) + 1))
```

3.128.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(69) = 138$.

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.49

$$\int \left(a + b\mathrm{sech}^2(c + dx)\right)^3 \, dx = \frac{15\,(dx + c)a^3 - \frac{2\,(45\,a^2be^{(8\,dx+8\,c)} + 180\,a^2be^{(6\,dx+6\,c)} + 90\,ab^2e^{(6\,dx+6\,c)} + 270\,a^2be^{(4\,dx+4\,c)} + 210\,ab^2e^{(4\,dx+4\,c)} + 80\,b^3e^{(4\,dx+4\,c)} + 180\,a^2b}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

input
```
integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)*a^3 - 2*(45*a^2*b*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x
+ 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 210*a*b^2*
e^(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) + 150
*a*b^2*e^(2*d*x + 2*c) + 40*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 30*a*b^2 + 8*
b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.128.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 502, normalized size of antiderivative = 6.88

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$= a^3\,x-\frac{\frac{2\,\left(9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{15\,d}+\frac{12\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,a^2\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}$$
$$-\frac{\frac{6\,a^2\,b}{5\,d}+\frac{24\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{24\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{5\,d}+\frac{6\,a^2\,b\,{\mathrm{e}}^{8\,c+8\,d\,x}}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}$$
$$-\frac{\frac{6\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{18\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{5\,d}+\frac{6\,a^2\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}$$
$$-\frac{\frac{6\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,a^2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{6\,a^2\,b}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^3,x)
```

output
```
a^3*x - ((2*(12*a*b^2 + 9*a^2*b + 8*b^3))/(15*d) + (12*exp(2*c + 2*d*x)*(a
*b^2 + a^2*b))/(5*d) + (6*a^2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*
x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((6*a^2*b)/(5*d) + (24*e
xp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d) + (24*exp(6*c + 6*d*x)*(a*b^2 + a^2
*b))/(5*d) + (4*exp(4*c + 4*d*x)*(12*a*b^2 + 9*a^2*b + 8*b^3))/(5*d) + (6*
a^2*b*exp(8*c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) +
 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((6*
(a*b^2 + a^2*b))/(5*d) + (18*exp(4*c + 4*d*x)*(a*b^2 + a^2*b))/(5*d) + (2*
exp(2*c + 2*d*x)*(12*a*b^2 + 9*a^2*b + 8*b^3))/(5*d) + (6*a^2*b*exp(6*c +
6*d*x))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*
x) + exp(8*c + 8*d*x) + 1) - ((6*(a*b^2 + a^2*b))/(5*d) + (6*a^2*b*exp(2*c
 + 2*d*x))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (6*a^2*b)/
(5*d*(exp(2*c + 2*d*x) + 1))
```

3.129 $\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

3.129.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = -\frac{b(3a^2+3ab+b^2)\log(\cosh(c+dx))}{d} + \frac{(a+b)^3\log(\sinh(c+dx))}{d} + \frac{b^2(3a+b)\text{sech}^2(c+dx)}{2d} + \frac{b^3\text{sech}^4(c+dx)}{4d}$$

output
```
-b*(3*a^2+3*a*b+b^2)*ln(cosh(d*x+c))/d+(a+b)^3*ln(sinh(d*x+c))/d+1/2*b^2*(
3*a+b)*sech(d*x+c)^2/d+1/4*b^3*sech(d*x+c)^4/d
```

3.129.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = -\frac{2\cosh^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\left(4b(3a^2+3ab+b^2)\log(\cosh(c+dx))-4(a+b)^3\log(\sinh(c+d\right.}{d(a+2b+a\cosh(2c+2dx))^3}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-2*Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(4*b*(3*a^2 + 3*a*b + b^2)*L
og[Cosh[c + d*x]] - 4*(a + b)^3*Log[Sinh[c + d*x]] - 2*b^2*(3*a + b)*Sech[
c + d*x]^2 - b^3*Sech[c + d*x]^4))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

3.129.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sec(ic+idx)^2\right)^3}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sec(ic+idx)^2+a\right)^3}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^3\text{sech}^5(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^3\text{sech}^3(c+dx)}{1-\cosh^2(c+dx)}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(-\frac{(a+b)^3}{\cosh^2(c+dx)-1}+b^3\text{sech}^3(c+dx)+b^2(3a+b)\text{sech}^2(c+dx)+b\left(3a^2+3ba+b^2\right)\text{sech}(c+dx)\right)d\cosh^2(c+d}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(3a^2+3ab+b^2\right)\log\left(\cosh^2(c+dx)\right)-b^2(3a+b)\text{sech}(c+dx)-(a+b)^3\log\left(1-\cosh^2(c+dx)\right)-\frac{1}{2}b^3\text{sech}^2(c}{2d}$$

3.129. $\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

input
```
Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/2*(b*(3*a^2 + 3*a*b + b^2)*Log[Cosh[c + d*x]^2] - (a + b)^3*Log[1 - Cos
h[c + d*x]^2] - b^2*(3*a + b)*Sech[c + d*x] - (b^3*Sech[c + d*x]^2)/2)/d
```

3.129.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.129.4 Maple [A] (verified)

Time = 46.66 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a^3 \ln(\sinh(dx+c))+3\ln(\tanh(dx+c))a^2b+3a\,b^2\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^3\left(\frac{1}{4\cosh(dx+c)^4}+\frac{1}{2\cosh(dx+c)^2}+\ln\right.}{d}$
default	$\frac{a^3 \ln(\sinh(dx+c))+3\ln(\tanh(dx+c))a^2b+3a\,b^2\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^3\left(\frac{1}{4\cosh(dx+c)^4}+\frac{1}{2\cosh(dx+c)^2}+\ln\right.}{d}$
risch	$-a^3x-\frac{2a^3c}{d}+\frac{2b^2 e^{2dx+2c}\left(3a\,e^{4dx+4c}+b\,e^{4dx+4c}+6\,e^{2dx+2c}a+4b\,e^{2dx+2c}+3a+b\right)}{d\left(e^{2dx+2c}+1\right)^4}-\frac{3b\ln\left(e^{2dx+2c}+1\right)a^2}{d}-$ 3

input
```
int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*ln(sinh(d*x+c))+3*ln(tanh(d*x+c))*a^2*b+3*a*b^2*(1/2/cosh(d*x+c)^
2+ln(tanh(d*x+c)))+b^3*(1/4/cosh(d*x+c)^4+1/2/cosh(d*x+c)^2+ln(tanh(d*x+c)
)))
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2376 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 2376, normalized size of antiderivative = 28.29

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*
d*x*sinh(d*x + c)^8 + 2*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^6 + 2*(1
4*a^3*d*x*cosh(d*x + c)^2 + 2*a^3*d*x - 3*a*b^2 - b^3)*sinh(d*x + c)^6 + 4
*(14*a^3*d*x*cosh(d*x + c)^3 + 3*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)
)*sinh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*cosh(d*x + c
)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 + 3*a^3*d*x - 6*a*b^2 - 4*b^3 + 15*(2*
a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*c
osh(d*x + c)^5 + 5*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + (3*a^3*d*
x - 6*a*b^2 - 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(2*a^3*d*x - 3*a*b
^2 - b^3)*cosh(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 + 2*a^3*d*x + 15
*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3 + 6*(3*a^3*d*
x - 6*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)^8 + 8*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(
d*x + c)^7 + (3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(3*a^2*b + 3*a*
b^2 + b^3)*cosh(d*x + c)^6 + 4*(3*a^2*b + 3*a*b^2 + b^3 + 7*(3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 6*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(3*a^2*b + 3
*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2*b + 3*a*b^...
```

3.129.6 Sympy [F]

$$\int \coth(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx=\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^3\coth\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**3*coth(c + d*x), x)
```

3.129.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(80) = 160$.

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.57

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$= b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\left(e^{(-2\,dx-2\,c)}+4\,e^{(-4\,dx-4\,c}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-}}\right.$$
$$+3\,ab^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right.$$
$$+3\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}\right)$$
$$+\frac{a^3\log\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
b^3*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2
*c) + 1)/d + 2*(e^(-2*d*x - 2*c) + 4*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/
(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d
*x - 8*c) + 1))) + 3*a*b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1
)/d - log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2
*c) + e^(-4*d*x - 4*c) + 1))) + 3*a^2*b*(log(e^(-d*x - c) + 1)/d + log(e^(
-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d) + a^3*log(sinh(d*x + c))/d
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(80) = 160$.

Time = 0.32 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.37

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx =$$
$$-\frac{2\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,c}\right.}{}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) +
 2) - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x -
2*c) - 2) - (9*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 9*a*b^2*(e^(
2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x -
2*c))^2 + 36*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 60*a*b^2*(e^(2*d
*x + 2*c) + e^(-2*d*x - 2*c)) + 20*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)
) + 36*a^2*b + 84*a*b^2 + 44*b^3)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)
^2)/d
```

3.129.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.29

$$\int \coth(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx = \frac{2\,(b^3+3\,a\,b^2)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - a^3\,x$$
$$-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(a^3\,\sqrt{-d^2}+2\,b^3\,\sqrt{-d^2}+6\,a\,b^2\,\sqrt{-d^2}+6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+12\,a^5\,b+48\,a^4\,b^2+76\,a^3\,b^3+60\,a^2\,b^4+24\,a\,b^5+4\,b^6}}\right)\,\sqrt{a^6+12\,a^5\,b+48\,a^4\,b^2+76\,a^3\,b^3+60\,a^2\,b^4+24}{\sqrt{-d^2}}$$
$$+\frac{a^3\,\ln\left(\mathrm{e}^{4\,c+4\,d\,x}-1\right)}{2\,d} - \frac{8\,b^3}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}$$
$$-\frac{2\,(3\,a\,b^2-b^3)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} + \frac{4\,b^3}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$

input
```
int(coth(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(2*(3*a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - a^3*x - (atan((exp(2*c)*e
xp(2*d*x)*(a^3*(-d^2)^(1/2) + 2*b^3*(-d^2)^(1/2) + 6*a*b^2*(-d^2)^(1/2) +
6*a^2*b*(-d^2)^(1/2)))/(d*(24*a*b^5 + 12*a^5*b + a^6 + 4*b^6 + 60*a^2*b^4
+ 76*a^3*b^3 + 48*a^4*b^2)^(1/2)))*(24*a*b^5 + 12*a^5*b + a^6 + 4*b^6 + 60
*a^2*b^4 + 76*a^3*b^3 + 48*a^4*b^2)^(1/2))/(-d^2)^(1/2) + (a^3*log(exp(4*c
 + 4*d*x) - 1))/(2*d) - (8*b^3)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x
) + exp(6*c + 6*d*x) + 1)) - (2*(3*a*b^2 - b^3))/(d*(2*exp(2*c + 2*d*x) +
exp(4*c + 4*d*x) + 1)) + (4*b^3)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*
x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.130 $\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx$

3.130.1 Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx=a^3x-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

output
```
a^3*x-(a+b)^3*coth(d*x+c)/d-b^2*(3*a+2*b)*tanh(d*x+c)/d+1/3*b^3*tanh(d*x+c
)^3/d
```

3.130.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(61) = 122.

Time = 6.79 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.66

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx$$
$$=\frac{8(a\cosh(c+dx)+b\text{sech}(c+dx))^3\left(3a^3dx\cosh^3(c+dx)-b^3\text{sech}(c)\sinh(dx)+\frac{1}{2}\cosh(c+dx)\left((3a^3+9\right.\right.}{3d(a}$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(8*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(3*a^3*d*x*Cosh[c + d*x]^3 - b^3*
Sech[c]*Sinh[d*x] + (Cosh[c + d*x]*((3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*C
osh[d*x] + (3*a^3 + 9*a^2*b - 2*b^3)*Cosh[2*c + d*x])*Coth[c + d*x]*Csch[c
]*Sech[c]*Sinh[d*x])/2 - b^3*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cos
h[2*(c + d*x)])^3)
```

3.130.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 4629, 25, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+b\sec(ic+idx)^2\right)^3}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sec(ic+idx)^2+a\right)^3}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\coth^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(-\frac{a^3}{\tanh^2(c+dx)-1} + (a+b)^3 \coth^2(c+dx) + b^3 \tanh^2(c+dx) - b^2(3a+2b)\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3(-\text{arctanh}(\tanh(c+dx))) + b^2(3a+2b)\tanh(c+dx) + (a+b)^3\coth(c+dx) - \frac{1}{3}b^3\tanh^3(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-((-(a^3*ArcTanh[Tanh[c + d*x]]) + (a + b)^3*Coth[c + d*x] + b^2*(3*a + 2*
b)*Tanh[c + d*x] - (b^3*Tanh[c + d*x]^3)/3)/d)
```

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.130.4 Maple [A] (verified)

Time = 47.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{a^3(dx+c-\coth(dx+c))-3a^2b\coth(dx+c)+3ab^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^3\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^3}\right.}{d}$
default	$\frac{a^3(dx+c-\coth(dx+c))-3a^2b\coth(dx+c)+3ab^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^3\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^3}\right.}{d}$
risch	$a^3x-\frac{2(3a^3e^{6dx+6c}+9a^2be^{6dx+6c}+9a^3e^{4dx+4c}+27a^2be^{4dx+4c}+18ab^2e^{4dx+4c}+9a^3e^{2dx+2c}+27a^2be^{2dx+2c}+36e^{2c}}{3d(e^{2dx+2c}+1)^3(e^{2dx+2c}-1)}$

input
```
int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c-coth(d*x+c))-3*a^2*b*coth(d*x+c)+3*a*b^2*(-1/sinh(d*x+c)/c
osh(d*x+c)-2*tanh(d*x+c))+b^3*(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sec
h(d*x+c)^2)*tanh(d*x+c)))
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 5.89

$$\int \coth^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$
$$-\frac{(3a^3+9a^2b+18ab^2+8b^3)\cosh(dx+c)^4-4(3a^3dx+3a^3+9a^2b+18ab^2+8b^3)\cosh(dx+c)\sin}{}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/12*((3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x
 + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*
a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*sinh(d*x + c)^4 + 9*a^3 + 27*a^2*b + 18*
a*b^2 + 4*(3*a^3 + 9*a^2*b + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(6*a^3 +
 18*a^2*b + 18*a*b^2 + 8*b^3 + 3*(3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 - 4*((3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2
+ 8*b^3)*cosh(d*x + c)^3 + (3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3
)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh
(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \coth^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.130.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(59) = 118.

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.82

$$\int \coth^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=a^3\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)-\frac{16}{3}\,b^3\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1)}+\frac{1}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1)}\right)+\frac{6\,a^2b}{d(e^{(-2\,dx-2\,c)}-1)}+\frac{12\,ab^2}{d(e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) - 16/3*b^3*(2*e^(-2*d*x - 2*c
)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1)) + 1
/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1))) + 6
*a^2*b/(d*(e^(-2*d*x - 2*c) - 1)) + 12*a*b^2/(d*(e^(-4*d*x - 4*c) - 1))
```

3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(59) = 118$.

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.21

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{3\,(dx+c)a^3 - \frac{6\,(a^3+3\,a^2b+3\,ab^2+b^3)}{e^{(2\,dx+2\,c)}-1} + \frac{2\,(9\,ab^2e^{(4\,dx+4\,c)}+3\,b^3e^{(4\,dx+4\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+12\,b^3e^{(2\,dx+2\,c)}+9\,ab^2+5\,b^3)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^3 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(e^(2*d*x + 2*c)
- 1) + 2*(9*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*
d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 5*b^3)/(e^(2*d*x + 2*c) +
1)^3)/d
```

3.130.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.84

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\frac{2\,(b^3+3\,a\,b^2)}{3\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(b^3+a\,b^2)}{d}+\frac{2\,e^{4\,c+4\,d\,x}\,(b^3+3\,a\,b^2)}{3\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1} + a^3\,x + \frac{\frac{2\,(b^3+a\,b^2)}{d}+\frac{2\,e^{2\,c+2\,d\,x}\,(b^3+3\,a\,b^2)}{3\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1} + \frac{2\,(b^3+3\,a\,b^2)}{3\,d\,(e^{2\,c+2\,d\,x}+1)} - \frac{2\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{d\,(e^{2\,c+2\,d\,x}-1)}$$

input
```
int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
((2*(3*a*b^2 + b^3))/(3*d) + (4*exp(2*c + 2*d*x)*(a*b^2 + b^3))/d + (2*exp
(4*c + 4*d*x)*(3*a*b^2 + b^3))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) + 1) + a^3*x + ((2*(a*b^2 + b^3))/d + (2*exp(2*c +
 2*d*x)*(3*a*b^2 + b^3))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1
) + (2*(3*a*b^2 + b^3))/(3*d*(exp(2*c + 2*d*x) + 1)) - (2*(3*a*b^2 + 3*a^2
*b + a^3 + b^3))/(d*(exp(2*c + 2*d*x) - 1))
```

3.131 $\int \coth^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx$

3.131.1 Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \coth^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx = -\frac{(a+b)^3\mathrm{csch}^2(c+dx)}{2d} + \frac{b^2(3a+2b)\log(\cosh(c+dx))}{d} + \frac{(a-2b)(a+b)^2\log(\sinh(c+dx))}{d} - \frac{b^3\mathrm{sech}^2(c+dx)}{2d}$$

output
```
-1/2*(a+b)^3*csch(d*x+c)^2/d+b^2*(3*a+2*b)*ln(cosh(d*x+c))/d+(a-2*b)*(a+b)
^2*ln(sinh(d*x+c))/d-1/2*b^3*sech(d*x+c)^2/d
```

3.131.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36

$$\int \coth^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{4\cosh^6(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\left((a+b)^3\mathrm{csch}^2(c+dx)-2b^2(3a+2b)\log(\cosh(c+dx))-2(a-\right.}{d(a+2b+a\cosh(2c+2dx))^3}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-4*Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*((a + b)^3*Csch[c + d*x]^2 -
 2*b^2*(3*a + 2*b)*Log[Cosh[c + d*x]] - 2*(a - 2*b)*(a + b)^2*Log[Sinh[c +
 d*x]] + b^3*Sech[c + d*x]^2))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

3.131.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\left(a+b\sec(ic+idx)^2\right)^3}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\left(b\sec(ic+idx)^2+a\right)^3}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{ 4626}$$

$$\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^3\text{sech}^3(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^3\text{sech}^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(\text{sech}^2(c+dx)b^3+(3a+2b)\text{sech}(c+dx)b^2+\frac{(a-2b)(a+b)^2}{\cosh^2(c+dx)-1}+\frac{(a+b)^3}{\left(\cosh^2(c+dx)-1\right)^2}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2(3a+2b)\log\left(\cosh^2(c+dx)\right)+\frac{(a+b)^3}{1-\cosh^2(c+dx)}+(a-2b)(a+b)^2\log\left(1-\cosh^2(c+dx)\right)+b^3(-\text{sech}(c+dx))}{2d}$$

input
```
Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + b)^3/(1 - Cosh[c + d*x]^2) + b^2*(3*a + 2*b)*Log[Cosh[c + d*x]^2] +
(a - 2*b)*(a + b)^2*Log[1 - Cosh[c + d*x]^2] - b^3*Sech[c + d*x])/(2*d)
```

3.131.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.131.4 Maple [A] (verified)

Time = 46.88 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)-\frac{3a^2b}{2\sinh(dx+c)^2}+3ab^2\left(-\frac{1}{2\sinh(dx+c)^2}-\ln(\tanh(dx+c))\right)+b^3\left(-\frac{1}{2\sinh(dx+c)^2\cosh(d}\right.}{d}$
default	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)-\frac{3a^2b}{2\sinh(dx+c)^2}+3ab^2\left(-\frac{1}{2\sinh(dx+c)^2}-\ln(\tanh(dx+c))\right)+b^3\left(-\frac{1}{2\sinh(dx+c)^2\cosh(d}\right.}{d}$
risch	$-a^3x-\frac{2a^3c}{d}-\frac{2\,e^{2dx+2c}\left(a^3e^{4dx+4c}+3a^2b\,e^{4dx+4c}+3a\,b^2e^{4dx+4c}+2\,e^{4dx+4c}b^3+2a^3e^{2dx+2c}+6a^2b\,e^{2dx+2c}+6\,e^{2c}\right.}{d\left(e^{2dx+2c}+1\right)^2\left(e^{2dx+2c}-1\right)^2}$

input
```
int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)-3/2*a^2*b/sinh(d*x+c)^2+3*a*b
^2*(-1/2/sinh(d*x+c)^2-ln(tanh(d*x+c)))+b^3*(-1/2/sinh(d*x+c)^2/cosh(d*x+c
)^2-1/cosh(d*x+c)^2-2*ln(tanh(d*x+c))))
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1701 vs. $2(77) = 154$.

Time = 0.28 (sec) , antiderivative size = 1701, normalized size of antiderivative = 21.00

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*
d*x*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^6
+ 2*(14*a^3*d*x*cosh(d*x + c)^2 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*
x + c)^6 + 4*(14*a^3*d*x*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + 2*
b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + a^3*d*x - 2*(a^3*d*x - 2*a^3 - 6*a^2
*b - 6*a*b^2)*cosh(d*x + c)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 - a^3*d*x +
2*a^3 + 6*a^2*b + 6*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b
+ 3*a*b^2 + 2*b^3)*cosh(d*x + c)^3 - (a^3*d*x - 2*a^3 - 6*a^2*b - 6*a*b^2)
*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh
(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 3*a*b^2
+ 2*b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3 - 6*(a^3*d*x -
2*a^3 - 6*a^2*b - 6*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 +
2*b^3)*cosh(d*x + c)^8 + 56*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^3*sinh(d*x + c
)^5 + 28*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 +
2*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a*b^2 + 2*b^3)*sinh(d*x + c)^8 -
 2*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^4 + 2*(35*(3*a*b^2 + 2*b^3)*cosh(d*x +
c)^4 - 3*a*b^2 - 2*b^3)*sinh(d*x + c)^4 + 8*(7*(3*a*b^2 + 2*b^3)*cosh(d*x
+ c)^5 - (3*a*b^2 + 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*b^2 + 2*b^
3 + 4*(7*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^6 - 3*(3*a*b^2 + 2*b^3)*cosh(d...
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \coth^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.131.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(77) = 154$.

Time = 0.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.88

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$= a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-2\,b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\left(e^{(-2\,dx-2\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(2\,e^{(-4\,dx-4\,c)}-e^{(-8\,dx-8\,c)}-1)}\right.$$
$$-3\,ab^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1}\right.$$
$$-\frac{6\,a^2b}{d\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2
*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*b^3*(log(
e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/
d - 2*(e^(-2*d*x - 2*c) + e^(-6*d*x - 6*c))/(d*(2*e^(-4*d*x - 4*c) - e^(-8
*d*x - 8*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) -
 1)/d - log(e^(-2*d*x - 2*c) + 1)/d - 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x -
 2*c) - e^(-4*d*x - 4*c) - 1))) - 6*a^2*b/(d*(e^(d*x + c) - e^(-d*x - c))^
2)
```

3.131.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(77) = 154$.

Time = 0.40 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.05

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{2\left(3\,ab^2+2\,b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+2\left(a^3-3\,ab^2-2\,b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\cdots}{}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*(3*a*b^2 + 2*b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(
a^3 - 3*a*b^2 - 2*b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^3*
(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 8*a^3*(e^(2*d*x + 2*c) + e^(-2*d*
x - 2*c)) + 24*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 24*a*b^2*(e^(2
*d*x + 2*c) + e^(-2*d*x - 2*c)) + 16*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*
c)) + 12*a^3 + 48*a^2*b + 48*a*b^2)/((e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^
2 - 4))/d
```

3.131.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 324, normalized size of antiderivative = 4.00

$$\int \coth^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3 dx$$
$$= \frac{\mathrm{atan}\left(\frac{e^{2c}e^{2dx}\left(4b^3\sqrt{-d^2}-a^3\sqrt{-d^2}+6ab^2\sqrt{-d^2}\right)}{d\sqrt{a^6-12a^4b^2-8a^3b^3+36a^2b^4+48ab^5+16b^6}}\right)\sqrt{a^6-12a^4b^2-8a^3b^3+36a^2b^4+48ab^5+16b^6}}{\sqrt{-d^2}} - \frac{\frac{4(a^3+3a^2b+3ab^2)}{d}+\frac{2e^{2c+2dx}(a^3+3a^2b+3ab^2+2b^3)}{d}}{e^{4c+4dx}-1} - \frac{\frac{4(a^3+3a^2b+3ab^2)}{d}+\frac{4e^{2c+2dx}(a^3+3a^2b+3ab^2+2b^3)}{d}}{e^{8c+8dx}-2e^{4c+4dx}+1} - a^3x + \frac{a^3\ln\left(e^{4c+4dx}-1\right)}{2d}$$

input
```
int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(atan((exp(2*c)*exp(2*d*x)*(4*b^3*(-d^2)^(1/2) - a^3*(-d^2)^(1/2) + 6*a*b^
2*(-d^2)^(1/2)))/(d*(48*a*b^5 + a^6 + 16*b^6 + 36*a^2*b^4 - 8*a^3*b^3 - 12
*a^4*b^2)^(1/2)))*(48*a*b^5 + a^6 + 16*b^6 + 36*a^2*b^4 - 8*a^3*b^3 - 12*a
^4*b^2)^(1/2))/(-d^2)^(1/2) - ((4*(3*a*b^2 + 3*a^2*b + a^3))/d + (2*exp(2*
c + 2*d*x)*(3*a*b^2 + 3*a^2*b + a^3 + 2*b^3))/d)/(exp(4*c + 4*d*x) - 1) -
((4*(3*a*b^2 + 3*a^2*b + a^3))/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b
+ a^3 + 2*b^3))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - a^3*x + (
a^3*log(exp(4*c + 4*d*x) - 1))/(2*d)
```

3.132 $\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

3.132.1 Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = a^3x-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{b^3\tanh(c+dx)}{d}$$

output
```
a^3*x-(a-2*b)*(a+b)^2*coth(d*x+c)/d-1/3*(a+b)^3*coth(d*x+c)^3/d+b^3*tanh(d
*x+c)/d
```

3.132.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(60) = 120.

Time = 3.65 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.72

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{csch}(c)\text{csch}^3(c+dx)\text{sech}(c)\text{sech}(c+dx)\left(6a^3dx\cosh(2dx)-3a^3dx\cosh(2(c+2dx))-6a^3dx\cosh(4c+2\right.}{}$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Csch[c]*Csch[c + d*x]^3*Sech[c]*Sech[c + d*x]*(6*a^3*d*x*Cosh[2*d*x] - 3*
a^3*d*x*Cosh[2*(c + 2*d*x)] - 6*a^3*d*x*Cosh[4*c + 2*d*x] + 3*a^3*d*x*Cosh
[6*c + 4*d*x] - 18*a^2*b*Sinh[2*c] - 36*a*b^2*Sinh[2*c] - 4*a^3*Sinh[2*d*x
] + 6*a^2*b*Sinh[2*d*x] + 24*a*b^2*Sinh[2*d*x] + 32*b^3*Sinh[2*d*x] - 16*a
^3*Sinh[2*(c + d*x)] - 12*a^2*b*Sinh[2*(c + d*x)] + 24*a*b^2*Sinh[2*(c + d
*x)] + 8*b^3*Sinh[2*(c + d*x)] + 8*a^3*Sinh[4*(c + d*x)] + 6*a^2*b*Sinh[4*
(c + d*x)] - 12*a*b^2*Sinh[4*(c + d*x)] - 4*b^3*Sinh[4*(c + d*x)] + 8*a^3*
Sinh[2*(c + 2*d*x)] + 6*a^2*b*Sinh[2*(c + 2*d*x)] - 12*a*b^2*Sinh[2*(c + 2
*d*x)] - 16*b^3*Sinh[2*(c + 2*d*x)] - 12*a^3*Sinh[4*c + 2*d*x] - 18*a^2*b*
Sinh[4*c + 2*d*x]))/(96*d)
```

3.132.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = $\{3042, 4629, 2075, 364, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(ic+idx)^2\right)^3}{\tan(ic+idx)^4}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left((a+b)^3\coth^4(c+dx)+(a-2b)(a+b)^2\coth^2(c+dx)+b^3-\frac{a^3}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\mathrm{arctanh}(\tanh(c+dx)) - \frac{1}{3}(a+b)^3\coth^3(c+dx) - (a-2b)(a+b)^2\coth(c+dx) + b^3\tanh(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(a^3*ArcTanh[Tanh[c + d*x]] - (a - 2*b)*(a + b)^2*Coth[c + d*x] - ((a + b)
^3*Coth[c + d*x]^3)/3 + b^3*Tanh[c + d*x])/d
```

3.132.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.132.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(58) = 116.

Time = 67.92 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.48

method	result
derivativedivides	$\frac{a^3\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}\right)+3a^2b\left(-\frac{\cosh(dx+c)}{2\sinh(dx+c)^3}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)}{2}\right)+3a\,b^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right.}{d}$
default	$\frac{a^3\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}\right)+3a^2b\left(-\frac{\cosh(dx+c)}{2\sinh(dx+c)^3}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)}{2}\right)+3a\,b^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right.}{d}$
risch	$a^3x-\frac{2\left(6a^3e^{6dx+6c}+9a^2b\,e^{6dx+6c}+9a^2b\,e^{4dx+4c}+18a\,b^2e^{4dx+4c}-2a^3e^{2dx+2c}+3a^2b\,e^{2dx+2c}+12\,e^{2dx+2c}a\,b^2+16\,e^{2}\right.}{3d\left(e^{2dx+2c}-1\right)^3\left(e^{2dx+2c}+1\right)}$

input
```
int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+3*a^2*b*(-1/2/sinh(d*x+c)^3
*cosh(d*x+c)-1/2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))+3*a*b^2*(2/3-1/3*csc
h(d*x+c)^2)*coth(d*x+c)+b^3*(-1/3/sinh(d*x+c)^3/cosh(d*x+c)+4/3/sinh(d*x+c
)/cosh(d*x+c)+8/3*tanh(d*x+c)))
```

3.132.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 354, normalized size of antiderivative = 5.90

$$\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$
$$-\frac{\left(4\,a^3+3\,a^2b-6\,ab^2-8\,b^3\right)\cosh\left(dx+c\right)^4-4\left(3\,a^3dx+4\,a^3+3\,a^2b-6\,ab^2-8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(\right.}{}$$

input
```
integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/12*((4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x
+ 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^
3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*sinh(d*x + c)^4 + 9*a^2*b + 18*a*b^2 + 4*(a
^3 + 3*a^2*b + 3*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(2*a^3 + 6*a^2*b + 6*a
*b^2 + 8*b^3 + 3*(4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 - 4*((3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x
+ c)^3 - (3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*si
nh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cos
h(d*x + c))*sinh(d*x + c))
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.132.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(58) = 116$.

Time = 0.20 (sec) , antiderivative size = 366, normalized size of antiderivative = 6.10

$$\begin{aligned}&\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx\\&=\frac{1}{3}a^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)\\&\quad+4ab^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right.\\&\quad+\frac{16}{3}b^3\left(\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}+e^{(-8dx-8c)}-1)}-\frac{1}{d(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}+e^{(-8dx-8c)}-1)}\right.\\&\quad+2a^2b\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}+\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right.\end{aligned}$$

input
```
integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*
(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b
^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) - 1))) + 16/3*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) -
2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2
*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1))) + 2*a^2*b*(3*e^(-4*d*x - 4*c)/
(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) + 1/(
d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

3.132.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(58) = 116$.

Time = 0.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.63

$$\int \coth^4(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{3\,(dx+c)a^3 - \frac{6\,b^3}{e^{(2\,dx+2\,c)}+1} - \frac{2\,\left(6\,a^3e^{(4\,dx+4\,c)}+9\,a^2be^{(4\,dx+4\,c)}-3\,b^3e^{(4\,dx+4\,c)}-6\,a^3e^{(2\,dx+2\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+12\,b^3e^{(2\,dx+2\,c)}+4\,a}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^3 - 6*b^3/(e^(2*d*x + 2*c) + 1) - 2*(6*a^3*e^(4*d*x + 4
*c) + 9*a^2*b*e^(4*d*x + 4*c) - 3*b^3*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x + 2
*c) + 18*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 4*a^3 + 3*a^2*b
- 6*a*b^2 - 5*b^3)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.132.9 Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 260, normalized size of antiderivative = 4.33

$$\int \coth^4(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx$$
$$= a^3\,x-\frac{\frac{2\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}$$
$$-\frac{\frac{2\,\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}$$
$$-\frac{2\,b^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
a^3*x - ((2*(2*a*b^2 + a^2*b + b^3))/d + (2*exp(2*c + 2*d*x)*(3*a^2*b + 2*
a^3 - b^3))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*(3*a^
2*b + 2*a^3 - b^3))/(3*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b + 2*a^3 - b^3))/(
3*d) + (4*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b + b^3))/d)/(3*exp(2*c + 2*d*x)
 - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (2*b^3)/(d*(exp(2*c + 2*d*
x) + 1)) - (2*(3*a^2*b + 2*a^3 - b^3))/(3*d*(exp(2*c + 2*d*x) - 1))
```

3.133 $\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

3.133.1 Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = -\frac{(2a-b)(a+b)^2\text{csch}^2(c+dx)}{2d} - \frac{(a+b)^3\text{csch}^4(c+dx)}{4d} - \frac{b^3\log(\cosh(c+dx))}{d} + \frac{(a^3+b^3)\log(\sinh(c+dx))}{d}$$

output
```
-1/2*(2*a-b)*(a+b)^2*csch(d*x+c)^2/d-1/4*(a+b)^3*csch(d*x+c)^4/d-b^3*ln(co
sh(d*x+c))/d+(a^3+b^3)*ln(sinh(d*x+c))/d
```

3.133.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = -\frac{2\left(b+a\cosh^2(c+dx)\right)^3\left(2(2a-b)(a+b)^2\text{csch}^2(c+dx)+(a+b)^3\text{csch}^4(c+dx)+4b^3\log(\cosh(c+dx\right.}{d(a+2b+a\cosh(2(c+dx)))^3}$$

input
```
Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-2*(b + a*Cosh[c + d*x]^2)^3*(2*(2*a - b)*(a + b)^2*Csch[c + d*x]^2 + (a
+ b)^3*Csch[c + d*x]^4 + 4*b^3*Log[Cosh[c + d*x]] - 4*(a^3 + b^3)*Log[Sinh
[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

3.133.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sec(ic+idx)^2\right)^3}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sec(ic+idx)^2+a\right)^3}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^3\text{sech}(c+dx)}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^3\text{sech}(c+dx)}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\text{sech}(c+dx)b^3+\frac{-a^3-b^3}{\cosh^2(c+dx)-1}-\frac{(2a-b)(a+b)^2}{\left(\cosh^2(c+dx)-1\right)^2}-\frac{(a+b)^3}{\left(\cosh^2(c+dx)-1\right)^3}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\left(a^3+b^3\right)\log\left(1-\cosh^2(c+dx)\right)-\frac{(2a-b)(a+b)^2}{1-\cosh^2(c+dx)}+\frac{(a+b)^3}{2\left(1-\cosh^2(c+dx)\right)^2}+b^3\log\left(\cosh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/2*((a + b)^3/(2*(1 - Cosh[c + d*x]^2)^2) - ((2*a - b)*(a + b)^2)/(1 - C
osh[c + d*x]^2) + b^3*Log[Cosh[c + d*x]^2] - (a^3 + b^3)*Log[1 - Cosh[c +
d*x]^2])/d
```

3.133.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.133.4 Maple [A] (verified)

Time = 94.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+3a^2b\left(-\frac{\cosh(dx+c)^2}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)-\frac{3a\,b^2}{4\sinh(dx+c)^4}+b^3\left(-\frac{1}{4\sinh(dx}\right.}{d}$
default	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+3a^2b\left(-\frac{\cosh(dx+c)^2}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)-\frac{3a\,b^2}{4\sinh(dx+c)^4}+b^3\left(-\frac{1}{4\sinh(dx}\right.}{d}$
risch	$-a^3x-\frac{2a^3c}{d}-\frac{2\,e^{2dx+2c}\left(2a^3e^{4dx+4c}+3a^2b\,e^{4dx+4c}-e^{4dx+4c}b^3-2a^3e^{2dx+2c}+6\,e^{2dx+2c}a\,b^2+4\,e^{2dx+2c}b^3+2a^3+\right.}{d\left(e^{2dx+2c}-1\right)^4}$

input
```
int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+3*a^2*b*(-1
/2/sinh(d*x+c)^4*cosh(d*x+c)^2+1/4/sinh(d*x+c)^4)-3/4*a*b^2/sinh(d*x+c)^4+
b^3*(-1/4/sinh(d*x+c)^4+1/2/sinh(d*x+c)^2+ln(tanh(d*x+c))))
```

3.133.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1830 vs. $2(77) = 154$.

Time = 0.26 (sec) , antiderivative size = 1830, normalized size of antiderivative = 22.59

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*
d*x*sinh(d*x + c)^8 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^
6 + 2*(14*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x + 2*a^3 + 3*a^2*b - b^3)*sin
h(d*x + c)^6 + 4*(14*a^3*d*x*cosh(d*x + c)^3 - 3*(2*a^3*d*x - 2*a^3 - 3*a^
2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 2*a^3
 + 6*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 + 3*a^
3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3 - 15*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x + c)^5 - 5*(2*a^3*
d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^3 + (3*a^3*d*x - 2*a^3 + 6*a*b^
2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b
 + b^3)*cosh(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 - 2*a^3*d*x - 15*(
2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^4 + 2*a^3 + 3*a^2*b - b^3
 + 6*(3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + (b^3*cosh(d*x + c)^8 + 8*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + b^3*sinh(
d*x + c)^8 - 4*b^3*cosh(d*x + c)^6 + 6*b^3*cosh(d*x + c)^4 + 4*(7*b^3*cosh
(d*x + c)^2 - b^3)*sinh(d*x + c)^6 + 8*(7*b^3*cosh(d*x + c)^3 - 3*b^3*cosh
(d*x + c))*sinh(d*x + c)^5 - 4*b^3*cosh(d*x + c)^2 + 2*(35*b^3*cosh(d*x +
c)^4 - 30*b^3*cosh(d*x + c)^2 + 3*b^3)*sinh(d*x + c)^4 + 8*(7*b^3*cosh(d*x
 + c)^5 - 10*b^3*cosh(d*x + c)^3 + 3*b^3*cosh(d*x + c))*sinh(d*x + c)^3 +
b^3 + 4*(7*b^3*cosh(d*x + c)^6 - 15*b^3*cosh(d*x + c)^4 + 9*b^3*cosh(d*...
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.133.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(77) = 154$.

Time = 0.29 (sec) , antiderivative size = 422, normalized size of antiderivative = 5.21

$$\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx$$
$$= a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8}}\right.$$
$$+b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\left(e^{(-2\,dx-2\,c)}-4\,e^{(-4\,dx-}\right.}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e}\right.$$
$$+6\,a^2b\left(\frac{e^{(-2\,dx-2\,c)}}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}+\frac{e^{(-6\,dx-}}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4}\right.$$
$$-\frac{12\,ab^2}{d\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4}$$

input
```
integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c)
 - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + b^3
*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c)
 + 1)/d - 2*(e^(-2*d*x - 2*c) - 4*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*
(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x
- 8*c) - 1))) + 6*a^2*b*(e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4
*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*
c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-
8*d*x - 8*c) - 1))) - 12*a*b^2/(d*(e^(d*x + c) - e^(-d*x - c))^4)
```

3.133.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(77) = 154$.

Time = 0.42 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.80

$$\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx =$$
$$-\frac{2\,b^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-2\left(a^3+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+\frac{3\,a^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-}\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/4*(2*b^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - 2*(a^3 + b^3)*lo
g(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + (3*a^3*(e^(2*d*x + 2*c) + e^(-
2*d*x - 2*c))^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 4*a^3*(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 24*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x
- 2*c)) - 20*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*a^3 + 48*a*b^2 +
 44*b^3)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)^2)/d
```

3.133.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.74

$$\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$$
$$= -a^3x - \frac{2\,(4\,a^3+9\,a^2\,b+6\,a\,b^2+b^3)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} - \frac{\ln\left(e^{4\,c+4\,d\,x}-1\right)\,\left(b^3\,d-d\,(a^3+b^3)\right)}{2\,d^2}$$
$$- \frac{\operatorname{atan}\left(\frac{e^{2\,c}\,e^{2\,d\,x}\,\left(a^6\,\sqrt{-d^2}+4\,b^6\,\sqrt{-d^2}+4\,a^3\,b^3\,\sqrt{-d^2}\right)}{a^3\,d\,\sqrt{a^6+4\,a^3\,b^3+4\,b^6}+2\,b^3\,d\,\sqrt{a^6+4\,a^3\,b^3+4\,b^6}}\right)\,\sqrt{a^6+4\,a^3\,b^3+4\,b^6}}{\sqrt{-d^2}}$$
$$- \frac{2\,(2\,a^3+3\,a^2\,b-b^3)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} - \frac{8\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}$$
$$- \frac{4\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$

input
```
int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
- a^3*x - (2*(6*a*b^2 + 9*a^2*b + 4*a^3 + b^3))/(d*(exp(4*c + 4*d*x) - 2*e
xp(2*c + 2*d*x) + 1)) - (log(exp(4*c + 4*d*x) - 1)*(b^3*d - d*(a^3 + b^3))
)/(2*d^2) - (atan((exp(2*c)*exp(2*d*x)*(a^6*(-d^2)^(1/2) + 4*b^6*(-d^2)^(1
/2) + 4*a^3*b^3*(-d^2)^(1/2)))/(a^3*d*(a^6 + 4*b^6 + 4*a^3*b^3)^(1/2) + 2*
b^3*d*(a^6 + 4*b^6 + 4*a^3*b^3)^(1/2)))*(a^6 + 4*b^6 + 4*a^3*b^3)^(1/2))/(
-d^2)^(1/2) - (2*(3*a^2*b + 2*a^3 - b^3))/(d*(exp(2*c + 2*d*x) - 1)) - (8*
(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) - 1)) - (4*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(6*ex
p(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x
) + 1))
```

3.134 $\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = a^3x-\frac{(a^3+b^3)\coth(c+dx)}{d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}-\frac{(a+b)^3\coth^5(c+dx)}{5d}$$

output
```
a^3*x-(a^3+b^3)*coth(d*x+c)/d-1/3*(a-2*b)*(a+b)^2*coth(d*x+c)^3/d-1/5*(a+b
)^3*coth(d*x+c)^5/d
```

3.134.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 303 vs. 2(69) = 138.

Time = 2.90 (sec) , antiderivative size = 303, normalized size of antiderivative = 4.39

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$=\frac{\text{csch}(c)\text{csch}^5(c+dx)\left(-150a^3dx\cosh(dx)+150a^3dx\cosh(2c+dx)+75a^3dx\cosh(2c+3dx)-75a^3dx\cos\right.}{}$$

input
```
Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(Csch[c]*Csch[c + d*x]^5*(-150*a^3*d*x*Cosh[d*x] + 150*a^3*d*x*Cosh[2*c +
d*x] + 75*a^3*d*x*Cosh[2*c + 3*d*x] - 75*a^3*d*x*Cosh[4*c + 3*d*x] - 15*a^
3*d*x*Cosh[4*c + 5*d*x] + 15*a^3*d*x*Cosh[6*c + 5*d*x] + 280*a^3*Sinh[d*x]
 + 180*a^2*b*Sinh[d*x] + 60*a*b^2*Sinh[d*x] + 160*b^3*Sinh[d*x] + 180*a^3*
Sinh[2*c + d*x] - 180*a*b^2*Sinh[2*c + d*x] - 140*a^3*Sinh[2*c + 3*d*x] +
60*a*b^2*Sinh[2*c + 3*d*x] - 80*b^3*Sinh[2*c + 3*d*x] - 90*a^3*Sinh[4*c +
3*d*x] - 90*a^2*b*Sinh[4*c + 3*d*x] + 46*a^3*Sinh[4*c + 5*d*x] + 18*a^2*b*
Sinh[4*c + 5*d*x] - 12*a*b^2*Sinh[4*c + 5*d*x] + 16*b^3*Sinh[4*c + 5*d*x])
)/(480*d)
```

3.134.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 4629, 25, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+b\sec(ic+idx)^2\right)^3}{\tan(ic+idx)^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sec(ic+idx)^2+a\right)^3}{\tan(ic+idx)^6}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\coth^6(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^6(c+dx)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^6(c+dx)\left(-b\tanh^2(c+dx)+a+b\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 364$$

$$\frac{\int \left((a+b)^3\coth^6(c+dx)+(a-2b)(a+b)^2\coth^4(c+dx)+\left(a^3+b^3\right)\coth^2(c+dx)-\frac{a^3}{\tanh^2(c+dx)-1}\right)d\tanh(c+$$

$$d$$

$$\Big\downarrow \; 2009$$

$$-\frac{-a^3\text{arctanh}(\tanh(c+dx))+\left(a^3+b^3\right)\coth(c+dx)+\frac{1}{5}(a+b)^3\coth^5(c+dx)+\frac{1}{3}(a-2b)(a+b)^2\coth^3(c+dx}{d}$$

input
```
Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-((-(a^3*ArcTanh[Tanh[c + d*x]]) + (a^3 + b^3)*Coth[c + d*x] + ((a - 2*b)*
(a + b)^2*Coth[c + d*x]^3)/3 + ((a + b)^3*Coth[c + d*x]^5)/5)/d)
```

3.134.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.134.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(65) = 130$.

Time = 130.20 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.88

method	result
derivativedivides	$\frac{a^3\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}-\frac{\coth(dx+c)^5}{5}\right)+3a^2b\left(-\frac{\cosh(dx+c)^3}{2\sinh(dx+c)^5}+\frac{3\cosh(dx+c)}{8\sinh(dx+c)^5}+\frac{3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csc}}{}\right.}{8}\right.}{}$
default	$\frac{a^3\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}-\frac{\coth(dx+c)^5}{5}\right)+3a^2b\left(-\frac{\cosh(dx+c)^3}{2\sinh(dx+c)^5}+\frac{3\cosh(dx+c)}{8\sinh(dx+c)^5}+\frac{3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csc}}{}\right.}{8}\right.}{}$
risch	$a^3x-\frac{2\left(45a^3e^{8dx+8c}+45a^2b\,e^{8dx+8c}-90a^3e^{6dx+6c}+90a\,b^2e^{6dx+6c}+140a^3e^{4dx+4c}+90a^2b\,e^{4dx+4c}+30a\,b^2e^{4dx+4c}+\right.}{15d\left(e^{2dx+2c}-1\right)^5}$

input
```
int(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+3*a^2*b*(
-1/2/sinh(d*x+c)^5*cosh(d*x+c)^3+3/8/sinh(d*x+c)^5*cosh(d*x+c)+3/8*(-8/15-
1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))+3*a*b^2*(-1/4/sinh(d*x+
c)^5*cosh(d*x+c)-1/4*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x
+c))+b^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(65) = 130$.

Time = 0.26 (sec) , antiderivative size = 521, normalized size of antiderivative = 7.55

$$\int \coth^6(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx =$$

$$-\frac{\left(23\,a^3+9\,a^2b-6\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)^5+5\left(23\,a^3+9\,a^2b-6\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right.}{}$$

input
```
integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/15*((23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 5*(23*a^3 +
9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^3*d*x + 2
3*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*sinh(d*x + c)^5 - 5*(5*a^3 - 9*a^2*b -
6*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 5*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*
b^2 + 8*b^3 - 2*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^3 + 5*(2*(23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(
d*x + c)^3 - 3*(5*a^3 - 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^2 + 10*(5*a^3 + 9*a^2*b + 12*a*b^2 + 8*b^3)*cosh(d*x + c) - 5*(30*a^
3*d*x + (15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^4
+ 46*a^3 + 18*a^2*b - 12*a*b^2 + 16*b^3 - 3*(15*a^3*d*x + 23*a^3 + 9*a^2*b
 - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5
*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*co
sh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.134.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(65) = 130$.

Time = 0.21 (sec) , antiderivative size = 826, normalized size of antiderivative = 11.97

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/15*a^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) - 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) - 1)) + 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x
- 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1
)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10
*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(
5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d
*x - 8*c) + e^(-10*d*x - 10*c) - 1))) - 16/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(
5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d
*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))) +
6/5*a^2*b*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) +
5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6
*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 1/(d*(5*e...
```

3.134.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(65) = 130$.

Time = 0.38 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.09

$$\int \coth^6(c+dx) \left(a+b\text{sech}^2(c+dx)\right)^3 dx$$
$$= \frac{15\,(dx+c)a^3 - \frac{2\left(45\,a^3e^{(8\,dx+8\,c)}+45\,a^2be^{(8\,dx+8\,c)}-90\,a^3e^{(6\,dx+6\,c)}+90\,ab^2e^{(6\,dx+6\,c)}+140\,a^3e^{(4\,dx+4\,c)}+90\,a^2be^{(4\,dx+4\,c)}+30\,ab^2e^{(4\,dx}}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

input
```
integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)*a^3 - 2*(45*a^3*e^(8*d*x + 8*c) + 45*a^2*b*e^(8*d*x + 8
*c) - 90*a^3*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 140*a^3*e^(4*d*x
 + 4*c) + 90*a^2*b*e^(4*d*x + 4*c) + 30*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(
4*d*x + 4*c) - 70*a^3*e^(2*d*x + 2*c) + 30*a*b^2*e^(2*d*x + 2*c) - 40*b^3*
e^(2*d*x + 2*c) + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)/(e^(2*d*x + 2*c) - 1
)^5)/d
```

3.134.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 547, normalized size of antiderivative = 7.93

$$\int \coth^6(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx = a^3\,x$$
$$-\frac{\frac{6\,(a^3+b\,a^2)}{5\,d}+\frac{24\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}+\frac{24\,\mathrm{e}^{6\,c+6\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}+\frac{6\,\mathrm{e}^{8\,c+8\,d\,x}\,(a^3+b\,a^2)}{5\,d}+\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(5\,a^3+9\,a^2\,b+12\,a\,b^2+8\,b^3)}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1}$$
$$-\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}+\frac{6\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^3+b\,a^2)}{5\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}$$
$$-\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}+\frac{18\,\mathrm{e}^{4\,c+4\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}+\frac{6\,\mathrm{e}^{6\,c+6\,d\,x}\,(a^3+b\,a^2)}{5\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(5\,a^3+9\,a^2\,b+12\,a\,b^2+8\,b^3)}{5\,d}}{6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}$$
$$-\frac{\frac{2\,(5\,a^3+9\,a^2\,b+12\,a\,b^2+8\,b^3)}{15\,d}+\frac{12\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}+\frac{6\,\mathrm{e}^{4\,c+4\,d\,x}\,(a^3+b\,a^2)}{5\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}-\frac{6\,(a^3+b\,a^2)}{5\,d\,(\mathrm{e}^{2\,c+2\,d\,x}-1)}$$

input
```
int(coth(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
a^3*x - ((6*(a^2*b + a^3))/(5*d) + (24*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(
5*d) + (24*exp(6*c + 6*d*x)*(a*b^2 + a^2*b))/(5*d) + (6*exp(8*c + 8*d*x)*(
a^2*b + a^3))/(5*d) + (4*exp(4*c + 4*d*x)*(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*
b^3))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((6*(a*b^2 + a^2*b))/(
5*d) + (6*exp(2*c + 2*d*x)*(a^2*b + a^3))/(5*d))/(exp(4*c + 4*d*x) - 2*exp
(2*c + 2*d*x) + 1) - ((6*(a*b^2 + a^2*b))/(5*d) + (18*exp(4*c + 4*d*x)*(a*
b^2 + a^2*b))/(5*d) + (6*exp(6*c + 6*d*x)*(a^2*b + a^3))/(5*d) + (2*exp(2*
c + 2*d*x)*(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*b^3))/(5*d))/(6*exp(4*c + 4*d*x
) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*
(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*b^3))/(15*d) + (12*exp(2*c + 2*d*x)*(a*b^2
 + a^2*b))/(5*d) + (6*exp(4*c + 4*d*x)*(a^2*b + a^3))/(5*d))/(3*exp(2*c +
2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (6*(a^2*b + a^3))/(5
*d*(exp(2*c + 2*d*x) - 1))
```

3.135 $\int \coth^7(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx$

3.135.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \coth^7(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx=-\frac{3a^2(a+b)\mathrm{csch}^2(c+dx)}{2d}-\frac{3a(a+b)^2\mathrm{csch}^4(c+dx)}{4d}-\frac{(a+b)^3\mathrm{csch}^6(c+dx)}{6d}+\frac{a^3\log(\sinh(c+dx))}{d}$$

output
```
-3/2*a^2*(a+b)*csch(d*x+c)^2/d-3/4*a*(a+b)^2*csch(d*x+c)^4/d-1/6*(a+b)^3*c
sch(d*x+c)^6/d+a^3*ln(sinh(d*x+c))/d
```

3.135.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \coth^7(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\,dx=-\frac{2\left(b+a\cosh^2(c+dx)\right)^3\left(18a^2(a+b)\mathrm{csch}^2(c+dx)+9a(a+b)^2\mathrm{csch}^4(c+dx)+2(a+b)^3\mathrm{csch}^6(c+dx)\right.}{3d(a+2b+a\cosh(2(c+dx)))^3}$$

input
```
Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-2*(b + a*Cosh[c + d*x]^2)^3*(18*a^2*(a + b)*Csch[c + d*x]^2 + 9*a*(a + b
)^2*Csch[c + d*x]^4 + 2*(a + b)^3*Csch[c + d*x]^6 - 12*a^3*Log[Sinh[c + d*
x]]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

3.135.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a+b\sec(ic+idx)^2\right)^3}{\tan(ic+idx)^7}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(b\sec(ic+idx)^2+a\right)^3}{\tan(ic+idx)^7}dx$$

$$\Big\downarrow \text{4626}$$

$$\frac{\int \frac{\cosh(c+dx)\left(a\cosh^2(c+dx)+b\right)^3}{\left(1-\cosh^2(c+dx)\right)^4}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{\left(a\cosh^2(c+dx)+b\right)^3}{\left(1-\cosh^2(c+dx)\right)^4}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{a^3}{\cosh^2(c+dx)-1}+\frac{3(a+b)a^2}{\left(\cosh^2(c+dx)-1\right)^2}+\frac{3(a+b)^2a}{\left(\cosh^2(c+dx)-1\right)^3}+\frac{(a+b)^3}{\left(\cosh^2(c+dx)-1\right)^4}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\log\left(1-\cosh^2(c+dx)\right)+\frac{3a^2(a+b)}{1-\cosh^2(c+dx)}-\frac{3a(a+b)^2}{2\left(1-\cosh^2(c+dx)\right)^2}+\frac{(a+b)^3}{3\left(1-\cosh^2(c+dx)\right)^3}}{2d}$$

input
```
Int[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + b)^3/(3*(1 - Cosh[c + d*x]^2)^3) - (3*a*(a + b)^2)/(2*(1 - Cosh[c +
d*x]^2)^2) + (3*a^2*(a + b))/(1 - Cosh[c + d*x]^2) + a^3*Log[1 - Cosh[c +
d*x]^2])/(2*d)
```

3.135.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.135.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(71) = 142.

Time = 173.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.94

method	result
derivativedivides	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}-\frac{\coth(dx+c)^6}{6}\right)+3a^2b\left(-\frac{\cosh(dx+c)^4}{2\sinh(dx+c)^6}+\frac{\cosh(dx+c)^2}{2\sinh(dx+c)^6}-\frac{1}{6\sinh(dx+c)^6}\right)+}{d}$
default	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}-\frac{\coth(dx+c)^6}{6}\right)+3a^2b\left(-\frac{\cosh(dx+c)^4}{2\sinh(dx+c)^6}+\frac{\cosh(dx+c)^2}{2\sinh(dx+c)^6}-\frac{1}{6\sinh(dx+c)^6}\right)+}{d}$
risch	$-a^3x-\frac{2a^3c}{d}-\frac{2\,e^{2dx+2c}(9a^3e^{8dx+8c}+9a^2b\,e^{8dx+8c}-18a^3e^{6dx+6c}+18a\,b^2e^{6dx+6c}+34a^3e^{4dx+4c}+30a^2b\,e^{4dx+4c}}{3d(e^{2dx+2c}-1)^6}$

input
```
int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4-1/6*coth(d*x
+c)^6)+3*a^2*b*(-1/2/sinh(d*x+c)^6*cosh(d*x+c)^4+1/2/sinh(d*x+c)^6*cosh(d*
x+c)^2-1/6/sinh(d*x+c)^6)+3*a*b^2*(-1/4/sinh(d*x+c)^6*cosh(d*x+c)^2+1/12/s
inh(d*x+c)^6)-1/6*b^3/sinh(d*x+c)^6)
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2632 vs. 2(71) = 142.

Time = 0.28 (sec) , antiderivative size = 2632, normalized size of antiderivative = 34.18

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^
11 + 3*a^3*d*x*sinh(d*x + c)^12 - 18*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)
^10 + 18*(11*a^3*d*x*cosh(d*x + c)^2 - a^3*d*x + a^3 + a^2*b)*sinh(d*x + c
)^10 + 60*(11*a^3*d*x*cosh(d*x + c)^3 - 3*(a^3*d*x - a^3 - a^2*b)*cosh(d*x
 + c))*sinh(d*x + c)^9 + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^8 +
 9*(165*a^3*d*x*cosh(d*x + c)^4 + 5*a^3*d*x - 4*a^3 + 4*a*b^2 - 90*(a^3*d*
x - a^3 - a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*
x + c)^5 - 30*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^3 + (5*a^3*d*x - 4*a^3
 + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^3*d*x - 17*a^3 - 15*a
^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 4*(693*a^3*d*x*cosh(d*x + c)^6 -
 15*a^3*d*x - 945*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^4 + 17*a^3 + 15*a^
2*b + 6*a*b^2 + 8*b^3 + 63*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^6 + 24*(99*a^3*d*x*cosh(d*x + c)^7 - 189*(a^3*d*x - a^3 - a^
2*b)*cosh(d*x + c)^5 + 21*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^3 -
(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 3*a^3*d*x + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 + 3*
(495*a^3*d*x*cosh(d*x + c)^8 - 1260*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^
6 + 15*a^3*d*x + 210*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 - 12*a^
3 + 12*a*b^2 - 20*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 - 540*(a^3...
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \coth^7(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.135.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(71) = 142$.

Time = 0.21 (sec) , antiderivative size = 727, normalized size of antiderivative = 9.44

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{3}a^3\left(3x + \frac{3c}{d} + \frac{3\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(9e^{(-2dx-2c)} - 18e^{(-4dx-4c)} + \right.}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx}}\right.$$
$$+2a^2b\left(\frac{3e^{(-2dx-2c)}}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)}\right.$$
$$+4ab^2\left(\frac{3e^{(-4dx-4c)}}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)}\right.$$
$$- \frac{32b^3}{3d(e^{(dx+c)} - e^{(-dx-c)})^6}$$

input
```
integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)
/d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 1
8*e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(
-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x -
 10*c) - e^(-12*d*x - 12*c) - 1))) + 2*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(
-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x -
 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 10*e^(-6*d*x - 6
*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 1
5*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e
^(-10*d*x - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*
d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*
c) - 1))) + 4*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10
*c) - e^(-12*d*x - 12*c) - 1)) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c)
 - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(
-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-
2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x -
8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - 32/3*b^3/(d*(e^(
d*x + c) - e^(-d*x - c))^6)
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(71) = 142$.

Time = 0.44 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.14

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = -\frac{60\,(dx+c)a^3 - 60\,a^3\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{147\,a^3e^{(12\,dx+12\,c)} - 522\,a^3e^{(10\,dx+10\,c)} + 360\,a^2be^{(10\,dx+10\,c)} + 1485\,a^3e^{(8\,dx+8\,c)}}{}}{}$$

input
```
integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/60*(60*(d*x + c)*a^3 - 60*a^3*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^3*
e^(12*d*x + 12*c) - 522*a^3*e^(10*d*x + 10*c) + 360*a^2*b*e^(10*d*x + 10*c
) + 1485*a^3*e^(8*d*x + 8*c) + 720*a*b^2*e^(8*d*x + 8*c) - 1580*a^3*e^(6*d
*x + 6*c) + 1200*a^2*b*e^(6*d*x + 6*c) + 480*a*b^2*e^(6*d*x + 6*c) + 640*b
^3*e^(6*d*x + 6*c) + 1485*a^3*e^(4*d*x + 4*c) + 720*a*b^2*e^(4*d*x + 4*c)
- 522*a^3*e^(2*d*x + 2*c) + 360*a^2*b*e^(2*d*x + 2*c) + 147*a^3)/(e^(2*d*x
 + 2*c) - 1)^6)/d
```

3.135.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 411, normalized size of antiderivative = 5.34

$$\begin{aligned}
&\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx \\
&= \frac{a^3\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)}{d} \\
&- \frac{32\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1\right)} \\
&- \frac{32\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{3\,d\,\left(15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{2\,c+2\,d\,x}-20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}-6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)} \\
&- \frac{6\,\left(a^3+b\,a^2\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{6\,\left(3\,a^3+5\,a^2\,b+2\,a\,b^2\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)} \\
&- \frac{8\,\left(13\,a^3+30\,a^2\,b+21\,a\,b^2+4\,b^3\right)}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)} \\
&- \frac{4\,\left(11\,a^3+30\,a^2\,b+27\,a\,b^2+8\,b^3\right)}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)} - a^3\,x
\end{aligned}$$

input
```
int(coth(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
(a^3*log(exp(2*c)*exp(2*d*x) - 1))/d - (32*(3*a*b^2 + 3*a^2*b + a^3 + b^3)
)/(d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*e
xp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (32*(3*a*b^2 + 3*a^2*b + a^3
+ b^3))/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*
x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1))
 - (6*(a^2*b + a^3))/(d*(exp(2*c + 2*d*x) - 1)) - (6*(2*a*b^2 + 5*a^2*b +
3*a^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*(21*a*b^2 + 3
0*a^2*b + 13*a^3 + 4*b^3))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) +
 exp(6*c + 6*d*x) - 1)) - (4*(27*a*b^2 + 30*a^2*b + 11*a^3 + 8*b^3))/(d*(6
*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*
d*x) + 1)) - a^3*x
```

3.135. $\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

3.136 $\int \left(a+b\text{sech}^2(c+dx)\right)^4 \, dx$

3.136.1 Optimal result

Integrand size = 14, antiderivative size = 111

$$\int \left(a+b\text{sech}^2(c+dx)\right)^4 \, dx = a^4x+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2(6a^2+8ab+3b^2)\tanh^3(c+dx)}{3d} + \frac{b^3(4a+3b)\tanh^5(c+dx)}{5d} - \frac{b^4\tanh^7(c+dx)}{7d}$$

output
```
a^4*x+b*(2*a+b)*(2*a^2+2*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(6*a^2+8*a*b+3*b^2
)*tanh(d*x+c)^3/d+1/5*b^3*(4*a+3*b)*tanh(d*x+c)^5/d-1/7*b^4*tanh(d*x+c)^7/
d
```

3.136.2 Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.58

$$\int \left(a+b\text{sech}^2(c+dx)\right)^4 \, dx = a^4x+\frac{4a^3b\tanh(c+dx)}{d}+\frac{6a^2b^2\tanh(c+dx)}{d} + \frac{4ab^3\tanh(c+dx)}{d}+\frac{b^4\tanh(c+dx)}{d}-\frac{2a^2b^2\tanh^3(c+dx)}{d} - \frac{8ab^3\tanh^3(c+dx)}{3d}-\frac{b^4\tanh^3(c+dx)}{d} + \frac{4ab^3\tanh^5(c+dx)}{5d}+\frac{3b^4\tanh^5(c+dx)}{5d}-\frac{b^4\tanh^7(c+dx)}{7d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^4,x]
```

output
```
a^4*x + (4*a^3*b*Tanh[c + d*x])/d + (6*a^2*b^2*Tanh[c + d*x])/d + (4*a*b^3
*Tanh[c + d*x])/d + (b^4*Tanh[c + d*x])/d - (2*a^2*b^2*Tanh[c + d*x]^3)/d
- (8*a*b^3*Tanh[c + d*x]^3)/(3*d) - (b^4*Tanh[c + d*x]^3)/d + (4*a*b^3*Tan
h[c + d*x]^5)/(5*d) + (3*b^4*Tanh[c + d*x]^5)/(5*d) - (b^4*Tanh[c + d*x]^7
)/(7*d)
```

3.136.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^2(c+dx)\right)^4 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a+b\sec(ic+idx)^2\right)^4 dx$$

$$\Big\downarrow \text{ 4616}$$

$$\frac{\int \frac{\left(-b\tanh^2(c+dx)+a+b\right)^4}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(-b^4\tanh^6(c+dx)+b^3(4a+3b)\tanh^4(c+dx)-b^2\left(6a^2+8ba+3b^2\right)\tanh^2(c+dx)+b(2a+b)\left(2a^2+2ba+\right.\right.}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^4\text{arctanh}(\tanh(c+dx))-\frac{1}{3}b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)+b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)+\frac{1}{5}b^3(}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^4,x]
```

output
```
(a^4*ArcTanh[Tanh[c + d*x]] + b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d
*x] - (b^2*(6*a^2 + 8*a*b + 3*b^2)*Tanh[c + d*x]^3)/3 + (b^3*(4*a + 3*b)*T
anh[c + d*x]^5)/5 - (b^4*Tanh[c + d*x]^7)/7)/d
```

3.136.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.136.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{a^4(dx+c)+4a^3b\tanh(dx+c)+6a^2b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+4a\,b^3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+}{d}$
default	$\frac{a^4(dx+c)+4a^3b\tanh(dx+c)+6a^2b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+4a\,b^3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+}{d}$
parts	$x\,a^4+\frac{b^4\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}(dx+c)^4}{35}+\frac{8\operatorname{sech}(dx+c)^2}{35}\right)\tanh(dx+c)}{d}+\frac{4a^3b\tanh(dx+c)}{d}+\frac{6a^2b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+}{3}\right.}{d}$
risch	$x\,a^4-\frac{8b(105a^3e^{12dx+12c}+630a^3e^{10dx+10c}+315a^2b\,e^{10dx+10c}+1575a^3e^{8dx+8c}+1365a^2b\,e^{8dx+8c}+560a\,b^2e^{8dx+8c}+}{}$
parallelrisch	$\frac{105x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}a^4d+(840a^3b+1260a^2b^2+840a\,b^3+210b^4)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}+735x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}a^4d+(5040a^3b+}{}$

input
```
int((a+b*sech(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(d*x+c)+4*a^3*b*tanh(d*x+c)+6*a^2*b^2*(2/3+1/3*sech(d*x+c)^2)*tan
h(d*x+c)+4*a*b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b
^4*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*
x+c))
```

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 941, normalized size of antiderivative = 8.48

$$\int \left(a + b\text{sech}^2(c + dx)\right)^4 dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d
*x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)
*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 1
2*b^4)*sinh(d*x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*
b^3 - 48*b^4)*cosh(d*x + c)^5 + 28*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12
*b^4 + 3*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*si
nh(d*x + c)^5 + 35*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 4
8*b^4)*cosh(d*x + c)^3 + (105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^
3 - 48*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^4*d*x - 420*a^3*b -
 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 28*(5*(105*a^3*b + 10
5*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 135*a^3*b + 225*a^2*b^2 +
 168*a*b^3 + 36*b^4 + 10*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2
 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^5 + 10*(105*a^4*d*x - 420*a^3*b - 420
*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 9*(105*a^4*d*x - 420*a^3*
b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*
(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)
 + 28*((105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^6 + 5*(
75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 75*a^3*b + 1
35*a^2*b^2 + 120*a*b^3 + 60*b^4 + 9*(45*a^3*b + 75*a^2*b^2 + 56*a*b^3 +...
```

3.136.6 Sympy [F]

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^4\,dx=\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^4\,dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**4,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**4, x)
```

3.136.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(105) = 210.

Time = 0.20 (sec) , antiderivative size = 703, normalized size of antiderivative = 6.33

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^4\,dx=a^4x$$
$$+\frac{32}{35}\,b^4\left(\frac{7\,e^{(-2\,dx-2\,c)}}{d(7\,e^{(-2\,dx-2\,c)}+21\,e^{(-4\,dx-4\,c)}+35\,e^{(-6\,dx-6\,c)}+35\,e^{(-8\,dx-8\,c)}+21\,e^{(-10\,dx-10\,c)}+7\,e^{(-12\,dx-12\,c)}\ldots}\right.$$
$$+\frac{64}{15}\,ab^3\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{\ldots}{d(5\,e^{(-2\,dx-2\ldots}}\right.$$
$$+8\,a^2b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\ldots}\right.$$
$$+\frac{8\,a^3b}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
a^4*x + 32/35*b^4*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*
x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*
c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)
/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e
^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*
x - 14*c) + 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*
x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*
c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x -
 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) +
 64/15*a*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4
*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))
+ 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^
(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e
^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1))) + 8*a^2*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e
^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^
(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 8*a^3*b/(d
*(e^(-2*d*x - 2*c) + 1))
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(105) = 210$.

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.01

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^4\,dx$$

$$= \frac{105\,(dx+c)a^4 - \frac{8\,(105\,a^3be^{(12\,dx+12\,c)}+630\,a^3be^{(10\,dx+10\,c)}+315\,a^2b^2e^{(10\,dx+10\,c)}+1575\,a^3be^{(8\,dx+8\,c)}+1365\,a^2b^2e^{(8\,dx+8\,c)}+560\,ab^3e^{(8}}{}}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/105*(105*(d*x + c)*a^4 - 8*(105*a^3*b*e^(12*d*x + 12*c) + 630*a^3*b*e^(1
0*d*x + 10*c) + 315*a^2*b^2*e^(10*d*x + 10*c) + 1575*a^3*b*e^(8*d*x + 8*c)
 + 1365*a^2*b^2*e^(8*d*x + 8*c) + 560*a*b^3*e^(8*d*x + 8*c) + 2100*a^3*b*e
^(6*d*x + 6*c) + 2310*a^2*b^2*e^(6*d*x + 6*c) + 1400*a*b^3*e^(6*d*x + 6*c)
 + 420*b^4*e^(6*d*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 1890*a^2*b^2*e^(
4*d*x + 4*c) + 1176*a*b^3*e^(4*d*x + 4*c) + 252*b^4*e^(4*d*x + 4*c) + 630*
a^3*b*e^(2*d*x + 2*c) + 735*a^2*b^2*e^(2*d*x + 2*c) + 392*a*b^3*e^(2*d*x +
 2*c) + 84*b^4*e^(2*d*x + 2*c) + 105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b
^4)/(e^(2*d*x + 2*c) + 1)^7)/d
```

3.136.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 1083, normalized size of antiderivative = 9.76

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^4\,dx = a^4\,x$$
$$-\frac{\frac{8\left(a^3\,b+a^2\,b^2\right)}{7\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{21\,d}+\frac{16\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{21\,d}+\frac{16\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(5\,a^3\,b+9\,a^2\,b^2+8\,a\,b^3+\cdots\right)}{7\,d}\cdots}{6\,{\mathrm{e}}^{2\,c+2\,d\,x}+15\,{\mathrm{e}}^{4\,c+4\,d\,x}+20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}+6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+\cdots}}$$
$$-\frac{\frac{8\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{7\,d}+\frac{8\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{7\,d}+\frac{8\,a^3\,b}{7\,d}+\frac{32\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(5\,a^3\,b+9\,a^2\,b^2+8\,a\,b^3+4\,b^4\right)}{7\,d}+\cdots}{7\,{\mathrm{e}}^{2\,c+2\,d\,x}+21\,{\mathrm{e}}^{4\,c+4\,d\,x}+35\,{\mathrm{e}}^{6\,c+6\,d\,x}+35\,{\mathrm{e}}^{8\,c+8\,d\,x}+21\,{\mathrm{e}}^{10\,c+10\,d\,x}+7\,{\mathrm{e}}^{1\cdots}}$$
$$-\frac{\frac{8\left(a^3\,b+a^2\,b^2\right)}{7\,d}+\frac{8\,a^3\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{7\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}$$
$$-\frac{\frac{8\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{105\,d}+\frac{16\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{35\,d}+\frac{32\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(5\,a^3\,b+9\,a^2\,b^2+8\,a\,b^3+4\,b^4\right)}{35\,d}+\frac{32\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a^3\,b+\cdots\right)}{7\,d}\cdots}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}$$
$$-\frac{\frac{8\left(5\,a^3\,b+9\,a^2\,b^2+8\,a\,b^3+4\,b^4\right)}{35\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{35\,d}+\frac{24\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^3\,b+a^2\,b^2\right)}{7\,d}+\frac{8\,a^3\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}}{7\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}$$
$$-\frac{\frac{8\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{105\,d}+\frac{16\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^3\,b+a^2\,b^2\right)}{7\,d}+\frac{8\,a^3\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{7\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{8\,a^3\,b}{7\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x)^2)^4,x)
```

output
```
a^4*x - ((8*(a^3*b + a^2*b^2))/(7*d) + (8*exp(2*c + 2*d*x)*(16*a*b^3 + 15*
a^3*b + 24*a^2*b^2))/(21*d) + (16*exp(6*c + 6*d*x)*(16*a*b^3 + 15*a^3*b +
24*a^2*b^2))/(21*d) + (16*exp(4*c + 4*d*x)*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9*
a^2*b^2))/(7*d) + (40*exp(8*c + 8*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b
*exp(10*c + 10*d*x))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20
*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c
+ 12*d*x) + 1) - ((8*exp(4*c + 4*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/
(7*d) + (8*exp(8*c + 8*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(7*d) + (8
*a^3*b)/(7*d) + (32*exp(6*c + 6*d*x)*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9*a^2*b^
2))/(7*d) + (48*exp(2*c + 2*d*x)*(a^3*b + a^2*b^2))/(7*d) + (48*exp(10*c +
 10*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(12*c + 12*d*x))/(7*d))/(7
*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c
 + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d
*x) + 1) - ((8*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(2*c + 2*d*x))/(7*d)
)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((8*(16*a*b^3 + 15*a^3*b +
 24*a^2*b^2))/(105*d) + (16*exp(4*c + 4*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2
*b^2))/(35*d) + (32*exp(2*c + 2*d*x)*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9*a^2*b^
2))/(35*d) + (32*exp(6*c + 6*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(
8*c + 8*d*x))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*
c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((8*(8*a*b^...
```

3.136. $\int \left(a + b\operatorname{sech}^2(c + dx)\right)^4 \, dx$

3.137 $\int \left(a+b\text{sech}^2(c+dx)\right)^5\,dx$

3.137.1 Optimal result

Integrand size = 14, antiderivative size = 163

$$\begin{aligned}\int \left(a+b\text{sech}^2(c+dx)\right)^5\,dx = a^5x &+ \frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d}\\ &- \frac{b^2(10a^3+20a^2b+15ab^2+4b^3)\tanh^3(c+dx)}{3d}\\ &+ \frac{b^3(10a^2+15ab+6b^2)\tanh^5(c+dx)}{5d}\\ &- \frac{b^4(5a+4b)\tanh^7(c+dx)}{7d} + \frac{b^5\tanh^9(c+dx)}{9d}\end{aligned}$$

output
```
a^5*x+b*(5*a^4+10*a^3*b+10*a^2*b^2+5*a*b^3+b^4)*tanh(d*x+c)/d-1/3*b^2*(10*
a^3+20*a^2*b+15*a*b^2+4*b^3)*tanh(d*x+c)^3/d+1/5*b^3*(10*a^2+15*a*b+6*b^2)
*tanh(d*x+c)^5/d-1/7*b^4*(5*a+4*b)*tanh(d*x+c)^7/d+1/9*b^5*tanh(d*x+c)^9/d
```

3.137.2 Mathematica [A] (verified)

Time = 5.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.65

$$\int \left(a+b\text{sech}^2(c+dx)\right)^5\,dx = a^5x+\frac{5a^4b\tanh(c+dx)}{d}+\frac{10a^3b^2\tanh(c+dx)}{d}+\frac{10a^2b^3\tanh(c+dx)}{d}+\frac{5ab^4\tanh(c+dx)}{d}+\frac{b^5\tanh(c+dx)}{d}-\frac{10a^3b^2\tanh^3(c+dx)}{3d}-\frac{20a^2b^3\tanh^3(c+dx)}{3d}-\frac{5ab^4\tanh^3(c+dx)}{d}-\frac{4b^5\tanh^3(c+dx)}{3d}+\frac{2a^2b^3\tanh^5(c+dx)}{d}+\frac{3ab^4\tanh^5(c+dx)}{d}+\frac{6b^5\tanh^5(c+dx)}{5d}-\frac{5ab^4\tanh^7(c+dx)}{7d}-\frac{4b^5\tanh^7(c+dx)}{7d}+\frac{b^5\tanh^9(c+dx)}{9d}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^5,x]
```

output
```
a^5*x + (5*a^4*b*Tanh[c + d*x])/d + (10*a^3*b^2*Tanh[c + d*x])/d + (10*a^2
*b^3*Tanh[c + d*x])/d + (5*a*b^4*Tanh[c + d*x])/d + (b^5*Tanh[c + d*x])/d
- (10*a^3*b^2*Tanh[c + d*x]^3)/(3*d) - (20*a^2*b^3*Tanh[c + d*x]^3)/(3*d)
- (5*a*b^4*Tanh[c + d*x]^3)/d - (4*b^5*Tanh[c + d*x]^3)/(3*d) + (2*a^2*b^3
*Tanh[c + d*x]^5)/d + (3*a*b^4*Tanh[c + d*x]^5)/d + (6*b^5*Tanh[c + d*x]^5
)/(5*d) - (5*a*b^4*Tanh[c + d*x]^7)/(7*d) - (4*b^5*Tanh[c + d*x]^7)/(7*d)
+ (b^5*Tanh[c + d*x]^9)/(9*d)
```

3.137.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^2(c+dx)\right)^5\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\sec(ic+idx)^2\right)^5 dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\left(-b\tanh^2(c+dx)+a+b\right)^5}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^5\tanh^8(c+dx)-b^4(5a+4b)\tanh^6(c+dx)+b^3\left(10a^2+15ba+6b^2\right)\tanh^4(c+dx)-b^2\left(10a^3+20ba^2+15b\right.\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\text{arctanh}(\tanh(c+dx))+\frac{1}{5}b^3\left(10a^2+15ab+6b^2\right)\tanh^5(c+dx)-\frac{1}{3}b^2\left(10a^3+20a^2b+15ab^2+4b^3\right)\tanh^3(c+}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^5,x]
```

output
```
(a^5*ArcTanh[Tanh[c + d*x]] + b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 +
 b^4)*Tanh[c + d*x] - (b^2*(10*a^3 + 20*a^2*b + 15*a*b^2 + 4*b^3)*Tanh[c +
 d*x]^3)/3 + (b^3*(10*a^2 + 15*a*b + 6*b^2)*Tanh[c + d*x]^5)/5 - (b^4*(5*a
 + 4*b)*Tanh[c + d*x]^7)/7 + (b^5*Tanh[c + d*x]^9)/9)/d
```

3.137.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.137.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{a^5(dx+c)+5a^4b\tanh(dx+c)+10a^3b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+10a^2b^3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(d}{}$
default	$\frac{a^5(dx+c)+5a^4b\tanh(dx+c)+10a^3b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+10a^2b^3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(d}{}$
parts	$a^5x+\frac{b^5\left(\frac{128}{315}+\frac{\operatorname{sech}(dx+c)^8}{9}+\frac{8\operatorname{sech}(dx+c)^6}{63}+\frac{16\operatorname{sech}(dx+c)^4}{105}+\frac{64\operatorname{sech}(dx+c)^2}{315}\right)\tanh(dx+c)}{d}+\frac{5a^4b\tanh(dx+c)}{d}+\frac{1(}{}$
parallelrisch	$\frac{(44100a^4b+96600a^3b^2+107520a^2b^3+60480a\,b^4+10752b^5)\sinh(3dx+3c)+(31500a^4b+63000a^3b^2+60480a^2b^3+25920a}{}$
risch	$a^5x-\frac{2b(88200a^4e^{6dx+6c}+10752b^4e^{6dx+6c}+88200a^4e^{10dx+10c}+18900a^3b\,e^{2dx+2c}+15120a^2b^2e^{2dx+2c}+6480a\,b^3e^{2c}}{}$

input
```
int((a+b*sech(d*x+c)^2)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^5*(d*x+c)+5*a^4*b*tanh(d*x+c)+10*a^3*b^2*(2/3+1/3*sech(d*x+c)^2)*ta
nh(d*x+c)+10*a^2*b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+
c)+5*a*b^4*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)
*tanh(d*x+c)+b^5*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)^6+16/105*sech
(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c))
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1652 vs. 2(155) = 310.

Time = 0.27 (sec) , antiderivative size = 1652, normalized size of antiderivative = 10.13

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^5\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="fricas")
```

output
```
1/315*((315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4
 - 128*b^5)*cosh(d*x + c)^9 + 9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 -
 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)*sinh(d*x + c)^8 + (1575
*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*sinh(d*x + c)^
9 + 9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4
- 128*b^5)*cosh(d*x + c)^7 + 9*(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 +
 720*a*b^4 + 128*b^5 + 4*(1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a
*b^4 + 128*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(315*a^5*d*x - 15
75*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c
)^3 + 3*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^
4 - 128*b^5)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*(315*a^5*d*x - 1575*a^4*b
 - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^5 + 9*
(3500*a^4*b + 7000*a^3*b^2 + 6720*a^2*b^3 + 2880*a*b^4 + 512*b^5 + 14*(157
5*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)
^4 + 21*(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*c
osh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(315*a^5*d*x - 1575*a^4*b - 2100*a
^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^5 + 35*(315*a^5
*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cos
h(d*x + c)^3 + 20*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3
- 720*a*b^4 - 128*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(315*a^5*d*x...
```

3.137.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)^5 dx = \int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)^5 dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**5,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**5, x)
```

3.137.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(155) = 310$.

Time = 0.20 (sec) , antiderivative size = 1277, normalized size of antiderivative = 7.83

$$\int \left(a+b\text{sech}^2(c+dx)\right)^5 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="maxima")
```

output
```
a^5*x + 256/315*b^5*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*
d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x -
 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16
*c) + e^(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c
) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126
*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^
(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^
(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x
 - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x -
 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 126*e^(-8*d*x -
 8*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) +
 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 3
6*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 1
/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*
e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(
-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))) + 32/7*
a*b^4*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 3
5*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-1
2*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2
*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x ...
```

3.137.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(155) = 310$.

Time = 0.32 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.29

$$\int \left(a+b\text{sech}^2(c+dx)\right)^5 \, dx$$
$$= \frac{315\,(dx+c)a^5 - \frac{2\,(1575\,a^4be^{(16\,dx+16\,c)}+12600\,a^4be^{(14\,dx+14\,c)}+6300\,a^3b^2e^{(14\,dx+14\,c)}+44100\,a^4be^{(12\,dx+12\,c)}+39900\,a^3b^2e^{(12\,dx+12\,c)}}{}}{}$$

input
```
integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="giac")
```

output
```
1/315*(315*(d*x + c)*a^5 - 2*(1575*a^4*b*e^(16*d*x + 16*c) + 12600*a^4*b*e
^(14*d*x + 14*c) + 6300*a^3*b^2*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x
+ 12*c) + 39900*a^3*b^2*e^(12*d*x + 12*c) + 16800*a^2*b^3*e^(12*d*x + 12*c
) + 88200*a^4*b*e^(10*d*x + 10*c) + 107100*a^3*b^2*e^(10*d*x + 10*c) + 756
00*a^2*b^3*e^(10*d*x + 10*c) + 25200*a*b^4*e^(10*d*x + 10*c) + 110250*a^4*
b*e^(8*d*x + 8*c) + 157500*a^3*b^2*e^(8*d*x + 8*c) + 136080*a^2*b^3*e^(8*d
*x + 8*c) + 65520*a*b^4*e^(8*d*x + 8*c) + 16128*b^5*e^(8*d*x + 8*c) + 8820
0*a^4*b*e^(6*d*x + 6*c) + 136500*a^3*b^2*e^(6*d*x + 6*c) + 124320*a^2*b^3*
e^(6*d*x + 6*c) + 60480*a*b^4*e^(6*d*x + 6*c) + 10752*b^5*e^(6*d*x + 6*c)
+ 44100*a^4*b*e^(4*d*x + 4*c) + 69300*a^3*b^2*e^(4*d*x + 4*c) + 60480*a^2*
b^3*e^(4*d*x + 4*c) + 25920*a*b^4*e^(4*d*x + 4*c) + 4608*b^5*e^(4*d*x + 4*
c) + 12600*a^4*b*e^(2*d*x + 2*c) + 18900*a^3*b^2*e^(2*d*x + 2*c) + 15120*a
^2*b^3*e^(2*d*x + 2*c) + 6480*a*b^4*e^(2*d*x + 2*c) + 1152*b^5*e^(2*d*x +
2*c) + 1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)/(e^
(2*d*x + 2*c) + 1)^9)/d
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1952, normalized size of antiderivative = 11.98

$$\int \left(a+b\text{sech}^2(c+dx)\right)^5\,dx=\text{Too large to display}$$

input
```
int((a + b/cosh(c + d*x)^2)^5,x)
```

output
```
a^5*x - ((10*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(63*d) + (10*e
xp(2*c + 2*d*x)*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(21*d) + (10*exp(4*c +
 4*d*x)*(a^4*b + a^3*b^2))/(3*d) + (10*a^4*b*exp(6*c + 6*d*x))/(9*d))/(4*e
xp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*
x) + 1) - ((80*exp(6*c + 6*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b
^2))/(9*d) + (80*exp(10*c + 10*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a
^3*b^2))/(9*d) + (4*exp(8*c + 8*d*x)*(320*a*b^4 + 175*a^4*b + 128*b^5 + 48
0*a^2*b^3 + 400*a^3*b^2))/(9*d) + (10*a^4*b)/(9*d) + (40*exp(4*c + 4*d*x)*
(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(9*d) + (40*exp(12*c + 12*d*x)*(7*a^4*
b + 8*a^2*b^3 + 12*a^3*b^2))/(9*d) + (80*exp(2*c + 2*d*x)*(a^4*b + a^3*b^2
))/(9*d) + (80*exp(14*c + 14*d*x)*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*exp
(16*c + 16*d*x))/(9*d))/(9*exp(2*c + 2*d*x) + 36*exp(4*c + 4*d*x) + 84*exp
(6*c + 6*d*x) + 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) + 84*exp(12*
c + 12*d*x) + 36*exp(14*c + 14*d*x) + 9*exp(16*c + 16*d*x) + exp(18*c + 18
*d*x) + 1) - ((10*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*exp(2*c + 2*d*x))/(
9*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((10*(7*a^4*b + 8*a^2*
b^3 + 12*a^3*b^2))/(63*d) + (20*exp(2*c + 2*d*x)*(8*a*b^4 + 7*a^4*b + 16*a
^2*b^3 + 15*a^3*b^2))/(21*d) + (200*exp(6*c + 6*d*x)*(8*a*b^4 + 7*a^4*b +
16*a^2*b^3 + 15*a^3*b^2))/(63*d) + (2*exp(4*c + 4*d*x)*(320*a*b^4 + 175*a^
4*b + 128*b^5 + 480*a^2*b^3 + 400*a^3*b^2))/(21*d) + (50*exp(8*c + 8*d*...
```

3.137. $\int \left(a+b\text{sech}^2(c+dx)\right)^5\,dx$

3.138 $\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{(a+2b)\log(\cosh(c+dx))}{b^2d} + \frac{(a+b)^2\log\left(b+a\cosh^2(c+dx)\right)}{2ab^2d} - \frac{\text{sech}^2(c+dx)}{2bd}$$

output
```
-(a+2*b)*ln(cosh(d*x+c))/b^2/d+1/2*(a+b)^2*ln(b+a*cosh(d*x+c)^2)/a/b^2/d-1
/2*sech(d*x+c)^2/b/d
```

3.138.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(2a(a+2b)\log(\cosh(c+dx)) - (a+b)^2\log\left(a+b+a\sinh^2\right.\right.}{4ab^2d\left(a+b\text{sech}^2(c+dx)\right)}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/4*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(2*a*(a + 2*b)*Log[C
osh[c + d*x]] - (a + b)^2*Log[a + b + a*Sinh[c + d*x]^2] + a*b*Sech[c + d*
x]^2))/(a*b^2*d*(a + b*Sech[c + d*x]^2))
```

3.138.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\tan(ic+idx)^5}{a+b\sec(ic+idx)^2}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\tan(ic+idx)^5}{b\sec(ic+idx)^2+a}dx$$
$$\Big\downarrow \text{4626}$$
$$\frac{\int \frac{(1-\cosh^2(c+dx))^2\text{sech}^3(c+dx)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{d}$$
$$\Big\downarrow \text{354}$$
$$\frac{\int \frac{(1-\cosh^2(c+dx))^2\text{sech}^2(c+dx)}{a\cosh^2(c+dx)+b}d\cosh^2(c+dx)}{2d}$$
$$\Big\downarrow \text{99}$$
$$\frac{\int \left(\frac{(a+b)^2}{b^2(a\cosh^2(c+dx)+b)}+\frac{\text{sech}^2(c+dx)}{b}+\frac{(-a-2b)\text{sech}(c+dx)}{b^2}\right)d\cosh^2(c+dx)}{2d}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{(a+b)^2\log(a\cosh^2(c+dx)+b)}{ab^2}-\frac{(a+2b)\log(\cosh^2(c+dx))}{b^2}-\frac{\text{sech}(c+dx)}{b}}{2d}$$

input
```
Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-(((a + 2*b)*Log[Cosh[c + d*x]^2])/b^2) + ((a + b)^2*Log[b + a*Cosh[c + d
*x]^2])/(a*b^2) - Sech[c + d*x]/b)/(2*d)
```

3.138. $\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}dx$

3.138.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.138. $\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.138.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(66) = 132$.

Time = 4.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.61

method	result
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}a^2+\frac{1}{2}ab+\frac{1}{4}b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b\right)}{a\,b^2}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-}{d}$
default	$\frac{\frac{2\left(\frac{1}{4}a^2+\frac{1}{2}ab+\frac{1}{4}b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b\right)}{a\,b^2}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-}{d}$
risch	$-\frac{x}{a}-\frac{2c}{da}-\frac{2\,e^{2dx+2c}}{bd\left(e^{2dx+2c}+1\right)^2}-\frac{\ln(e^{2dx+2c}+1)a}{b^2d}-\frac{2\ln(e^{2dx+2c}+1)}{bd}+\frac{a\ln\left(e^{4dx+4c}+\frac{2(a+2b)e^{2dx+2c}}{a}+1\right)}{2b^2d}+$

input
```
int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a/b^2*(1/4*a^2+1/2*a*b+1/4*b^2)*ln(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2
*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1
/a*ln(1+tanh(1/2*d*x+1/2*c))-1/b^2*((a+2*b)*ln(tanh(1/2*d*x+1/2*c)^2+1)+2*
b/(tanh(1/2*d*x+1/2*c)^2+1)^2-2*b/(tanh(1/2*d*x+1/2*c)^2+1))-1/a*ln(tanh(1
/2*d*x+1/2*c)-1))
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(66) = 132$.

Time = 0.33 (sec) , antiderivative size = 736, normalized size of antiderivative = 10.51

$$\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx =$$

$$-\frac{2\,b^2dx\cosh\left(dx+c\right)^4+8\,b^2dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,b^2dx\sinh\left(dx+c\right)^4+2\,b^2dx+4\left(b^2dx+\right.}{}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x + a*b)*cosh(d*x + c)^
2 + 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x + a*b)*sinh(d*x + c)^2 - ((a^2
+ 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(
d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b +
b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(
d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 +
4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c
)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2
 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x
+ c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 + (b^2*d*x +
a*b)*cosh(d*x + c))*sinh(d*x + c))/(a*b^2*d*cosh(d*x + c)^4 + 4*a*b^2*d*co
sh(d*x + c)*sinh(d*x + c)^3 + a*b^2*d*sinh(d*x + c)^4 + 2*a*b^2*d*cosh(d*x
 + c)^2 + a*b^2*d + 2*(3*a*b^2*d*cosh(d*x + c)^2 + a*b^2*d)*sinh(d*x + c)^
2 + 4*(a*b^2*d*cosh(d*x + c)^3 + a*b^2*d*cosh(d*x + c))*sinh(d*x + c))
```

3.138.6 Sympy [F]

$$\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\tanh^5(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$$

input
```
integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2), x)
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.87

$$\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{dx+c}{ad} - \frac{2\,e^{(-2\,dx-2\,c)}}{\left(2\,be^{(-2\,dx-2\,c)} + be^{(-4\,dx-4\,c)} + b\right)d} - \frac{(a+2\,b)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d} + \frac{\left(a^2+2\,ab+b^2\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)} + ae^{(-4\,dx-4\,c)} + a\right)}{2\,ab^2d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) - 2*e^(-2*d*x - 2*c)/((2*b*e^(-2*d*x - 2*c) + b*e^(-4*d*x
- 4*c) + b)*d) - (a + 2*b)*log(e^(-2*d*x - 2*c) + 1)/(b^2*d) + 1/2*(a^2 +
2*a*b + b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a
*b^2*d)
```

3.138.8 Giac [F]

$$\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\tanh(dx+c)^5}{b\,\text{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.138.9 Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 421, normalized size of antiderivative = 6.01

$$\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{2}{b\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} - \frac{2}{b\,d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{x}{a} - \frac{\ln\left(39\,a\,b^7+243\,a^7\,b+27\,a^8+2\,b^8+289\,a^2\,b^6+1017\,a^3\,b^5+1791\,a^4\,b^4+1701\,a^5\,b^3+891\,a^6\,b^2+27\,a^8 \cdots\right.}{\cdots} + \frac{\ln\left(a\,b^2+6\,a^2\,b+3\,a^3+6\,a^3\,e^{2\,c}\,e^{2\,d\,x}+3\,a^3\,e^{4\,c}\,e^{4\,d\,x}+4\,b^3\,e^{2\,c}\,e^{2\,d\,x}+26\,a\,b^2\,e^{2\,c}\,e^{2\,d\,x}+24\,a^2\,b\,e^{2\,c}\,e^{2\,d\,x} \cdots\right.}{2\,a\,b^2\,d}$$

input
```
int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2),x)
```

output
```
2/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - 2/(b*d*(exp(2*c + 2*
d*x) + 1)) - x/a - (log(39*a*b^7 + 243*a^7*b + 27*a^8 + 2*b^8 + 289*a^2*b^
6 + 1017*a^3*b^5 + 1791*a^4*b^4 + 1701*a^5*b^3 + 891*a^6*b^2 + 27*a^8*exp(
2*c)*exp(2*d*x) + 2*b^8*exp(2*c)*exp(2*d*x) + 39*a*b^7*exp(2*c)*exp(2*d*x)
 + 243*a^7*b*exp(2*c)*exp(2*d*x) + 289*a^2*b^6*exp(2*c)*exp(2*d*x) + 1017*
a^3*b^5*exp(2*c)*exp(2*d*x) + 1791*a^4*b^4*exp(2*c)*exp(2*d*x) + 1701*a^5*
b^3*exp(2*c)*exp(2*d*x) + 891*a^6*b^2*exp(2*c)*exp(2*d*x))*(a + 2*b))/(b^2
*d) + (log(a*b^2 + 6*a^2*b + 3*a^3 + 6*a^3*exp(2*c)*exp(2*d*x) + 3*a^3*exp
(4*c)*exp(4*d*x) + 4*b^3*exp(2*c)*exp(2*d*x) + 26*a*b^2*exp(2*c)*exp(2*d*x
) + 24*a^2*b*exp(2*c)*exp(2*d*x) + a*b^2*exp(4*c)*exp(4*d*x) + 6*a^2*b*exp
(4*c)*exp(4*d*x))*(2*a*b + a^2 + b^2))/(2*a*b^2*d)
```

3.138. $\int \frac{\tanh^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.139 $\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\frac{x}{a}-\frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d}+\frac{\tanh(c+dx)}{bd}$$

output
```
x/a-(a+b)^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/b^(3/2)/d+tanh(
d*x+c)/b/d
```

3.139.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 196 vs. 2(59) = 118.

Time = 1.87 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.32

$$\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$

$$=\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left((a+b)^2\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.\right.}{2ab\sqrt{a+b}d\left(a+b\text{sech}^2(c+d\right.}$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*((a + b)^2*ArcTanh[(Sech[
d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2
*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sq
rt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(b*d*x + a*Sech[c]*Sech[c + d*x]*S
inh[d*x])))/(2*a*b*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] -
 Sinh[c])^4])
```

3.139.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4629, 2075, 381, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(ic+idx)^4}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \ 4629$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))(a+b(1-\tanh^2(c+dx)))}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 2075$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 381$$

$$\frac{\frac{\tanh(c+dx)}{b}-\frac{\int \frac{-\left((a+2b)\tanh^2(c+dx)\right)+a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\tanh(c+dx)}{b}-\frac{\frac{(a+b)^2\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}-\frac{b\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}}{b}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\tanh(c+dx)}{b}-\frac{\frac{(a+b)^2\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}-\frac{b\text{arctanh}(\tanh(c+dx))}{a}}{b}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\tanh(c+dx)}{b}-\frac{\frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}-\frac{b\text{arctanh}(\tanh(c+dx))}{a}}{b}}{d}$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-((-((b*ArcTanh[Tanh[c + d*x]])/a) + ((a + b)^(3/2)*ArcTanh[(Sqrt[b]*Tanh
[c + d*x])/Sqrt[a + b]])/(a*Sqrt[b]))/b) + Tanh[c + d*x]/b)/d
```

3.139.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.139.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(51) = 102$.

Time = 2.88 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.12

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2\left(a^2+2ab+b^2\right)\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+\cdots}\right.}{\cdots}\right.}{ab}\cdots}{d}$
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2\left(a^2+2ab+b^2\right)\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+\cdots}\right.}{\cdots}\right.}{ab}\cdots}{d}$
risch	$\frac{x}{a}-\frac{2}{bd\left(e^{2dx+2c}+1\right)}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}+a+2b}{a}\right)}{2b^2d}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}+a+2b}{a}\right)}{2bda}-\cdots$

input
```
int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(1+tanh(1/2*d*x+1/2*c))-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+2/a/b*(a^
2+2*a*b+b^2)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^
2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
)+2/b*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1))
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(51) = 102$.

Time = 0.32 (sec) , antiderivative size = 683, normalized size of antiderivative = 11.58

$$\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$= \left[\frac{2\,bdx\cosh\left(dx+c\right)^2 + 4\,bdx\cosh\left(dx+c\right)\sinh\left(dx+c\right) + 2\,bdx\sinh\left(dx+c\right)^2 + 2\,bdx + \left(\left(a+b\right)\cosh\left(d\right.\right.}{}\right.$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*
d*x*sinh(d*x + c)^2 + 2*b*d*x + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(
d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*sqrt((a + b)/b)*
log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(
d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 +
a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^
3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 +
2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqr
t((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*s
inh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a
+ 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*s
inh(d*x + c) + a)) - 4*a)/(a*b*d*cosh(d*x + c)^2 + 2*a*b*d*cosh(d*x + c)*s
inh(d*x + c) + a*b*d*sinh(d*x + c)^2 + a*b*d), (b*d*x*cosh(d*x + c)^2 + 2*
b*d*x*cosh(d*x + c)*sinh(d*x + c) + b*d*x*sinh(d*x + c)^2 + b*d*x - ((a +
b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(
d*x + c)^2 + a + b)*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*c
osh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-(a + b)/b)
/(a + b)) - 2*a)/(a*b*d*cosh(d*x + c)^2 + 2*a*b*d*cosh(d*x + c)*sinh(d*x +
 c) + a*b*d*sinh(d*x + c)^2 + a*b*d)]
```

3.139.6 Sympy [F]

$$\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\tanh^4(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$$

input
```
integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.139.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(51) = 102$.

Time = 0.36 (sec) , antiderivative size = 637, normalized size of antiderivative = 10.80

$$\begin{aligned}
\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= -\frac{(a+2\,b)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}bd} \\
&+ \frac{(a+2\,b)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}bd} \\
&+ \frac{3\,a\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}bd} \\
&+ \frac{(a+2\,b)\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{8\,abd} \\
&+ \frac{\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,bd} \\
&- \frac{(a+2\,b)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{8\,abd} \\
&- \frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,bd} \\
&- \frac{3\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{4\,bd} + \frac{3\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{4\,bd} \\
&- \frac{(a^2+8\,ab+8\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{32\,\sqrt{(a+b)b}abd} \\
&+ \frac{(a^2+8\,ab+8\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{32\,\sqrt{(a+b)b}abd} \\
&- \frac{5}{8\,(be^{(2\,dx+2\,c)}+b)d} + \frac{11}{8\,(be^{(-2\,dx-2\,c)}+b)d}
\end{aligned}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*(a + 2*b)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^
(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 1/8*
(a + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*
d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 3/16*a*
log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c)
 + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 1/8*(a + 2*b)*log
(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a*b*d) + 1/4*log(a*
e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(b*d) - 1/8*(a + 2*b)*l
og(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a*b*d) - 1/4*lo
g(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(b*d) - 3/4*log(e
^(2*d*x + 2*c) + 1)/(b*d) + 3/4*log(e^(-2*d*x - 2*c) + 1)/(b*d) - 1/32*(a^
2 + 8*a*b + 8*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(
a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*b*d)
+ 1/32*(a^2 + 8*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a
 + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b
)*b)*a*b*d) - 5/8/((b*e^(2*d*x + 2*c) + b)*d) + 11/8/((b*e^(-2*d*x - 2*c)
+ b)*d)
```

3.139.8 Giac [F]

$$\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\tanh\left(dx+c\right)^4}{b\,\text{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.139.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.10

$$\int \frac{\tanh^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx = \frac{x}{a} - \frac{2}{b\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} + \frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)^2}{a^2\,b} - \frac{2\,(a+b)^{3/2}\,\left(a+a\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^2\,b^{3/2}}\right)\,(a+b)^{3/2}}{2\,a\,b^{3/2}\,d} - \frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)^2}{a^2\,b} + \frac{2\,(a+b)^{3/2}\,\left(a+a\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^2\,b^{3/2}}\right)\,(a+b)^{3/2}}{2\,a\,b^{3/2}\,d}$$

input
```
int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

output
```
x/a - 2/(b*d*(exp(2*c + 2*d*x) + 1)) + (log((4*exp(2*c + 2*d*x)*(a + b)^2)
/(a^2*b) - (2*(a + b)^(3/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2*c + 2*d*x)
))/(a^2*b^(3/2)))*(a + b)^(3/2))/(2*a*b^(3/2)*d) - (log((4*exp(2*c + 2*d*x
)*(a + b)^2)/(a^2*b) + (2*(a + b)^(3/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(
2*c + 2*d*x)))/(a^2*b^(3/2)))*(a + b)^(3/2))/(2*a*b^(3/2)*d)
```

3.140 $\int \frac{\tanh^3(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

3.140.1 Optimal result

Integrand size = 23, antiderivative size = 45

$$\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=-\frac{\log(\cosh(c+dx))}{bd}+\frac{(a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2abd}$$

output
```
-ln(cosh(d*x+c))/b/d+1/2*(a+b)*ln(b+a*cosh(d*x+c)^2)/a/b/d
```

3.140.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\frac{-2a\log(\cosh(c+dx))+(a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2abd}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-2*a*Log[Cosh[c + d*x]] + (a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a*b*d)
```

3.140.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ic+idx)^3}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ic+idx)^3}{b\sec(ic+idx)^2+a}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{(1-\cosh^2(c+dx))\text{sech}(c+dx)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{(1-\cosh^2(c+dx))\text{sech}(c+dx)}{a\cosh^2(c+dx)+b}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{86}$$

$$-\frac{\int \left(\frac{-a-b}{b(a\cosh^2(c+dx)+b)}+\frac{\text{sech}(c+dx)}{b}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\log(\cosh^2(c+dx))}{b}-\frac{(a+b)\log(a\cosh^2(c+dx)+b)}{ab}}{2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/2*(Log[Cosh[c + d*x]^2]/b - ((a + b)*Log[b + a*Cosh[c + d*x]^2])/(a*b))
/d
```

3.140.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.140.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(43) = 86.

Time = 1.74 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.56

method	result
risch	$-\frac{x}{a}-\frac{2c}{da}-\frac{\ln\left(e^{2dx+2c}+1\right)}{bd}+\frac{\ln\left(e^{4dx+4c}+\frac{2(a+2b)e^{2dx+2c}}{a}+1\right)}{2bd}+\frac{\ln\left(e^{4dx+4c}+\frac{2(a+2b)e^{2dx+2c}}{a}+1\right)}{2ad}$
derivativedivides	$\frac{\frac{2\left(\frac{a}{4}+\frac{b}{4}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}{ab}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{a}}{d}$
default	$\frac{\frac{2\left(\frac{a}{4}+\frac{b}{4}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}{ab}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{a}}{d}$

input
```
int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-x/a-2/d/a*c-1/b/d*ln(exp(2*d*x+2*c)+1)+1/2/b/d*ln(exp(4*d*x+4*c)+2*(a+2*b
)/a*exp(2*d*x+2*c)+1)+1/2/a/d*ln(exp(4*d*x+4*c)+2*(a+2*b)/a*exp(2*d*x+2*c)
+1)
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(43) = 86.

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{2\,bdx-(a+b)\log\left(\frac{2\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2\,b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)+2\,a\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,abd}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x - (a + b)*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a +
 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2))
 + 2*a*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(a*b*d)
```

3.140.6 Sympy [F]

$$\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\tanh^3(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$$

input
```
integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.71

$$\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{dx+c}{ad} + \frac{(a+b)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)} + ae^{(-4\,dx-4\,c)} + a\right)}{2\,abd} - \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{bd}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) + 1/2*(a + b)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d
*x - 4*c) + a)/(a*b*d) - log(e^(-2*d*x - 2*c) + 1)/(b*d)
```

3.140.8 Giac [F]

$$\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\tanh(dx+c)^3}{b\,\text{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.140.9 Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.29

$$\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$= \frac{\ln\left(a\,b+3\,a^2+6\,a^2\,e^{2\,c}\,e^{2\,d\,x}+3\,a^2\,e^{4\,c}\,e^{4\,d\,x}+4\,b^2\,e^{2\,c}\,e^{2\,d\,x}+14\,a\,b\,e^{2\,c}\,e^{2\,d\,x}+a\,b\,e^{4\,c}\,e^{4\,d\,x}\right)\,(a+b)}{2\,a\,b\,d}$$
$$- \frac{\ln\left(21\,a\,b^4+108\,a^4\,b+27\,a^5+2\,b^5+82\,a^2\,b^3+144\,a^3\,b^2+27\,a^5\,e^{2\,c}\,e^{2\,d\,x}+2\,b^5\,e^{2\,c}\,e^{2\,d\,x}+21\,a\,b^4\,e^{2\,c}\,e^{2\,c}\right.}{b\,d}$$
$$- \frac{x}{a}$$

input
```
int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

output
```
(log(a*b + 3*a^2 + 6*a^2*exp(2*c)*exp(2*d*x) + 3*a^2*exp(4*c)*exp(4*d*x) +
 4*b^2*exp(2*c)*exp(2*d*x) + 14*a*b*exp(2*c)*exp(2*d*x) + a*b*exp(4*c)*exp
(4*d*x))*(a + b))/(2*a*b*d) - log(21*a*b^4 + 108*a^4*b + 27*a^5 + 2*b^5 +
82*a^2*b^3 + 144*a^3*b^2 + 27*a^5*exp(2*c)*exp(2*d*x) + 2*b^5*exp(2*c)*exp
(2*d*x) + 21*a*b^4*exp(2*c)*exp(2*d*x) + 108*a^4*b*exp(2*c)*exp(2*d*x) + 8
2*a^2*b^3*exp(2*c)*exp(2*d*x) + 144*a^3*b^2*exp(2*c)*exp(2*d*x))/(b*d) - x
/a
```

3.141 $\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.141.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\frac{x}{a}-\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}d}$$

output
```
x/a-arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b)^(1/2)/a/d/b^(1/2)
```

3.141.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 174 vs. $2(46) = 92$.

Time = 0.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.78

$$\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$=\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(\sqrt{a+b}dx\sqrt{b(\cosh(c)-\sinh(c))^4}+(a+b)\text{arctanh}\left(\frac{\text{sech}(dx)(\cdots}{\cdots}\right.\right.}{2a\sqrt{a+b}d\left(a+b\text{sech}^2(c+dx)\right)\sqrt{b(\cosh(c)-\cdots}}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(
Cosh[c] - Sinh[c])^4] + (a + b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])
*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c]
 - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c])))/(2*a*Sqrt[a + b]*d*(a + b*Sech
[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

3.141.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 25, 4629, 25, 2075, 383, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ic+idx)^2}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ic+idx)^2}{b\sec(ic+idx)^2+a}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(a+b(1-\tanh^2(c+dx)))}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(a+b(1-\tanh^2(c+dx)))}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{383}$$

$$-\frac{\frac{(a+b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}-\frac{\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}}{d}$$

$$\downarrow \text{219}$$

$$-\frac{\frac{(a+b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}-\frac{\text{arctanh}(\tanh(c+dx))}{a}}{d}$$

$$\downarrow \text{221}$$

$$-\frac{\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}-\frac{\text{arctanh}(\tanh(c+dx))}{a}}{d}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
-((-(ArcTanh[Tanh[c + d*x]]/a) + (Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a + b]])/(a*Sqrt[b]))/d)
```

3.141.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.141. $\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.141.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(38) = 76$.

Time = 1.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.26

method	result
risch	$\frac{x}{a}+\frac{\sqrt{(a+b)b}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{(a+b)b}+a+2b}{a}\right)}{2bda}-\frac{\sqrt{(a+b)b}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{(a+b)b}-a-2b}{a}\right)}{2bda}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2(a+b)\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\tanh\left(\frac{dx}{2}\cdots\right.\right.}{4\sqrt{b}\,\sqrt{a+b}}\cdots\right.}{a}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2(a+b)\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\tanh\left(\frac{dx}{2}\cdots\right.\right.}{4\sqrt{b}\,\sqrt{a+b}}\cdots\right.}{a}}{d}$

input
```
int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
x/a+1/2/b*((a+b)*b)^(1/2)/d/a*ln(exp(2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/
a)-1/2/b*((a+b)*b)^(1/2)/d/a*ln(exp(2*d*x+2*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a
)
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 419, normalized size of antiderivative = 9.11

$$\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$= \left[\frac{2\,dx + \sqrt{\frac{a+b}{b}}\log\left(\frac{a^2\cosh(dx+c)^4+4\,a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\,(a^2+2\,ab)\cosh(dx+c)^2+2\left(3\,a^2\cosh(dx+c)^2+a^2\cdots\right.}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+\cdots}\right.}{\cdots}\right.$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*d*x + sqrt((a + b)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x +
c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2
 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b +
 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x +
c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh
(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(
d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2
 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^
3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/(a*d), (d*x - sqrt(-(a +
 b)/b)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a
*sinh(d*x + c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)))/(a*d)]
```

3.141.6 Sympy [F]

$$\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\tanh^2{(c+dx)}}{a+b\,\text{sech}^2{(c+dx)}}\,dx$$

input
```
integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.141.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 6.33

$$\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{(a+2b)\log\left(\frac{ae^{(2dx+2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(2dx+2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}ad} + \frac{\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{4\sqrt{(a+b)b}d} + \frac{(a+2b)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}ad} + \frac{\log\left(ae^{(4dx+4c)}+2(a+2b)e^{(2dx+2c)}+a\right)}{4ad} - \frac{\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{4ad}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*(a + 2*b)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^
(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 1/4*
log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c)
 + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*d) + 1/8*(a + 2*b)*log((
a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
+ 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 1/4*log(a*e^(4*d*x + 4
*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a*d) - 1/4*log(2*(a + 2*b)*e^(-2*d
*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a*d)
```

3.141.8 Giac [F]

$$\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\tanh(dx+c)^2}{b\,\text{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.141.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.28

$$\int \frac{\tanh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{x}{a}+\frac{\text{atan}\left(\frac{\sqrt{-a^2\,b\,d^2}}{a\,d\,\sqrt{a+b}}+\frac{\sqrt{-a^2\,b\,d^2}}{2\,b\,d\,\sqrt{a+b}}+\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-a^2\,b\,d^2}}{2\,b\,d\,\sqrt{a+b}}\right)\,\sqrt{a+b}}{\sqrt{-a^2\,b\,d^2}}$$

input

```
int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

output

```
x/a + (atan((-a^2*b*d^2)^(1/2)/(a*d*(a + b)^(1/2)) + (-a^2*b*d^2)^(1/2)/(2
*b*d*(a + b)^(1/2)) + (exp(2*c)*exp(2*d*x)*(-a^2*b*d^2)^(1/2))/(2*b*d*(a +
 b)^(1/2)))*(a + b)^(1/2))/(-a^2*b*d^2)^(1/2)
```

3.142 $\int \frac{\tanh(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx$

3.142.1 Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\tanh(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2ad}$$

output
```
1/2*ln(b+a*cosh(d*x+c)^2)/a/d
```

3.142.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\tanh(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \frac{\log(a+2b+a\cosh(2(c+dx)))}{2ad}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
Log[a + 2*b + a*Cosh[2*(c + d*x)]]/(2*a*d)
```

3.142.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 26, 4626, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\tan(ic+idx)}{a+b\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\tan(ic+idx)}{b\sec(ic+idx)^2+a}dx$$

$$\Big\downarrow \text{ 4626}$$

$$\frac{\int \frac{\cosh(c+dx)}{a\cosh^2(c+dx)+b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2ad}$$

input
```
Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
Log[b + a*Cosh[c + d*x]^2]/(2*a*d)
```

3.142.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.142.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

method	result	size
derivativedivides	$\frac{\ln\left(a+b\,\mathrm{sech}(dx+c)^2\right)}{2da}-\frac{\ln(\mathrm{sech}(dx+c))}{da}$	38
default	$\frac{\ln\left(a+b\,\mathrm{sech}(dx+c)^2\right)}{2da}-\frac{\ln(\mathrm{sech}(dx+c))}{da}$	38
risch	$-\frac{x}{a}-\frac{2c}{da}+\frac{\ln\left(\mathrm{e}^{4dx+4c}+\frac{2(a+2b)\mathrm{e}^{2dx+2c}}{a}+1\right)}{2ad}$	56

input
```
int(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a*ln(a+b*sech(d*x+c)^2)-1/d/a*ln(sech(d*x+c))
```

3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(21) = 42$.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int \frac{\tanh(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = -\frac{2\,dx-\log\left(\frac{2\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2\,b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)}{2\,ad}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*d*x - log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cos
h(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/(a*d)
```

3.142.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(17) = 34$.

Time = 1.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.96

$$\int \frac{\tanh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x\tanh(c)}{\operatorname{sech}^2(c)} & \text{for } a=0\wedge b=0\wedge d=0 \\ \frac{1}{2bd\operatorname{sech}^2(c+dx)} & \text{for } a=0 \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}}{a} & \text{for } b=0 \\ \frac{x\tanh(c)}{a+b\operatorname{sech}^2(c)} & \text{for } d=0 \\ \frac{x}{a}+\frac{\log\left(-\sqrt{-\frac{a}{b}}+\operatorname{sech}(c+dx)\right)}{2ad}+\frac{\log\left(\sqrt{-\frac{a}{b}}+\operatorname{sech}(c+dx)\right)}{2ad}-\frac{\log(\tanh(c+dx)+1)}{ad} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x*tanh(c)/sech(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(
2*b*d*sech(c + d*x)**2), Eq(a, 0)), ((x - log(tanh(c + d*x) + 1)/d)/a, Eq(
b, 0)), (x*tanh(c)/(a + b*sech(c)**2), Eq(d, 0)), (x/a + log(-sqrt(-a/b) +
 sech(c + d*x))/(2*a*d) + log(sqrt(-a/b) + sech(c + d*x))/(2*a*d) - log(ta
nh(c + d*x) + 1)/(a*d), True))
```

3.142.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(21) = 42$.

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\tanh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{dx+c}{ad}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,ad}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c
) + a)/(a*d)
```

3.142.8 Giac [F]

$$\int \frac{\tanh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\tanh\left(dx+c\right)}{b\,\text{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\tanh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\ln\left(a+2\,a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+4\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)-2\,d\,x}{2\,a\,d}$$

input
```
int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

output
```
(log(a + 2*a*exp(2*c)*exp(2*d*x) + a*exp(4*c)*exp(4*d*x) + 4*b*exp(2*c)*ex
p(2*d*x)) - 2*d*x)/(2*a*d)
```

3.143 $\int \frac{1}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

3.143.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{a+b\,\mathrm{sech}^2(c+dx)}\,dx=\frac{x}{a}-\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}$$

output
```
x/a-arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a/d/(a+b)^(1/2)
```

3.143.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(46) = 92.

Time = 1.50 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.74

$$\int \frac{1}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$
$$=\frac{(a+2b+a\cosh(2(c+dx)))\mathrm{sech}^2(c+dx)\left(\sqrt{a+b}\,dx\sqrt{b(\cosh(c)-\sinh(c))^4}+b\,\mathrm{arctanh}\left(\frac{\mathrm{sech}(dx)(\cosh(2c)}{2\sqrt{}}\right.\right.}{2a\sqrt{a+b}\,d\left(a+b\,\mathrm{sech}^2(c+dx)\right)\sqrt{b(\cosh(c)-\sinh}}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^(-1),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(
Cosh[c] - Sinh[c])^4] + b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a +
 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(-Cosh[2*c] + Sinh[2*c])))/(2*a*Sqrt[a + b]*d*(a + b*Sech[c + d
*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

3.143.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4615, 3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{sech}^2(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+b\sec(ic+idx)^2}dx$$
$$\Big\downarrow \text{4615}$$
$$\frac{x}{a}-\frac{b\int \frac{1}{a\cosh^2(c+dx)+b}dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{x}{a}-\frac{b\int \frac{1}{a\sin\left(ic+idx+\frac{\pi}{2}\right)^2+b}dx}{a}$$
$$\Big\downarrow \text{3660}$$
$$\frac{x}{a}-\frac{b\int \frac{1}{b-(a+b)\coth^2(c+dx)}d\coth(c+dx)}{ad}$$
$$\Big\downarrow \text{221}$$
$$\frac{x}{a}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{a+b}\coth(c+dx)}{\sqrt{b}}\right)}{ad\sqrt{a+b}}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^(-1),x]
```

output
```
x/a - (Sqrt[b]*ArcTanh[(Sqrt[a + b]*Coth[c + d*x])/Sqrt[b]])/(a*Sqrt[a + b
]*d)
```

3.143.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 4615
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Simp[b/a   Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f},
x] && NeQ[a + b, 0]
```

3.143.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(38) = 76$.

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.35

method	result
risch	$\frac{x}{a}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}+a+2b}{a}\right)}{2(a+b)da}-\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+b)b}-a-2b}{a}\right)}{2(a+b)da}$
derivativedivides	$\frac{\frac{2b\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}\right)}{a}-\ln\left(\tanh\left(\right.\right.}{d}$
default	$\frac{\frac{2b\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}\right)}{a}-\ln\left(\tanh\left(\right.\right.}{d}$

input
```
int(1/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
x/a+1/2*((a+b)*b)^(1/2)/(a+b)/d/a*ln(exp(2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2
*b)/a)-1/2*((a+b)*b)^(1/2)/(a+b)/d/a*ln(exp(2*d*x+2*c)-(2*((a+b)*b)^(1/2)-
a-2*b)/a)
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(38) = 76$.

Time = 0.30 (sec) , antiderivative size = 436, normalized size of antiderivative = 9.48

$$\int \frac{1}{a + b\text{sech}^2(c + dx)}\,dx$$
$$= \left[\frac{2\,dx + \sqrt{\frac{b}{a+b}}\log\left(\frac{a^2\cosh(dx+c)^4 + 4\,a^2\cosh(dx+c)\sinh(dx+c)^3 + a^2\sinh(dx+c)^4 + 2\,(a^2+2\,ab)\cosh(dx+c)^2 + 2\,\left(3\,a^2\cosh(dx+c)^2 + a^2 \cdots\right.}{a\cosh(dx+c)^4 + 4\,a\cosh(dx+c)\sinh(dx+c)^3 + a\,\text{si}\cdots}\right.}{} \right.$$

input
```
integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*d*x + sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x +
c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2
 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b +
 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x +
c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x
 + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b))
)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c)
 + a)))/(a*d), (d*x - sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a
*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b
))/b))/(a*d)]
```

3.143.6 Sympy [F]

$$\int \frac{1}{a+b\mathrm{sech}^2(c+dx)}\,dx = \int \frac{1}{a+b\,\mathrm{sech}^2\left(c+dx\right)}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(1/(a + b*sech(c + d*x)**2), x)
```

3.143.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \frac{1}{a+b\mathrm{sech}^2(c+dx)}\,dx = \frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}ad} + \frac{dx+c}{ad}$$

input
```
integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + (d*x + c)/(
a*d)
```

3.143.8 Giac [F]

$$\int \frac{1}{a+b\mathrm{sech}^2(c+dx)}\,dx = \int \frac{1}{b\,\mathrm{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.143.9 Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 470, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\mathrm{sech}^2(c+dx)}\,dx = \frac{x}{a} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\left(a^5\sqrt{-a^3\,d^2-b\,a^2\,d^2}+a^4\,b\sqrt{-a^3\,d^2-b\,a^2\,d^2}\right)\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(\frac{2\left(a^2+8\,a\,b+8\,b^2\right)\left(8\,b^{5/2}\sqrt{-a^3\,d^2-b\,a^2\,d^2}+8\,a\,b^{3/2}\sqrt{-a^3\,d^2-b\,a^2\,d^2}+a\right.}{a^8\,d\,(a+b)^2\sqrt{-a^3\,d^2-b\,a^2\,d^2}}\right.\right.}{}\right.}{}$$

input

```
int(1/(a + b/cosh(c + d*x)^2),x)
```

output

```
x/a + (b^(1/2)*atan(((a^5*(- a^3*d^2 - a^2*b*d^2)^(1/2) + a^4*b*(- a^3*d^2
 - a^2*b*d^2)^(1/2))*(exp(2*c)*exp(2*d*x)*((2*(8*a*b + a^2 + 8*b^2)*(8*b^(
5/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2) + 8*a*b^(3/2)*(- a^3*d^2 - a^2*b*d^2)^(
1/2) + a^2*b^(1/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2)))/(a^8*d*(a + b)^2*(- a^3
*d^2 - a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(12*a^2*b^2*d + 8*a*b^3*
d + 4*a^3*b*d))/(a^7*(a + b)*(- a^3*d^2 - a^2*b*d^2)^(1/2)*(-a^2*d^2*(a +
b))^(1/2))) + (2*(2*a*b^(3/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2) + a^2*b^(1/2)*
(- a^3*d^2 - a^2*b*d^2)^(1/2))*(8*a*b + a^2 + 8*b^2))/(a^8*d*(a + b)^2*(-
a^3*d^2 - a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a^2*b^2*d + 2*a^3*b*d)*(2*a +
4*b))/(a^7*(a + b)*(- a^3*d^2 - a^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2))
))/(4*b)))/(- a^3*d^2 - a^2*b*d^2)^(1/2)
```

3.144 $\int \frac{\coth(c+dx)}{a+b\text{sech}^2(c+dx)} dx$

3.144.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\coth(c+dx)}{a+b\text{sech}^2(c+dx)} dx = \frac{b\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)d} + \frac{\log(\sinh(c+dx))}{(a+b)d}$$

output
```
1/2*b*ln(b+a*cosh(d*x+c)^2)/a/(a+b)/d+ln(sinh(d*x+c))/(a+b)/d
```

3.144.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\coth(c+dx)}{a+b\text{sech}^2(c+dx)} dx = \frac{2a\log(\sinh(c+dx)) + b\log\left(a+b+a\sinh^2(c+dx)\right)}{2a^2d+2abd}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
(2*a*Log[Sinh[c + d*x]] + b*Log[a + b + a*Sinh[c + d*x]^2])/(2*a^2*d + 2*a
*b*d)
```

3.144.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i}{\tan(ic+idx)\left(a+b\sec(ic+idx)^2\right)}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)\tan(ic+idx)}dx$$
$$\Big\downarrow \text{4626}$$
$$-\frac{\int \frac{\cosh^3(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}d\cosh(c+dx)}{d}$$
$$\Big\downarrow \text{354}$$
$$-\frac{\int \frac{\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)}d\cosh^2(c+dx)}{2d}$$
$$\Big\downarrow \text{86}$$
$$-\frac{\int \left(\frac{1}{(-a-b)\left(\cosh^2(c+dx)-1\right)}-\frac{b}{(a+b)\left(a\cosh^2(c+dx)+b\right)}\right)d\cosh^2(c+dx)}{2d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{-\frac{\log\left(1-\cosh^2(c+dx)\right)}{a+b}-\frac{b\log\left(a\cosh^2(c+dx)+b\right)}{a(a+b)}}{2d}$$

input
```
Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/2*(-(Log[1 - Cosh[c + d*x]^2]/(a + b)) - (b*Log[b + a*Cosh[c + d*x]^2])
/(a*(a + b)))/d
```

3.144.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.144.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(44) = 88.

Time = 1.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.59

method	result
risch	$\frac{x}{a}-\frac{2x}{a+b}-\frac{2c}{d(a+b)}-\frac{2bx}{a(a+b)}-\frac{2bc}{da(a+b)}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{d(a+b)}+\frac{b\ln\left(\mathrm{e}^{4dx+4c}+\frac{2(a+2b)\mathrm{e}^{2dx+2c}}{a}+1\right)}{2da(a+b)}$
derivativedivides	$\frac{-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-}{2a(a+b)}}{d}$
default	$\frac{-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-}{2a(a+b)}}{d}$

input
```
int(coth(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
x/a-2*x/(a+b)-2/d/(a+b)*c-2*b/a/(a+b)*x-2*b/d/a/(a+b)*c+1/d/(a+b)*ln(exp(2
*d*x+2*c)-1)+1/2*b/d/a/(a+b)*ln(exp(4*d*x+4*c)+2*(a+2*b)/a*exp(2*d*x+2*c)+
1)
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(44) = 88.

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

$$\int \frac{\coth(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{2\,(a+b)dx - b\log\left(\frac{2\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2\,b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right) - 2\,a\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,(a^2+ab)d}$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*(a + b)*d*x - b*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a +
 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2))
 - 2*a*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2 + a*b)*
d)
```

3.144.6 Sympy [F]

$$\int \frac{\coth(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\coth(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.144.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time $= 0.20$ (sec) , antiderivative size $= 100$, normalized size of antiderivative $= 2.17$

$$\int \frac{\coth(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,(a^2+ab)d} + \frac{dx+c}{ad} + \frac{\log\left(e^{(-dx-c)}+1\right)}{(a+b)d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{(a+b)d}$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^2 + a
*b)*d) + (d*x + c)/(a*d) + log(e^(-d*x - c) + 1)/((a + b)*d) + log(e^(-d*x
 - c) - 1)/((a + b)*d)
```

3.144.8 Giac [F]

$$\int \frac{\coth(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\coth(dx+c)}{b\,\text{sech}(dx+c)^2+a}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.144.9 Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.96

$$\int \frac{\coth(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$= \frac{\ln\left(8\,a\,b^5-b^6-24\,a^2\,b^4+32\,a^3\,b^3-16\,a^4\,b^2+b^6\,e^{2\,c}\,e^{2\,d\,x}-8\,a\,b^5\,e^{2\,c}\,e^{2\,d\,x}+24\,a^2\,b^4\,e^{2\,c}\,e^{2\,d\,x}-32\,a^3\,b^3\,e^{2\,c}\right.}{a\,d+b\,d}$$
$$-\frac{x}{a}$$
$$+\frac{b\,\ln\left(2\,a^2-a\,b+4\,a^2\,e^{2\,c}\,e^{2\,d\,x}+2\,a^2\,e^{4\,c}\,e^{4\,d\,x}-4\,b^2\,e^{2\,c}\,e^{2\,d\,x}+6\,a\,b\,e^{2\,c}\,e^{2\,d\,x}-a\,b\,e^{4\,c}\,e^{4\,d\,x}\right)}{2\,d\,a^2+2\,b\,d\,a}$$

input
```
int(coth(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

output
```
log(8*a*b^5 - b^6 - 24*a^2*b^4 + 32*a^3*b^3 - 16*a^4*b^2 + b^6*exp(2*c)*ex
p(2*d*x) - 8*a*b^5*exp(2*c)*exp(2*d*x) + 24*a^2*b^4*exp(2*c)*exp(2*d*x) -
32*a^3*b^3*exp(2*c)*exp(2*d*x) + 16*a^4*b^2*exp(2*c)*exp(2*d*x))/(a*d + b*
d) - x/a + (b*log(2*a^2 - a*b + 4*a^2*exp(2*c)*exp(2*d*x) + 2*a^2*exp(4*c)
*exp(4*d*x) - 4*b^2*exp(2*c)*exp(2*d*x) + 6*a*b*exp(2*c)*exp(2*d*x) - a*b*
exp(4*c)*exp(4*d*x)))/(2*a^2*d + 2*a*b*d)
```

3.145 $\int \frac{\coth^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

3.145.1 Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{\coth^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx=\frac{x}{a}-\frac{b^{3/2}\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}d}-\frac{\coth(c+dx)}{(a+b)d}$$

output
```
x/a-b^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/(a+b)^(3/2)/d-coth(
d*x+c)/(a+b)/d
```

3.145.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(62) = 124.

Time = 2.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.11

$$\int \frac{\coth^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$
$$=\frac{(a+2b+a\cosh(2(c+dx)))\mathrm{sech}^2(c+dx)\left(b^2\mathrm{arctanh}\left(\frac{\mathrm{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)\right.(}{2a(a+b)^{3/2}d\left(a+b\,\mathrm{sech}^2(c+\right.}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b^2*ArcTanh[(Sech[d*x]*(
Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a +
 b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*((a + b)*d*x + a*Csch[c]*Csch[c + d*x]*S
inh[d*x])))/(2*a*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] -
 Sinh[c])^4])
```

3.145.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 4629, 25, 2075, 382, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(a+b\left(1-\tanh^2(c+dx)\right)\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(a+b\left(1-\tanh^2(c+dx)\right)\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{382}$$

$$-\frac{\frac{\coth(c+dx)}{a+b}-\frac{\int \frac{-b\tanh^2(c+dx)+a+2b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}}{d}$$

$$\Big\downarrow \text{ 397}$$

$$-\frac{\frac{\coth(c+dx)}{a+b}-\frac{\frac{(a+b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b^2\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{\coth(c+dx)}{a+b}-\frac{\frac{(a+b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^2\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{\coth(c+dx)}{a+b}-\frac{\frac{(a+b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}}{d}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

output
```
-((-((((a + b)*ArcTanh[Tanh[c + d*x]])/a - (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[
c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) + Coth[c + d*x]/(a + b))
/d)
```

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.145. $\int \frac{\coth^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.145.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(54) = 108.

Time = 1.79 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.15

method	result
risch	$\frac{x}{a}-\frac{2}{d(a+b)(\mathrm{e}^{2dx+2c}-1)}+\frac{\sqrt{(a+b)b}\,b\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{(a+b)b}+a+2b}{a}\right)}{2(a+b)^2da}-\frac{\sqrt{(a+b)b}\,b\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{(a+b)b}-a-2b}{a}\right)}{2(a+b)^2da}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}-\frac{1}{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^2\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4\sqrt{}}\right.}{a(a+b)}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}-\frac{1}{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^2\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4\sqrt{}}\right.}{a(a+b)}}{d}$

input
```
int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
x/a-2/d/(a+b)/(exp(2*d*x+2*c)-1)+1/2*((a+b)*b)^(1/2)/(a+b)^2*b/d/a*ln(exp(
2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/a)-1/2*((a+b)*b)^(1/2)/(a+b)^2*b/d/a*
ln(exp(2*d*x+2*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a)
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(54) = 108.

Time = 0.31 (sec) , antiderivative size = 749, normalized size of antiderivative = 12.08

$$\int \frac{\coth^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$
$$=\left[\frac{2\,(a+b)dx\cosh\,(dx+c)^2+4\,(a+b)dx\cosh\,(dx+c)\sinh\,(dx+c)+2\,(a+b)dx\sinh\,(dx+c)^2-2\,(a+}{}\right.$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*(a + b)*d*x*cosh(d*x + c)^2 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
 + c) + 2*(a + b)*d*x*sinh(d*x + c)^2 - 2*(a + b)*d*x + (b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/(a + b)
)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sin
h(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2
+ a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c
)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x
 + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x
 + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*c
osh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x +
c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x +
 c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*a)/((a^2 + a*b)*d
*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*
b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d), ((a + b)*d*x*cosh(d*x + c)^2 + 2*(a
 + b)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a + b)*d*x*sinh(d*x + c)^2 - (a +
 b)*d*x - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 - b)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*
x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) -
2*a)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d
*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d)]
```

3.145.6 Sympy [F]

$$\int \frac{\coth^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\int \frac{\coth^2{(c+dx)}}{a+b\,\text{sech}^2{(c+dx)}}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.145.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(54) = 108.

Time = 0.30 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.92

$$\int \frac{\coth^2(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,(a^2+ab)d} - \frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,(a^2+ab)d} - \frac{(ab+2\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,(a^2+ab)\sqrt{(a+b)b}d} + \frac{(ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,(a^2+ab)\sqrt{(a+b)b}d} - \frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}(a+b)d} + \frac{\log\left(e^{(2\,dx+2\,c)}-1\right)}{2\,(a+b)d} - \frac{\log\left(e^{(-2\,dx-2\,c)}-1\right)}{2\,(a+b)d} - \frac{1}{2\,((a+b)e^{(2\,dx+2\,c)}-a-b)d} + \frac{3}{2\,((a+b)e^{(-2\,dx-2\,c)}-a-b)d}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^2 + a*b
)*d) - 1/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((
a^2 + a*b)*d) - 1/8*(a*b + 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqr
t((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + a
*b)*sqrt((a + b)*b)*d) + 1/8*(a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2
*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b))
)/((a^2 + a*b)*sqrt((a + b)*b)*d) - 1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*
b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))
/(sqrt((a + b)*b)*(a + b)*d) + 1/2*log(e^(2*d*x + 2*c) - 1)/((a + b)*d) -
1/2*log(e^(-2*d*x - 2*c) - 1)/((a + b)*d) - 1/2/(((a + b)*e^(2*d*x + 2*c)
- a - b)*d) + 3/2/(((a + b)*e^(-2*d*x - 2*c) - a - b)*d)
```

3.145.8 Giac [F]

$$\int \frac{\coth^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\int \frac{\coth\left(dx+c\right)^2}{b\,\text{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.145.9 Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 977, normalized size of antiderivative = 15.76

$$\int \frac{\coth^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx=\frac{x}{a}-\frac{2}{\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,\left(a\,d+b\,d\right)} + \frac{\mathrm{atan}\left(\frac{\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(\frac{8\,\left(a+2\,b\right)\,\left(4\,d\,a^4\,b^2+16\,d\,a^3\,b^3+20\,d\,a^2\,b^4+8\,d\,a\,b^5\right)}{a^6\,\left(a+b\right)\,\left(a^3+2\,a^2\,b+a\,b^2\right)\,\sqrt{-a^2\,d^2\,{\left(a+b\right)}^3}\,\sqrt{-a^5\,d^2-3\,a^4\,b\,d^2-3\,a^3\,b^2\,d^2-a^2\,b^3\,d^2}}+\frac{2\,\sqrt{b^3}\,\left(a^2+8\,a\,b+8\,b^2\right)\,\left(a^2\,\sqrt{b^3}\,\sqrt{-a^5\,d^2-3}\right.}{}\right.\right.}{}\right)}{}$$

input
```
int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

output
```
x/a - 2/((exp(2*c + 2*d*x) - 1)*(a*d + b*d)) + (atan(((exp(2*c)*exp(2*d*x)
*((8*(a + 2*b)*(20*a^2*b^4*d + 16*a^3*b^3*d + 4*a^4*b^2*d + 8*a*b^5*d))/(a
^6*(a + b)*(a*b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^3)^(1/2)*(- a^5*d^2 -
 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)) + (2*(b^3)^(1/2)*(8*a*b
 + a^2 + 8*b^2)*(a^2*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 -
3*a^3*b^2*d^2)^(1/2) + 8*b^2*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^
3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 8*a*b*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2
- a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))/(a^7*b^2*d*(a + b)^3*(a*b^2 + 2*a^2
*b + a^3)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))
+ (8*(a + 2*b)*(2*a^2*b^4*d + 4*a^3*b^3*d + 2*a^4*b^2*d))/(a^6*(a + b)*(a*
b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 -
 a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)) + (2*(a^2*(b^3)^(1/2)*(- a^5*d^2 - 3*
a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 2*a*b*(b^3)^(1/2)*(- a^5*
d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2))*(b^3)^(1/2)*(8*a*b
 + a^2 + 8*b^2))/(a^7*b^2*d*(a + b)^3*(a*b^2 + 2*a^2*b + a^3)*(- a^5*d^2 -
 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))*(a^7*(- a^5*d^2 - 3*a^
4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + a^4*b^3*(- a^5*d^2 - 3*a^4*
b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 3*a^5*b^2*(- a^5*d^2 - 3*a^4*
b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 3*a^6*b*(- a^5*d^2 - 3*a^4*b*
d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))/(4*(b^3)^(1/2)))*(b^3)^(1/2)...
```

3.145. $\int \frac{\coth^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

3.146 $\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.146.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{\text{csch}^2(c+dx)}{2(a+b)d}+\frac{b^2\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)^2d}+\frac{(a+2b)\log(\sinh(c+dx))}{(a+b)^2d}$$

output
```
-1/2*csch(d*x+c)^2/(a+b)/d+1/2*b^2*ln(b+a*cosh(d*x+c)^2)/a/(a+b)^2/d+(a+2*
b)*ln(sinh(d*x+c))/(a+b)^2/d
```

3.146.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{(a+2b+a\cosh(2(c+dx)))\left(a(a+b)\text{csch}^2(c+dx)-2a(a+2b)\log(\sinh(c+dx))-b^2\log\left(a+b+a\right.\right.}{4a(a+b)^2d\left(a+b\text{sech}^2(c+dx)\right)}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
-1/4*((a + 2*b + a*Cosh[2*(c + d*x)])*(a*(a + b)*Csch[c + d*x]^2 - 2*a*(a
+ 2*b)*Log[Sinh[c + d*x]] - b^2*Log[a + b + a*Sinh[c + d*x]^2])*Sech[c + d
*x]^2)/(a*(a + b)^2*d*(a + b*Sech[c + d*x]^2))
```

3.146.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{i}{\tan(ic+idx)^3\,(a+b\sec(ic+idx)^2)}dx$$
$$\Big\downarrow \; 26$$
$$-i\int \frac{1}{(b\sec(ic+idx)^2+a)\tan(ic+idx)^3}dx$$
$$\Big\downarrow \; 4626$$
$$\frac{\int \frac{\cosh^5(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2(a\cosh^2(c+dx)+b)}d\cosh(c+dx)}{d}$$
$$\Big\downarrow \; 354$$
$$\frac{\int \frac{\cosh^4(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2(a\cosh^2(c+dx)+b)}d\cosh^2(c+dx)}{2d}$$
$$\Big\downarrow \; 99$$
$$\frac{\int \left(\frac{b^2}{(a+b)^2(a\cosh^2(c+dx)+b)}+\frac{a+2b}{(a+b)^2(\cosh^2(c+dx)-1)}+\frac{1}{(a+b)\left(\cosh^2(c+dx)-1\right)^2}\right)d\cosh^2(c+dx)}{2d}$$
$$\Big\downarrow \; 2009$$
$$\frac{\frac{b^2\log(a\cosh^2(c+dx)+b)}{a(a+b)^2}+\frac{1}{(a+b)(1-\cosh^2(c+dx))}+\frac{(a+2b)\log(1-\cosh^2(c+dx))}{(a+b)^2}}{2d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

output
```
(1/((a + b)*(1 - Cosh[c + d*x]^2)) + ((a + 2*b)*Log[1 - Cosh[c + d*x]^2])/
(a + b)^2 + (b^2*Log[b + a*Cosh[c + d*x]^2])/(a*(a + b)^2))/(2*d)
```

3.146.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.146.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(69) = 138.

Time = 2.88 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.34

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8(a+b)}-\frac{1}{8(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(4a+8b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}+\frac{b^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{2a(a+b)^2}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8(a+b)}-\frac{1}{8(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(4a+8b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}+\frac{b^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{2a(a+b)^2}}{d}$
risch	$\frac{x}{a}-\frac{2ax}{a^2+2ab+b^2}-\frac{2ac}{d(a^2+2ab+b^2)}-\frac{4bx}{a^2+2ab+b^2}-\frac{4bc}{d(a^2+2ab+b^2)}-\frac{2b^2x}{a(a^2+2ab+b^2)}-\frac{2b^2c}{da(a^2+2ab+b^2)}-$

input
```
int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*tanh(1/2*d*x+1/2*c)^2/(a+b)-1/8/(a+b)/tanh(1/2*d*x+1/2*c)^2+1/4/
(a+b)^2*(4*a+8*b)*ln(tanh(1/2*d*x+1/2*c))+1/2*b^2/a/(a+b)^2*ln(tanh(1/2*d*
x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)-1/a*ln(1+tanh(1/2*d*x+1/2*c))-1/a*ln(tanh(1/2*d*x+1/2*
c)-1))
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. 2(69) = 138.

Time = 0.32 (sec) , antiderivative size = 862, normalized size of antiderivative = 11.81

$$\int \frac{\coth^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^
4 + 2*(a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*co
sh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*
b + b^2)*d*x + a^2 + a*b)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*c
osh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2
 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x
 + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*s
inh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)) - 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2
*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*
sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 +
2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))) + 8*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a
*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 2*a^2*b +
 a*b^2)*d*cosh(d*x + c)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh
(d*x + c)^3 + (a^3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)^4 - 2*(a^3 + 2*a^2*b
 + a*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)
^2 - (a^3 + 2*a^2*b + a*b^2)*d)*sinh(d*x + c)^2 + (a^3 + 2*a^2*b + a*b^...
```

3.146.6 Sympy [F]

$$\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\coth^3(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(69) = 138$.

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.56

$$\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{b^2\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,(a^3+2\,a^2b+ab^2)d} + \frac{(a+2\,b)\log\left(e^{(-dx-c)}+1\right)}{(a^2+2\,ab+b^2)d} + \frac{(a+2\,b)\log\left(e^{(-dx-c)}-1\right)}{(a^2+2\,ab+b^2)d} + \frac{dx+c}{ad} + \frac{2\,e^{(-2\,dx-2\,c)}}{(2\,(a+b)e^{(-2\,dx-2\,c)}-(a+b)e^{(-4\,dx-4\,c)}-a-b)d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b^2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 +
 2*a^2*b + a*b^2)*d) + (a + 2*b)*log(e^(-d*x - c) + 1)/((a^2 + 2*a*b + b^2
)*d) + (a + 2*b)*log(e^(-d*x - c) - 1)/((a^2 + 2*a*b + b^2)*d) + (d*x + c)
/(a*d) + 2*e^(-2*d*x - 2*c)/((2*(a + b)*e^(-2*d*x - 2*c) - (a + b)*e^(-4*d
*x - 4*c) - a - b)*d)
```

3.146.8 Giac [F]

$$\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\coth\left(dx+c\right)^3}{b\,\text{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.146.9 Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 523, normalized size of antiderivative = 7.16

$$\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$$
$$= \frac{\ln\left(23\,a\,b^7+8\,a^7\,b-2\,b^8-72\,a^2\,b^6-10\,a^3\,b^5+184\,a^4\,b^4+180\,a^5\,b^3+64\,a^6\,b^2+2\,b^8\,e^{2\,c}\,e^{2\,d\,x}-23\,a\,b^7\,e^{2\,c}\right.}{}$$
$$-\frac{x}{a}-\frac{2}{(a\,d+b\,d)\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$
$$+\frac{b^2\,\ln\left(a\,b^4+16\,a^4\,b+4\,a^5-8\,a^2\,b^3+12\,a^3\,b^2+8\,a^5\,e^{2\,c}\,e^{2\,d\,x}+4\,a^5\,e^{4\,c}\,e^{4\,d\,x}+4\,b^5\,e^{2\,c}\,e^{2\,d\,x}-30\,a\,b^4\,e^{2\,c}\right.}{}$$
$$-\frac{2\,(a^2+b\,a)}{a\,\left(e^{2\,c+2\,d\,x}-1\right)\,(a+b)\,(a\,d+b\,d)}$$

input
```
int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

output
```
(log(23*a*b^7 + 8*a^7*b - 2*b^8 - 72*a^2*b^6 - 10*a^3*b^5 + 184*a^4*b^4 +
180*a^5*b^3 + 64*a^6*b^2 + 2*b^8*exp(2*c)*exp(2*d*x) - 23*a*b^7*exp(2*c)*e
xp(2*d*x) - 8*a^7*b*exp(2*c)*exp(2*d*x) + 72*a^2*b^6*exp(2*c)*exp(2*d*x) +
 10*a^3*b^5*exp(2*c)*exp(2*d*x) - 184*a^4*b^4*exp(2*c)*exp(2*d*x) - 180*a^
5*b^3*exp(2*c)*exp(2*d*x) - 64*a^6*b^2*exp(2*c)*exp(2*d*x))*(a + 2*b))/(a^
2*d + b^2*d + 2*a*b*d) - x/a - 2/((a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*
c + 2*d*x) + 1)) + (b^2*log(a*b^4 + 16*a^4*b + 4*a^5 - 8*a^2*b^3 + 12*a^3*
b^2 + 8*a^5*exp(2*c)*exp(2*d*x) + 4*a^5*exp(4*c)*exp(4*d*x) + 4*b^5*exp(2*
c)*exp(2*d*x) - 30*a*b^4*exp(2*c)*exp(2*d*x) + 48*a^4*b*exp(2*c)*exp(2*d*x
) + a*b^4*exp(4*c)*exp(4*d*x) + 16*a^4*b*exp(4*c)*exp(4*d*x) + 32*a^2*b^3*
exp(2*c)*exp(2*d*x) + 88*a^3*b^2*exp(2*c)*exp(2*d*x) - 8*a^2*b^3*exp(4*c)*
exp(4*d*x) + 12*a^3*b^2*exp(4*c)*exp(4*d*x)))/(2*a^3*d + 2*a*b^2*d + 4*a^2
*b*d) - (2*(a*b + a^2))/(a*(exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d))
```

3.147 $\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.147.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{x}{a} - \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}d} - \frac{(a+2b)\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d}$$

output
```
x/a-b^(5/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/(a+b)^(5/2)/d-(a+2*
b)*coth(d*x+c)/(a+b)^2/d-1/3*coth(d*x+c)^3/(a+b)/d
```

3.147.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 380 vs. $2(87) = 174$.

Time = 4.37 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.37

$$\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(3b^3\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)\right.}{}$$

input
```
Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*b^3*ArcTanh[(Sech[d*x]
*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqr
t[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + (Sqrt[
a + b]*Csch[c]*Csch[c + d*x]^3*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(9*(a + b)^2*
d*x*Cosh[d*x] - 9*(a + b)^2*d*x*Cosh[2*c + d*x] - 3*a^2*d*x*Cosh[2*c + 3*d
*x] - 6*a*b*d*x*Cosh[2*c + 3*d*x] - 3*b^2*d*x*Cosh[2*c + 3*d*x] + 3*a^2*d*
x*Cosh[4*c + 3*d*x] + 6*a*b*d*x*Cosh[4*c + 3*d*x] + 3*b^2*d*x*Cosh[4*c + 3
*d*x] - 12*a^2*Sinh[d*x] - 24*a*b*Sinh[d*x] - 12*a^2*Sinh[2*c + d*x] - 18*
a*b*Sinh[2*c + d*x] + 8*a^2*Sinh[2*c + 3*d*x] + 14*a*b*Sinh[2*c + 3*d*x]))
/8))/(6*a*(a + b)^(5/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c
])^4])
```

3.147.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4629, 2075, 382, 27, 445, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\coth^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int \frac{3\coth^2(c+dx)\left(-b\tanh^2(c+dx)+a+2b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{3(a+b)}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\coth^2(c+dx)\left(-b\tanh^2(c+dx)+a+2b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

$\Big\downarrow$ 445

$$\frac{\frac{-\frac{\int -\frac{a^2+3ba+3b^2-b(a+2b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{(a+2b)\coth(c+dx)}{a+b}}{a+b}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{a^2+3ba+3b^2-b(a+2b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{(a+2b)\coth(c+dx)}{a+b}}{a+b}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{(a+b)^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b^3\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{(a+2b)\coth(c+dx)}{a+b}}{a+b}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\frac{(a+b)^2\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^3\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{(a+2b)\coth(c+dx)}{a+b}}{a+b}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{\frac{(a+b)^2\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{(a+2b)\coth(c+dx)}{a+b}}{a+b}-\frac{\coth^3(c+dx)}{3(a+b)}}{d}$$

input
```
Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

output
```
(-1/3*Coth[c + d*x]^3/(a + b) + ((((a + b)^2*ArcTanh[Tanh[c + d*x]])/a - (
b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a
+ b) - ((a + 2*b)*Coth[c + d*x])/(a + b))/(a + b))/d
```

3.147. $\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

3.147.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.147. $\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.147.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(77) = 154$.

Time = 4.69 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.21

method	result
risch	$\frac{x}{a}-\frac{2\left(6a\,{\mathrm e}^{4dx+4c}+9b\,{\mathrm e}^{4dx+4c}-6\,{\mathrm e}^{2dx+2c}a-12b\,{\mathrm e}^{2dx+2c}+4a+7b\right)}{3d\left(a+b\right)^{2}\left({\mathrm e}^{2dx+2c}-1\right)^{3}}+\frac{\sqrt{\left(a+b\right)b}\,b^{2}\ln\left({\mathrm e}^{2dx+2c}+\frac{2\sqrt{\left(a+b\right)b}+a+2b}{a}\right)}{2\left(a+b\right)^{3}da}-$
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b}{3}+5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+9b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a+b\right)^{2}}-\frac{1}{24\left(a+b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{5a+9b}{8\left(a+b\right)^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln(}{}$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}b}{3}+5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+9b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a+b\right)^{2}}-\frac{1}{24\left(a+b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{5a+9b}{8\left(a+b\right)^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln(}{}$

input
```
int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
x/a-2/3*(6*a*exp(4*d*x+4*c)+9*b*exp(4*d*x+4*c)-6*exp(2*d*x+2*c)*a-12*b*exp
(2*d*x+2*c)+4*a+7*b)/d/(a+b)^2/(exp(2*d*x+2*c)-1)^3+1/2*((a+b)*b)^(1/2)/(a
+b)^3*b^2/d/a*ln(exp(2*d*x+2*c)+(2*((a+b)*b)^(1/2)+a+2*b)/a)-1/2*((a+b)*b)
^(1/2)/(a+b)^3*b^2/d/a*ln(exp(2*d*x+2*c)-(2*((a+b)*b)^(1/2)-a-2*b)/a)
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(77) = 154$.

Time = 0.31 (sec) , antiderivative size = 2705, normalized size of antiderivative = 31.09

$$\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(6*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 36*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^5 + 6*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^6 - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^4 + 6*(15
*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x - 4*a
^2 - 6*a*b)*sinh(d*x + c)^4 + 24*(5*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^
3 - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x +
c)^3 - 6*(a^2 + 2*a*b + b^2)*d*x + 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 +
8*a*b)*cosh(d*x + c)^2 + 6*(15*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 3
*(a^2 + 2*a*b + b^2)*d*x - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*c
osh(d*x + c)^2 + 4*a^2 + 8*a*b)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 +
 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*
x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*
x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3
 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2
 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c
))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*
x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x +
c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a
*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*
x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*s...
```

3.147.6 Sympy [F]

$$\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\coth^4{(c+dx)}}{a+b\operatorname{sech}^2{(c+dx)}}\,dx$$

input
```
integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.147.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(77) = 154$.

Time = 0.39 (sec) , antiderivative size = 1435, normalized size of antiderivative = 16.49

$$\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

output
```
3/16*a*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d
*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b
)*b)*d) + 1/8*(a*b + 2*b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x +
 2*c) + a)/((a^3 + 2*a^2*b + a*b^2)*d) - 1/4*b*log(a*e^(4*d*x + 4*c) + 2*(
a + 2*b)*e^(2*d*x + 2*c) + a)/((a^2 + 2*a*b + b^2)*d) - 1/8*(a*b + 2*b^2)*
log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 + 2*a^2*b
 + a*b^2)*d) + 1/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c)
 + a)/((a^2 + 2*a*b + b^2)*d) + 1/4*(2*a + 3*b)*log(e^(2*d*x + 2*c) - 1)/(
(a^2 + 2*a*b + b^2)*d) + 1/2*b*log(e^(2*d*x + 2*c) - 1)/((a^2 + 2*a*b + b^
2)*d) - 1/4*(2*a + 3*b)*log(e^(-2*d*x - 2*c) - 1)/((a^2 + 2*a*b + b^2)*d)
- 1/2*b*log(e^(-2*d*x - 2*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1/32*(a^2*b +
8*a*b^2 + 8*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*
e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 2*a^2*b + a*b^2)*s
qrt((a + b)*b)*d) + 1/8*(a*b + 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2
*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2
 + 2*a*b + b^2)*sqrt((a + b)*b)*d) + 1/32*(a^2*b + 8*a*b^2 + 8*b^3)*log((a
*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a +
 2*b + 2*sqrt((a + b)*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*b)*d) - 1
/8*(a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a
*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*...
```

3.147.8 Giac [F]

$$\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \int \frac{\coth\left(dx+c\right)^4}{b\,\text{sech}\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.147.9 Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 779, normalized size of antiderivative = 8.95

$$\int \frac{\coth^4(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx = \frac{x}{a} - \frac{8}{3\,(a\,d+b\,d)\,(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1)}$$

$$-\frac{\mathrm{atan}\Bigg(\bigg(\Big(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\Big(\frac{4\,b^3}{a^3\,d\,(a+b)^2\,\sqrt{b^5}\,(a^3+2\,a^2\,b+a\,b^2)}+\frac{(a+2\,b)\,\left(a^4\,d\,\sqrt{b^5}+2\,a\,b^3\,d\,\sqrt{b^5}+4\,a^3\,b\,d\,\sqrt{b^5}+5\,a^2\,b^2\,d\,\sqrt{\cdots}\right.}{a^2\,b^3\,(a^3+2\,a^2\,b+a\,b^2)\,\sqrt{-a^2\,d^2\,(a+b)^5}\,\sqrt{-a^7\,d^2-5\,a^6\,b\,d^2-10\,a^5\,b^2\,d^2-10\,a^4\cdots}}}{}$$

$$-\frac{4\,(a^2+b\,a)}{a\,(a+b)\,(a\,d+b\,d)\,(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1)} - \frac{2\,(2\,a^2+3\,b\,a)}{a\,(\mathrm{e}^{2\,c+2\,d\,x}-1)\,(a+b)\,(a\,d+b\,d)}$$

input
```
int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

output
```
x/a - 8/(3*(a*d + b*d)*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
+ 6*d*x) - 1)) - (atan((exp(2*c)*exp(2*d*x)*((4*b^3)/(a^3*d*(a + b)^2*(b^5
)^(1/2)*(a*b^2 + 2*a^2*b + a^3)) + ((a + 2*b)*(a^4*d*(b^5)^(1/2) + 2*a*b^3
*d*(b^5)^(1/2) + 4*a^3*b*d*(b^5)^(1/2) + 5*a^2*b^2*d*(b^5)^(1/2)))/(a^2*b^
3*(a*b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^5)^(1/2)*(- a^7*d^2 - 5*a^6*b*
d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2)
)) + ((a + 2*b)*(a^4*d*(b^5)^(1/2) + 2*a^3*b*d*(b^5)^(1/2) + a^2*b^2*d*(b^
5)^(1/2)))/(a^2*b^3*(a*b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^5)^(1/2)*(-
a^7*d^2 - 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*
a^5*b^2*d^2)^(1/2)))*((a^4*(- a^7*d^2 - 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*
b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2))/2 + (a^2*b^2*(- a^7*d^2
- 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*
d^2)^(1/2))/2 + a^3*b*(- a^7*d^2 - 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d
^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2))*(b^5)^(1/2))/(- a^7*d^2 - 5*
a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)
^(1/2) - (4*(a*b + a^2))/(a*(a + b)*(a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(
2*c + 2*d*x) + 1)) - (2*(3*a*b + 2*a^2))/(a*(exp(2*c + 2*d*x) - 1)*(a + b)
*(a*d + b*d))
```

3.148 $\int \frac{\tanh^5(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

3.148.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx=\frac{(a+b)^2}{2a^2bd\left(b+a\cosh^2(c+dx)\right)}+\frac{\log(\cosh(c+dx))}{b^2d}+\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cosh^2(c+dx)\right)}{2d}$$

output
```
1/2*(a+b)^2/a^2/b/d/(b+a*cosh(d*x+c)^2)+ln(cosh(d*x+c))/b^2/d+1/2*(1/a^2-1
/b^2)*ln(b+a*cosh(d*x+c)^2)/d
```

3.148.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx=\frac{(a+2b+a\cosh(2c+2dx))^2\left(\frac{(a+b)^2}{a^2b(b+a\cosh^2(c+dx))}+\frac{2\log(\cosh(c+dx))}{b^2}+\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cosh^2(c+dx)\right)\right)\text{se}}{8d\left(a+b\,\text{sech}^2(c+dx)\right)^2}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*c + 2*d*x])^2*((a + b)^2/(a^2*b*(b + a*Cosh[c + d*x]^
2)) + (2*Log[Cosh[c + d*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*
x]^2])*Sech[c + d*x]^4)/(8*d*(a + b*Sech[c + d*x]^2)^2)
```

3.148.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)^5}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)^5}{\left(b\sec(ic+idx)^2+a\right)^2}dx$$

$$\Big\downarrow \text{4626}$$

$$\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^2\text{sech}(c+dx)}{\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^2\text{sech}(c+dx)}{\left(a\cosh^2(c+dx)+b\right)^2}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-\frac{(a+b)^2}{ab\left(a\cosh^2(c+dx)+b\right)^2}+\frac{\text{sech}(c+dx)}{b^2}+\frac{b^2-a^2}{ab^2\left(a\cosh^2(c+dx)+b\right)}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cosh^2(c+dx)+b\right)+\frac{(a+b)^2}{a^2b\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(\cosh^2(c+dx)\right)}{b^2}}{2d}$$

---

input
```
Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + b)^2/(a^2*b*(b + a*Cosh[c + d*x]^2)) + Log[Cosh[c + d*x]^2]/b^2 + (a
^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]^2])/(2*d)
```

3.148.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.148.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(72) = 144$.

Time = 22.78 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{x}{a^2}-\frac{2c}{a^2 d}+\frac{2(a^2+2ab+b^2)e^{2dx+2c}}{a^2 bd(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a)}+\frac{\ln(e^{2dx+2c}+1)}{b^2 d}-\frac{\ln\left(e^{4dx+4c}+\frac{2(a+2b)e^{2dx+2c}}{a}+1\right.}{2d\,b^2}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}{b^2}-\frac{(a+b)\left(\frac{2ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\right.}\right.}{}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}{b^2}-\frac{(a+b)\left(\frac{2ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\right.}\right.}{}}{d}$

input
```
int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/a^2-2/a^2/d*c+2/a^2/b*(a^2+2*a*b+b^2)/d*exp(2*d*x+2*c)/(a*exp(4*d*x+4*c
)+2*exp(2*d*x+2*c)*a+4*b*exp(2*d*x+2*c)+a)+1/b^2/d*ln(exp(2*d*x+2*c)+1)-1/
2/d/b^2*ln(exp(4*d*x+4*c)+2*(a+2*b)/a*exp(2*d*x+2*c)+1)+1/2/d/a^2*ln(exp(4
*d*x+4*c)+2*(a+2*b)/a*exp(2*d*x+2*c)+1)
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(72) = 144$.

Time = 0.32 (sec) , antiderivative size = 853, normalized size of antiderivative = 11.22

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx =$$

$$-\frac{2\,ab^2dx\cosh{(dx+c)}^4+8\,ab^2dx\cosh{(dx+c)}\sinh{(dx+c)}^3+2\,ab^2dx\sinh{(dx+c)}^4+2\,ab^2dx-4\,(a^2}{}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b^2*d*x*cosh(d*x + c)^4 + 8*a*b^2*d*x*cosh(d*x + c)*sinh(d*x + c
)^3 + 2*a*b^2*d*x*sinh(d*x + c)^4 + 2*a*b^2*d*x - 4*(a^2*b + 2*a*b^2 + b^3
 - (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*a*b^2*d*x*cosh(d*x + c)^2 -
 a^2*b - 2*a*b^2 - b^3 + (a*b^2 + 2*b^3)*d*x)*sinh(d*x + c)^2 + ((a^3 - a*
b^2)*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
3 - a*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^
3)*cosh(d*x + c)^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^3 + 3*(a^3 - a*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 +
 2*a^2*b - a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x
+ c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*s
inh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x
 + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh
(d*x + c)^2 + 2*(3*a^3*cosh(d*x + c)^2 + a^3 + 2*a^2*b)*sinh(d*x + c)^2 +
4*(a^3*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*log
(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a*b^2*d*x*cosh(d*x
+ c)^3 - (a^2*b + 2*a*b^2 + b^3 - (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh
(d*x + c))/(a^3*b^2*d*cosh(d*x + c)^4 + 4*a^3*b^2*d*cosh(d*x + c)*sinh(d*x
 + c)^3 + a^3*b^2*d*sinh(d*x + c)^4 + a^3*b^2*d + 2*(a^3*b^2 + 2*a^2*b^3)*
d*cosh(d*x + c)^2 + 2*(3*a^3*b^2*d*cosh(d*x + c)^2 + (a^3*b^2 + 2*a^2*b^3)
*d)*sinh(d*x + c)^2 + 4*(a^3*b^2*d*cosh(d*x + c)^3 + (a^3*b^2 + 2*a^2*b...
```

3.148.6 Sympy [F]

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\tanh^5{(c+dx)}}{\left(a+b\,\text{sech}^2{(c+dx)}\right)^2}\,dx$$

input
```
integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)
```

3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(72) = 144$.

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.03

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{2\,(a^2+2\,ab+b^2)e^{(-2\,dx-2\,c)}}{(a^3be^{(-4\,dx-4\,c)}+a^3b+2\,(a^3b+2\,a^2b^2)e^{(-2\,dx-2\,c)})d} + \frac{dx+c}{a^2d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d} - \frac{(a^2-b^2)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^2b^2d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c)/((a^3*b*e^(-4*d*x - 4*c) + a^3*b +
2*(a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + log(e^(-2
*d*x - 2*c) + 1)/(b^2*d) - 1/2*(a^2 - b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c
) + a*e^(-4*d*x - 4*c) + a)/(a^2*b^2*d)
```

3.148.8 Giac [F]

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\tanh\left(dx+c\right)^5}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^4\tanh(c+d\,x)^5}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^4*tanh(c + d*x)^5)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.148. $\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

3.149 $\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

3.149.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx=\frac{x}{a^2}+\frac{(a-2b)\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d}-\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^2+1/2*(a-2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b)^(1/2)/a^2
/b^(3/2)/d-1/2*(a+b)*tanh(d*x+c)/a/b/d/(a+b-b*tanh(d*x+c)^2)
```

3.149.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 228 vs. 2(91) = 182.

Time = 3.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.51

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$$

$$=\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^4(c+dx)\left(2x(a+2b+a\cosh(2(c+dx)))+\frac{(a^2-ab-2b^2)\text{arctanh}\left(\frac{\text{sech}(dx)}{\ldots}\right.}{\ldots}\right.}{8a^2\left(a+bs\ldots\right.}$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*
(c + d*x)]) + ((a^2 - a*b - 2*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*
c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh
[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]
))/(b*Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + ((a + b)*Sech[2*c]*((
a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*d)))/(8*a^2*(a + b*Sech[c + d*x]^2
)^2)
```

3.149.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4629, 2075, 372, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(ic+idx)^4}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{-\left((a-b)\tanh^2(c+dx)\right)+a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2ab}-\frac{(a+b)\tanh(c+dx)}{2ab\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2b\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}+\frac{(a-2b)(a+b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2ab}-\frac{(a+b)\tanh(c+dx)}{2ab(a-b\tanh^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{(a-2b)(a+b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}+\frac{2b\text{arctanh}(\tanh(c+dx))}{a}}{2ab}-\frac{(a+b)\tanh(c+dx)}{2ab(a-b\tanh^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{2b\text{arctanh}(\tanh(c+dx))}{a}+\frac{(a-2b)\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}}{2ab}-\frac{(a+b)\tanh(c+dx)}{2ab(a-b\tanh^2(c+dx)+b)}}{d}$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(((2*b*ArcTanh[Tanh[c + d*x]])/a + ((a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[b]
*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[b]))/(2*a*b) - ((a + b)*Tanh[c + d*x
])/(2*a*b*(a + b - b*Tanh[c + d*x]^2)))/d
```

3.149.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.149.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(79) = 158$.

Time = 15.77 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.85

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2\left(-\frac{a(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2b}-\frac{a(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b}+\cdots}{d}$
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2\left(-\frac{a(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2b}-\frac{a(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b}+\cdots}{d}$
risch	$\frac{x}{a^2}+\frac{a^2 e^{2dx+2c}+3ab\,e^{2dx+2c}+2\,e^{2dx+2c}b^2+a^2+ab}{a^2 bd\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+b)b}-a-2b}{a}\right)}{4b^2da}-\frac{\sqrt{(a+b)b}\,\ln\left(e^{2\cdots}\right.}{\cdots}$

3.149. $\int \frac{\tanh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

input
```
int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*ln(1+tanh(1/2*d*x+1/2*c))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/a^2
*((-1/2*a*(a+b)/b*tanh(1/2*d*x+1/2*c)^3-1/2*a*(a+b)/b*tanh(1/2*d*x+1/2*c))
/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2*(a^2-a*b-2*b^2)/b*(1/4/b^(1/2)/(a+b)^
(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(
a+b)^(1/2))-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2
*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))))
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(82) = 164.

Time = 0.29 (sec) , antiderivative size = 1479, normalized size of antiderivative = 16.25

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3
 + 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(a*b + 2*b^2)*d*x + a^2 +
3*a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(a*b + 2
*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 - ((a^2 - 2*a*b)*cosh(d*x
 + c)^4 + 4*(a^2 - 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b)*si
nh(d*x + c)^4 + 2*(a^2 - 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b)*cosh(
d*x + c)^2 + a^2 - 4*b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + 4*((a^2 - 2*a*b)
*cosh(d*x + c)^3 + (a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b
)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*
sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)
^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x
+ c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)
^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2
)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
+ a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
c))*sinh(d*x + c) + a)) + 4*a^2 + 4*a*b + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (
2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(
a^3*b*d*cosh(d*x + c)^4 + 4*a^3*b*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*
d*sinh(d*x + c)^4 + a^3*b*d + 2*(a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 +...
```

3.149.6 Sympy [F]

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\tanh^4(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$$

input
```
integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

3.149.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(82) = 164$.

Time = 0.46 (sec) , antiderivative size = 1053, normalized size of antiderivative = 11.57

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} \, dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/64*(a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b
- 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((
a^3*b + a^2*b^2)*sqrt((a + b)*b)*d) + 1/16*a*log((a*e^(2*d*x + 2*c) + a +
2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b))
)/(sqrt((a + b)*b)*(a*b + b^2)*d) - 1/64*(a^3 - 6*a^2*b - 24*a*b^2 - 16*b^
3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2
*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3*b + a^2*b^2)*sqrt((a + b)*b)*d)
- 3/32*(a + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a
*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b +
b^2)*d) - 1/16*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a
*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b +
b^2)*d) + 1/16*(a^3 + 8*a^2*b + 8*a*b^2 + (a^3 + 18*a^2*b + 48*a*b^2 + 32*
b^3)*e^(2*d*x + 2*c))/((a^4*b + a^3*b^2 + (a^4*b + a^3*b^2)*e^(4*d*x + 4*c
) + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*e^(2*d*x + 2*c))*d) - 1/16*(a^3 + 8*
a^2*b + 8*a*b^2 + (a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*e^(-2*d*x - 2*c))/(
(a^4*b + a^3*b^2 + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*e^(-2*d*x - 2*c) + (a
^4*b + a^3*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b + 8
*b^2)*e^(2*d*x + 2*c))/((a^3*b + a^2*b^2 + (a^3*b + a^2*b^2)*e^(4*d*x + 4*
c) + 2*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^(2*d*x + 2*c))*d) - 1/4*(a^2 + 2*a*
b + (a^2 + 8*a*b + 8*b^2)*e^(-2*d*x - 2*c))/((a^3*b + a^2*b^2 + 2*(a^3*...
```

3.149.8 Giac [F]

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} \, dx = \int \frac{\tanh\left(dx+c\right)^4}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^2} \, dx$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\left(\cosh(c+d\,x)^2-1\right)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^2 - 1)^2/(b + a*cosh(c + d*x)^2)^2, x)
```

3.150 $\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

3.150.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx=\frac{a+b}{2a^2d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

output
```
1/2*(a+b)/a^2/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^2/d
```

3.150.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.59

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx =\frac{2(a+b)+(a+2b)\log(a+2b+a\cosh(2(c+dx)))+a\cosh(2(c+dx))\log(a+2b+a\cosh(2(c+dx)))}{2a^2d(a+2b+a\cosh(2(c+dx)))}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c +
d*x)]*Log[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c
+ d*x)]))
```

3.150.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ic+idx)^3}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ic+idx)^3}{\left(b\sec(ic+idx)^2+a\right)^2}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cosh(c+dx)\left(1-\cosh^2(c+dx)\right)}{\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$-\frac{\int \frac{1-\cosh^2(c+dx)}{\left(a\cosh^2(c+dx)+b\right)^2}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(\frac{a+b}{a\left(a\cosh^2(c+dx)+b\right)^2}-\frac{1}{a\left(a\cosh^2(c+dx)+b\right)}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{a+b}{a^2\left(a\cosh^2(c+dx)+b\right)}-\frac{\log\left(a\cosh^2(c+dx)+b\right)}{a^2}}{2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-1/2*(-((a + b)/(a^2*(b + a*Cosh[c + d*x]^2))) - Log[b + a*Cosh[c + d*x]^2
]/a^2)/d
```

3.150. $\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$

3.150.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.150. $\int \frac{\tanh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

3.150.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(47) = 94$.

Time = 9.77 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.25

method	result
risch	$-\frac{x}{a^2}-\frac{2c}{a^2 d}+\frac{2(a+b)\mathrm{e}^{2dx+2c}}{a^2 d\left(a\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+4b\,\mathrm{e}^{2dx+2c}+a\right)}+\frac{\ln\left(\mathrm{e}^{4dx+4c}+\frac{2(a+2b)\mathrm{e}^{2dx+2c}}{a}+1\right)}{2d\,a^2}$
derivativedivides	$\frac{-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{-\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b}}{d}$
default	$\frac{-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{-\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b}}{d}$

input
```
int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/a^2-2/a^2/d*c+2/a^2*(a+b)/d*exp(2*d*x+2*c)/(a*exp(4*d*x+4*c)+2*exp(2*d*
x+2*c)*a+4*b*exp(2*d*x+2*c)+a)+1/2/d/a^2*ln(exp(4*d*x+4*c)+2*(a+2*b)/a*exp
(2*d*x+2*c)+1)
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 485, normalized size of antiderivative = 9.51

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx =$$

$$-\frac{2\,adx\cosh\left(dx+c\right)^4+8\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,adx\sinh\left(dx+c\right)^4+2\,adx+4\left((a+2\,b)d\right.}{}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*
a*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - a - b)*cosh(d*x + c)^
2 + 4*(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - a - b)*sinh(d*x + c)^2 -
(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4
 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
 a)*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)
^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x
 + c)^3 + ((a + 2*b)*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cos
h(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)
^4 + a^3*d + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c
)^2 + (a^3 + 2*a^2*b)*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3
 + 2*a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.150.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(47) = 94$.

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.12

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \frac{2\,(a+b)e^{(-2\,dx-2\,c)}}{(a^3e^{(-4\,dx-4\,c)}+a^3+2\,(a^3+2\,a^2b)e^{(-2\,dx-2\,c)})d} + \frac{dx+c}{a^2d} + \frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^2d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2*(a + b)*e^(-2*d*x - 2*c)/((a^3*e^(-4*d*x - 4*c) + a^3 + 2*(a^3 + 2*a^2*b
)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x
 - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

3.150.8 Giac [F]

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\tanh(dx+c)^3}{(b\,\text{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)^4\tanh(c+d\,x)^3}{(a\cosh(c+d\,x)^2+b)^2}\,dx$$

input
```
int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^4*tanh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.151 $\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

3.151.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{x}{a^2} - \frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}d} - \frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^2-1/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/d/b^(1/2)/(
a+b)^(1/2)-1/2*tanh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)
```

3.151.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 326 vs. $2(85) = 170$.

Time = 4.77 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.84

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{(a+2b+a\cosh(2(c+dx)))^2\text{sech}^4(c+dx)\left(\frac{16x}{a^2} - \frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}(a+b)^{3/2}d} + \frac{(a^3-6a^2b-24ab^2-16b^3)\text{arctan}}{}\right.}{}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((16*x)/a^2 - ((a + 2*b
)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*(a + b)^(3/2)*d)
+ ((a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sin
h[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(
Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(a^2*b*(a + b)^(3/2)*d*Sq
rt[b*(Cosh[c] - Sinh[c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)
*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x
)])) + (a*Sinh[2*(c + d*x)])/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))
))/(64*(a + b*Sech[c + d*x]^2)^2)
```

3.151.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 4629, 25, 2075, 373, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ic+idx)^2}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ic+idx)^2}{\left(b\sec(ic+idx)^2+a\right)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^2}d\tanh(c+dx)}{d}$$

$$\downarrow \text{373}$$

$$\frac{-\frac{\tanh(c+dx)}{2a(a-b\tanh^2(c+dx)+b)}-\frac{\int \frac{\tanh^2(c+dx)+1}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)}d\tanh(c+dx)}{2a}}{d}$$

$$\downarrow \text{397}$$

$$\frac{-\frac{\tanh(c+dx)}{2a(a-b\tanh^2(c+dx)+b)}-\frac{\frac{2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{(a+2b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a}}{d}$$

$$\downarrow \text{219}$$

$$\frac{-\frac{\tanh(c+dx)}{2a(a-b\tanh^2(c+dx)+b)}-\frac{\frac{2\text{arctanh}(\tanh(c+dx))}{a}-\frac{(a+2b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a}}{d}$$

$$\downarrow \text{221}$$

$$\frac{-\frac{\tanh(c+dx)}{2a(a-b\tanh^2(c+dx)+b)}-\frac{\frac{2\text{arctanh}(\tanh(c+dx))}{a}-\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\sqrt{a+b}}}{2a}}{d}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-((-1/2*((2*ArcTanh[Tanh[c + d*x]])/a - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c
 + d*x])/Sqrt[a + b]])/(a*Sqrt[b]*Sqrt[a + b]))/a + Tanh[c + d*x]/(2*a*(a
+ b - b*Tanh[c + d*x]^2)))/d)
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.151. $\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.151. $\int \frac{\tanh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

3.151.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(73) = 146.

Time = 6.53 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.78

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b}+(a}{d}$
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b}+(a}{d}$
risch	$\frac{x}{a^2}+\frac{e^{2dx+2c}a+2b\,e^{2dx+2c}+a}{d\,a^2\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}+2ab+2b^2}{a\sqrt{ab+b^2}}\right)}{4\sqrt{ab+b^2}\,da}+\frac{\ln\left(e^{2dx+2c}+\right.}{}$

input
```
int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*ln(1+tanh(1/2*d*x+1/2*c))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/a^2
*((-1/2*tanh(1/2*d*x+1/2*c)^3*a-1/2*tanh(1/2*d*x+1/2*c)*a)/(tanh(1/2*d*x+1
/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x
+1/2*c)^2*b+a+b)+1/2*(a+2*b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh
(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(
a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1
/2)+(a+b)^(1/2)))))
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. 2(76) = 152.

Time = 0.30 (sec) , antiderivative size = 1846, normalized size of antiderivative = 21.72

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b
 + 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b
 + 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d
*x + c)^2 + a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*sin
h(d*x + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b
^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4
*b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (
a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^
2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c
)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2
*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^
2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(
d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*
cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 +
2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x +
 c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)
) + 8*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b + 3*a*b^2 + 2*b^3 +
2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b ...
```

3.151.6 Sympy [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\mathrm{sech}^2(c+dx))^2}\,dx = \int \frac{\tanh^2{(c+dx)}}{\left(a+b\operatorname{sech}^2{(c+dx)}\right)^2}\,dx$$

input
```
integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.151.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(76) = 152.

Time = 0.35 (sec) , antiderivative size = 597, normalized size of antiderivative = 7.02

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx$$
$$= -\frac{(a^2+6\,ab+4\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^3+a^2b)\sqrt{(a+b)b}d} + \frac{(a^2+6\,ab+4\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^3+a^2b)\sqrt{(a+b)b}d} + \frac{a^2+2\,ab+(a^2+8\,ab+8\,b^2)e^{(2\,dx+2\,c)}}{4\,(a^4+a^3b+(a^4+a^3b)e^{(4\,dx+4\,c)}+2\,(a^4+3\,a^3b+2\,a^2b^2)e^{(2\,dx+2\,c)})d} - \frac{a^2+2\,ab+(a^2+8\,ab+8\,b^2)e^{(-2\,dx-2\,c)}}{4\,(a^4+a^3b+2\,(a^4+3\,a^3b+2\,a^2b^2)e^{(-2\,dx-2\,c)}+(a^4+a^3b)e^{(-4\,dx-4\,c)})d} - \frac{(a+2\,b)e^{(-2\,dx-2\,c)}+a}{2\,(a^3+a^2b+2\,(a^3+3\,a^2b+2\,ab^2)e^{(-2\,dx-2\,c)}+(a^3+a^2b)e^{(-4\,dx-4\,c)})d} + \frac{\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}(a+b)d} + \frac{\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^2d} - \frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,a^2d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/16*(a^2 + 6*a*b + 4*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a +
 b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*
sqrt((a + b)*b)*d) + 1/16*(a^2 + 6*a*b + 4*b^2)*log((a*e^(-2*d*x - 2*c) +
a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b
)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) + 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b
 + 8*b^2)*e^(2*d*x + 2*c))/((a^4 + a^3*b + (a^4 + a^3*b)*e^(4*d*x + 4*c) +
 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(2*d*x + 2*c))*d) - 1/4*(a^2 + 2*a*b + (a
^2 + 8*a*b + 8*b^2)*e^(-2*d*x - 2*c))/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2
*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) - 1/2*((a
+ 2*b)*e^(-2*d*x - 2*c) + a)/((a^3 + a^2*b + 2*(a^3 + 3*a^2*b + 2*a*b^2)*e
^(-2*d*x - 2*c) + (a^3 + a^2*b)*e^(-4*d*x - 4*c))*d) + 1/8*log((a*e^(-2*d*
x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*
sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a + b)*d) + 1/4*log(a*e^(4*d*x + 4*c)
+ 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x
 - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

3.151.8 Giac [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx=\int \frac{\tanh\left(dx+c\right)^2}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx=\int \frac{\cosh(c+d\,x)^2\,\left(\cosh(c+d\,x)^2-1\right)}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^2*(cosh(c + d*x)^2 - 1))/(b + a*cosh(c + d*x)^2)^2, x)
```

3.152 $\int \frac{\tanh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

3.152.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{\tanh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx=\frac{b}{2a^2d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

output
```
1/2*b/a^2/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^2/d
```

3.152.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.61

$$\int \frac{\tanh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx =\frac{2b+(a+2b)\log(a+2b+a\cosh(2(c+dx)))+a\cosh(2(c+dx))\log(a+2b+a\cosh(2(c+dx)))}{2a^2d(a+2b+a\cosh(2(c+dx)))}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c + d*x)]*
Log[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c + d*x)
]))
```

3.152.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}^2(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)}{(a+b\sec(ic+idx)^2)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)}{(b\sec(ic+idx)^2+a)^2}dx$$

$$\Big\downarrow \text{4626}$$

$$\frac{\int \frac{\cosh^3(c+dx)}{(a\cosh^2(c+dx)+b)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{\cosh^2(c+dx)}{(a\cosh^2(c+dx)+b)^2}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{1}{a(a\cosh^2(c+dx)+b)}-\frac{b}{a(a\cosh^2(c+dx)+b)^2}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b}{a^2(a\cosh^2(c+dx)+b)}+\frac{\log(a\cosh^2(c+dx)+b)}{a^2}}{2d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(b/(a^2*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/a^2)/(2*d)
```

3.152.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.152.4 Maple [A] (verified)

Time = 6.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$-\frac{1}{2da\left(a+b\,\mathrm{sech}(dx+c)^{2}\right)}+\frac{\ln\left(a+b\,\mathrm{sech}(dx+c)^{2}\right)}{2d\,a^{2}}-\frac{\ln(\mathrm{sech}(dx+c))}{d\,a^{2}}$	60
default	$-\frac{1}{2da\left(a+b\,\mathrm{sech}(dx+c)^{2}\right)}+\frac{\ln\left(a+b\,\mathrm{sech}(dx+c)^{2}\right)}{2d\,a^{2}}-\frac{\ln(\mathrm{sech}(dx+c))}{d\,a^{2}}$	60
risch	$-\frac{x}{a^{2}}-\frac{2c}{a^{2}d}+\frac{2b\,\mathrm{e}^{2dx+2c}}{a^{2}d\left(a\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+4b\,\mathrm{e}^{2dx+2c}+a\right)}+\frac{\ln\left(\mathrm{e}^{4dx+4c}+\frac{2(a+2b)\mathrm{e}^{2dx+2c}}{a}+1\right)}{2d\,a^{2}}$	113

input
```
int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/a/(a+b*sech(d*x+c)^2)+1/2/d/a^2*ln(a+b*sech(d*x+c)^2)-1/d/a^2*ln(se
ch(d*x+c))
```

3.152.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 476, normalized size of antiderivative = 9.71

$$\int \frac{\tanh(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^2}\,dx =$$

$$-\frac{2\,adx\cosh\left(dx+c\right)^4+8\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,adx\sinh\left(dx+c\right)^4+2\,adx+4\left(\left(a+2\,b\right)d\right.}{}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*
a*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - b)*cosh(d*x + c)^2 +
4*(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - b)*sinh(d*x + c)^2 - (a*cosh(
d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
+ 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*log(
2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x + c)^3
+ ((a + 2*b)*d*x - b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4
 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 + a^3*d +
 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c)^2 + (a^3 +
 2*a^2*b)*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*
d*cosh(d*x + c))*sinh(d*x + c))
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.152.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(45) = 90$.

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.16

$$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \frac{2\,be^{(-2\,dx-2\,c)}}{(a^3e^{(-4\,dx-4\,c)}+a^3+2\,(a^3+2\,a^2b)e^{(-2\,dx-2\,c)})d} + \frac{dx+c}{a^2d} + \frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^2d}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2*b*e^(-2*d*x - 2*c)/((a^3*e^(-4*d*x - 4*c) + a^3 + 2*(a^3 + 2*a^2*b)*e^(-
2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c
) + a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

3.152.8 Giac [F]

$$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx = \int \frac{\tanh(dx+c)}{(b\operatorname{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.152.9 Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{\tanh(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{\ln\left(\cosh(c+d\,x)^2\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)\right)}{2\,a^2\,d} - \frac{1}{2\,a\,d\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)}$$

input
```
int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
log(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2))/(2*a^2*d) - 1/(2*a*d*(a + b/c
osh(c + d*x)^2))
```

3.153 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

3.153.1 Optimal result

Integrand size = 14, antiderivative size = 93

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\frac{x}{a^2}-\frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d}-\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^2-1/2*(3*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^2/(
a+b)^(3/2)/d-1/2*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

3.153.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 221 vs. 2(93) = 186.

Time = 3.74 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.38

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

$$=\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^4(c+dx)\left(2x(a+2b+a\cosh(2(c+dx)))-\frac{b(3a+2b)\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx)(\cosh \cdots}{\cdots}\right)}{\cdots}\cdots\right.}{8a^2\left(a+b\operatorname{sech}\cdots\right.}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^(-2),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*
(c + d*x)]) - (b*(3*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((
a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/((a
 + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (b*Sech[2*c]*((a + 2*b)*Sin
h[2*c] - a*Sinh[2*d*x]))/((a + b)*d)))/(8*a^2*(a + b*Sech[c + d*x]^2)^2)
```

3.153.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4616, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{-\frac{\int -\frac{b\tanh^2(c+dx)+2a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\tanh^2(c+dx)+2a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2(a+b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b(3a+2b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{b\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{2(a+b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{b(3a+2b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{b\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2(a+b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^(-2),x]
```

output
```
(((2*(a + b)*ArcTanh[Tanh[c + d*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqr
t[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*Tanh
[c + d*x])/(2*a*(a + b)*(a + b - b*Tanh[c + d*x]^2)))/d
```

3.153.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.153.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(81) = 162$.

Time = 0.57 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.73

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2b\left(\frac{-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}$
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2b\left(\frac{-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a+b)}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}$
risch	$\frac{x}{a^2}+\frac{b\left(e^{2dx+2c}a+2b\,e^{2dx+2c}+a\right)}{a^2(a+b)d\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)}+\frac{3\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}+a+2b}{a}\right)}{4(a+b)^2da}+\frac{\sqrt{(a+b)b}\,\ln$

input
```
int(1/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*ln(1+tanh(1/2*d*x+1/2*c))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/a^2
*b*((-1/2*a/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/2*a/(a+b)*tanh(1/2*d*x+1/2*c))/(
tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2*(3*a+2*b)/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/
2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b
)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*ta
nh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))))
```

3.153.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(84) = 168$.

Time = 0.30 (sec) , antiderivative size = 1690, normalized size of antiderivative = 18.17

$$\int \frac{1}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)
*sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x +
 4*(2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*(a^2
 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*s
inh(d*x + c)^2 + ((3*a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + 2*a*b)*cosh
(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 8
*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a
^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 4*((3*a^2 + 2*a*b)*c
osh(d*x + c)^3 + (3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqr
t(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^
3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(
d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*c
osh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*
b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*
b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*
cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*a*b +
8*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b
+ 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^4...
```

3.153.6 Sympy [F]

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\text{sech}^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**(-2), x)
```

3.153.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(84) = 168.

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.01

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{(3\,ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,(a^3+a^2b)\sqrt{(a+b)b}d} - \frac{ab+(ab+2\,b^2)e^{(-2\,dx-2\,c)}}{(a^4+a^3b+2\,(a^4+3\,a^3b+2\,a^2b^2)e^{(-2\,dx-2\,c)}+(a^4+a^3b)e^{(-4\,dx-4\,c)})d} + \frac{dx+c}{a^2d}$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(3*a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))
/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((
a + b)*b)*d) - (a*b + (a*b + 2*b^2)*e^(-2*d*x - 2*c))/((a^4 + a^3*b + 2*(a
^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c
))*d) + (d*x + c)/(a^2*d)
```

3.153.8 Giac [F]

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \int \frac{1}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\mathrm{sech}^2(c+dx)\right)^2}\,dx=\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

input
```
int(1/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int(1/(a + b/cosh(c + d*x)^2)^2, x)
```

3.154 $\int \frac{\coth(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

3.154.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\coth(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{b^2}{2a^2(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{b(2a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2a^2(a+b)^2d} + \frac{\log(\sinh(c+dx))}{(a+b)^2d}$$

output
```
1/2*b^2/a^2/(a+b)/d/(b+a*cosh(d*x+c)^2)+1/2*b*(2*a+b)*ln(b+a*cosh(d*x+c)^2
)/a^2/(a+b)^2/d+ln(sinh(d*x+c))/(a+b)^2/d
```

3.154.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.39

$$\int \frac{\coth(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{(a+b)\left(2a^2\log(\sinh(c+dx))+b\left(b+(2a+b)\log\left(a+b+a\sinh^2(c+dx)\right)\right)\right)+a\left(2a^2\log(\sinh(c+dx))\right.}{a^2(a+b)^2d(a+2b+a\cosh(2(c+dx)))}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + b)*(2*a^2*Log[Sinh[c + d*x]] + b*(b + (2*a + b)*Log[a + b + a*Sinh[c
 + d*x]^2])) + a*(2*a^2*Log[Sinh[c + d*x]] + b*(2*a + b)*Log[a + b + a*Sin
h[c + d*x]^2])*Sinh[c + d*x]^2)/(a^2*(a + b)^2*d*(a + 2*b + a*Cosh[2*(c +
d*x)]))
```

3.154.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ic+idx)\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^2\tan(ic+idx)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cosh^5(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cosh^4(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^2}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{b^2}{a(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{(2a+b)b}{a(a+b)^2\left(a\cosh^2(c+dx)+b\right)}-\frac{1}{(a+b)^2\left(\cosh^2(c+dx)-1\right)}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b^2}{a^2(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{b(2a+b)\log\left(a\cosh^2(c+dx)+b\right)}{a^2(a+b)^2}-\frac{\log\left(1-\cosh^2(c+dx)\right)}{(a+b)^2}}{2d}$$

input
```
Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-1/2*(-(b^2/(a^2*(a + b)*(b + a*Cosh[c + d*x]^2))) - Log[1 - Cosh[c + d*x]
^2]/(a + b)^2 - (b*(2*a + b)*Log[b + a*Cosh[c + d*x]^2])/(a^2*(a + b)^2))/
d
```

3.154.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.154.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(79) = 158$.

Time = 10.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.48

method	result
derivativedivides	$\frac{-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{b\left(-\frac{2ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}+\frac{(2a+b)\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{}\right.}{a^2(a+b)^2}}{d}$
default	$\frac{-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{b\left(-\frac{2ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b}+\frac{(2a+b)\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{}\right.}{a^2(a+b)^2}}{d}$
risch	$\frac{x}{a^2}-\frac{2x}{a^2+2ab+b^2}-\frac{2c}{d(a^2+2ab+b^2)}-\frac{4bx}{a(a^2+2ab+b^2)}-\frac{4bc}{da(a^2+2ab+b^2)}-\frac{2b^2x}{a^2(a^2+2ab+b^2)}-\frac{2b^2c}{d\,a^2(a^2+2ab+b}$

input
```
int(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2*ln(1+tanh(1/2*d*x+1/2*c))+b/a^2/(a+b)^2*(-2*a*b*tanh(1/2*d*x+1
/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2*(2*a+b)*ln(tanh(1/2*d*x+1/2*c)
^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b))+1/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))-1/a^2*ln(tanh(1/2*d*x+1/2*c
)-1))
```

3.154.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(79) = 158$.

Time = 0.34 (sec) , antiderivative size = 1031, normalized size of antiderivative = 12.42

$$\int \frac{\coth(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^3 + 2*a^2*b + a
*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x*si
nh(d*x + c)^4 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(a*b^2 + b^3 - (a^3 + 4*
a^2*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + 2*a^2*b + a*b^
2)*d*x*cosh(d*x + c)^2 - a*b^2 - b^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d
*x)*sinh(d*x + c)^2 - ((2*a^2*b + a*b^2)*cosh(d*x + c)^4 + 4*(2*a^2*b + a*
b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b + a*b^2)*sinh(d*x + c)^4 + 2
*a^2*b + a*b^2 + 2*(2*a^2*b + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 2*(2*a^2*
b + 5*a*b^2 + 2*b^3 + 3*(2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 4*((2*a^2*b + a*b^2)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 + 2*b^3)*cosh
(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a
 + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)) - 2*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*si
nh(d*x + c)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(3*a^3*cosh(d*
x + c)^2 + a^3 + 2*a^2*b)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + (a^3
+ 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c
) - sinh(d*x + c))) + 8*((a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (a*
b^2 + b^3 - (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x
 + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a
^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*s...
```

3.154.6 Sympy [F]

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\coth\left(c+dx\right)}{\left(a+b\,\text{sech}^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

3.154.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(79) = 158.

Time = 0.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.52

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx$$
$$= \frac{2\,b^2e^{(-2\,dx-2\,c)}}{(a^4+a^3b+2\,(a^4+3\,a^3b+2\,a^2b^2)e^{(-2\,dx-2\,c)}+(a^4+a^3b)e^{(-4\,dx-4\,c)})d} + \frac{(2\,ab+b^2)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,(a^4+2\,a^3b+a^2b^2)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{(a^2+2\,ab+b^2)d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{(a^2+2\,ab+b^2)d} + \frac{dx+c}{a^2d}$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2*b^2*e^(-2*d*x - 2*c)/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2
*d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) + 1/2*(2*a*b + b^2)*log(2
*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 2*a^3*b + a^
2*b^2)*d) + log(e^(-d*x - c) + 1)/((a^2 + 2*a*b + b^2)*d) + log(e^(-d*x -
c) - 1)/((a^2 + 2*a*b + b^2)*d) + (d*x + c)/(a^2*d)
```

3.154.8 Giac [F]

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\coth(dx+c)}{(b\,\text{sech}\,(dx+c)^2+a)^2}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^4\coth(c+d\,x)}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(coth(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^4*coth(c + d*x))/(b + a*cosh(c + d*x)^2)^2, x)
```

3.155 $\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{x}{a^2}-\frac{b^{3/2}(5a+2b)\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}d} - \frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^2-1/2*b^(3/2)*(5*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/(
a+b)^(5/2)/d-1/2*(2*a-b)*coth(d*x+c)/a/(a+b)^2/d-1/2*b*coth(d*x+c)/a/(a+b)
/d/(a+b-b*tanh(d*x+c)^2)
```

3.155.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 268 vs. $2(121) = 242$.

Time = 4.12 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.21

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^4(c+dx)\left(\frac{2x(a+2b+a\cosh(2(c+dx)))}{a^2} - \frac{b^2(5a+2b)\text{arctanh}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c)) }{2\sqrt{a+b}\sqrt{b(\cos}}\right.}{a^2(}\right.}{}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((2*x*(a + 2*b + a*Cosh[2
*(c + d*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[
2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Co
sh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*
c]))/(a^2*(a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (2*(a + 2*b + a
*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a + b)^2*d) + (b^2*
Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2*(a + b)^2*d)))/(8*(a
 + b*Sech[c + d*x]^2)^2)
```

3.155.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 25, 4629, 25, 2075, 374, 25, 445, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ic+idx)^2\,(a+b\sec(ic+idx)^2)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{(b\sec(ic+idx)^2+a)^2\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(a+b(1-\tanh^2(c+dx))\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(a+b(1-\tanh^2(c+dx))\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{374}$$

$$-\frac{\frac{\int -\frac{\coth^2(c+dx)\left(3b\tanh^2(c+dx)+2a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{b\coth(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{b\coth(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\int \frac{\coth^2(c+dx)\left(3b\tanh^2(c+dx)+2a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}}{d}$$

$$\Big\downarrow \text{445}$$

$$-\frac{\frac{b\coth(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{-\frac{\int -\frac{2a^2+6ba+b^2-(2a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{(2a-b)\coth(c+dx)}{a+b}}{2a(a+b)}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{b\coth(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\int \frac{2a^2+6ba+b^2-(2a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{(2a-b)\coth(c+dx)}{a+b}}{2a(a+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{b\coth(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\frac{2(a+b)^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b^2(5a+2b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{(2a-b)\coth(c+dx)}{a+b}}{2a(a+b)}}{d}$$

$$\Big\downarrow \text{219}$$

3.155. $\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

$$-\frac{-\frac{b\coth(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\frac{2(a+b)^2\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^2(5a+2b)\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{(2a-b)\coth(c+dx)}{a+b}}{2a(a+b)}}{d}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{b\coth(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\frac{2(a+b)^2\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^{3/2}(5a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{(2a-b)\coth(c+dx)}{a+b}}{2a(a+b)}}{d}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
-((-1/2*(((2*(a + b)^2*ArcTanh[Tanh[c + d*x]])/a - (b^(3/2)*(5*a + 2*b)*Ar
cTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b) - ((2
*a - b)*Coth[c + d*x])/(a + b))/(a*(a + b)) + (b*Coth[c + d*x])/(2*a*(a +
b)*(a + b - b*Tanh[c + d*x]^2)))/d)
```

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

3.155. $\int \frac{\coth^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.155. $\int \frac{\coth^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

3.155.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(107) = 214$.

Time = 15.90 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.38

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+2ab+b^2\right)}-\frac{1}{2(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^2\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b}+\right.}{}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+2ab+b^2\right)}-\frac{1}{2(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^2\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a+b}+\right.}{}$
risch	$\frac{x}{a^2}-\frac{2a^3e^{4dx+4c}-ab^2e^{4dx+4c}-2e^{4dx+4c}b^3+4a^3e^{2dx+2c}+8a^2b\,e^{2dx+2c}+2\,e^{2dx+2c}b^3+2a^3+ab^2}{a^2d(a+b)^2\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)\left(e^{2dx+2c}-1\right)}+\frac{5\sqrt{(a+b)b}\,b\ln\left(\right.}{}$

input
```
int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/2/(a+b)^2/tanh(1/2*d*x+1/2
*c)+2*b^2/(a+b)^2/a^2*((-1/2*tanh(1/2*d*x+1/2*c)^3*a-1/2*tanh(1/2*d*x+1/2*
c)*a)/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2*(5*a+2*b)*(-1/4/b^(1/2)/(a+b)^(1
/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+
b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*t
anh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+
1/a^2*ln(1+tanh(1/2*d*x+1/2*c)))
```

3.155.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1674 vs. $2(110) = 220$.

Time = 0.31 (sec) , antiderivative size = 3624, normalized size of antiderivative = 29.95

$$\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^6 + 24*(a^3 + 2*a^2*b +
a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^3 + 2*a^2*b + a*b^2)*d*x*s
inh(d*x + c)^6 - 4*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b
^3)*d*x)*cosh(d*x + c)^4 + 4*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)
^2 - 2*a^3 + a*b^2 + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*sinh(d
*x + c)^4 + 16*(5*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (2*a^3 - a
*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c))*sinh(
d*x + c)^3 - 8*a^3 - 4*a*b^2 - 4*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(4*a^3 +
8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2 +
 4*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 - 4*a^3 - 8*a^2*b - 2*b
^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x - 6*(2*a^3 - a*b^2 - 2*b^3 - (a
^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((
5*a^2*b + 2*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)*s
inh(d*x + c)^5 + (5*a^2*b + 2*a*b^2)*sinh(d*x + c)^6 + (5*a^2*b + 22*a*b^2
 + 8*b^3)*cosh(d*x + c)^4 + (5*a^2*b + 22*a*b^2 + 8*b^3 + 15*(5*a^2*b + 2*
a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 2*a*b^2)*cosh(d*
x + c)^3 + (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5
*a^2*b - 2*a*b^2 - (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2 + (15*(5*a
^2*b + 2*a*b^2)*cosh(d*x + c)^4 - 5*a^2*b - 22*a*b^2 - 8*b^3 + 6*(5*a^2*b
+ 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + ...
```

3.155.6 Sympy [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\coth^2{(c+dx)}}{\left(a+b\operatorname{sech}^2{(c+dx)}\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1070 vs. 2(110) = 220.

Time = 0.36 (sec) , antiderivative size = 1070, normalized size of antiderivative = 8.84

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)
/((a^4 + 2*a^3*b + a^2*b^2)*d) - 1/4*(2*a*b + b^2)*log(2*(a + 2*b)*e^(-2*d
*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 2*a^3*b + a^2*b^2)*d) - 1/16*(
3*a^2*b + 10*a*b^2 + 4*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a +
 b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 2*a^3*b
 + a^2*b^2)*sqrt((a + b)*b)*d) + 1/16*(3*a^2*b + 10*a*b^2 + 4*b^3)*log((a*
e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a +
2*b + 2*sqrt((a + b)*b)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*b)*d) -
3/8*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b
)*d) + 1/4*(2*a^3 + a^2*b + 2*a*b^2 + (2*a^3 - a^2*b - 8*a*b^2 - 8*b^3)*e^
(4*d*x + 4*c) + 2*(2*a^3 + 4*a^2*b + 3*a*b^2 + 4*b^3)*e^(2*d*x + 2*c))/((a
^5 + 2*a^4*b + a^3*b^2 - (a^5 + 2*a^4*b + a^3*b^2)*e^(6*d*x + 6*c) - (a^5
+ 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(4*d*x + 4*c) + (a^5 + 6*a^4*b + 9*a^
3*b^2 + 4*a^2*b^3)*e^(2*d*x + 2*c))*d) - 1/4*(2*a^3 + a^2*b + 2*a*b^2 + 2*
(2*a^3 + 4*a^2*b + 3*a*b^2 + 4*b^3)*e^(-2*d*x - 2*c) + (2*a^3 - a^2*b - 8*
a*b^2 - 8*b^3)*e^(-4*d*x - 4*c))/((a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 6*a^4*
b + 9*a^3*b^2 + 4*a^2*b^3)*e^(-2*d*x - 2*c) - (a^5 + 6*a^4*b + 9*a^3*b^2 +
 4*a^2*b^3)*e^(-4*d*x - 4*c) - (a^5 + 2*a^4*b + a^3*b^2)*e^(-6*d*x - 6*c))
*d) - 1/2*(2*a^2 - a*b + 2*(2*a^2 + 4*a*b - b^2)*e^(-2*d*x - 2*c) + (2*...
```

3.155.8 Giac [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\coth(dx+c)^2}{(b\,\text{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^4\coth(c+d\,x)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^4*coth(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.156 $\int \frac{\coth^3(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2}\,dx$

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{b^3}{2a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{csch}^2(c+dx)}{2(a+b)^2d} + \frac{b^2(3a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2a^2(a+b)^3d} + \frac{(a+3b)\log(\sinh(c+dx))}{(a+b)^3d}$$

output
```
1/2*b^3/a^2/(a+b)^2/d/(b+a*cosh(d*x+c)^2)-1/2*csch(d*x+c)^2/(a+b)^2/d+1/2*
b^2*(3*a+b)*ln(b+a*cosh(d*x+c)^2)/a^2/(a+b)^3/d+(a+3*b)*ln(sinh(d*x+c))/(a
+b)^3/d
```

3.156.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))^2\text{sech}^4(c+dx)\left(-\left((a+b)\text{csch}^2(c+dx)\right)+2(a+3b)\log(\sinh(c+dx))+\frac{b^2(\cdots}{}\right.}{8(a+b)^3d\left(a+b\text{sech}^2(c+dx)\right)^2}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*(-((a + b)*Csch[c + d*x
]^2) + 2*(a + 3*b)*Log[Sinh[c + d*x]] + (b^2*(3*a + b)*Log[a + b + a*Sinh[
c + d*x]^2])/a^2 + (b^3*(a + b))/(a^2*(a + b + a*Sinh[c + d*x]^2))))/(8*(a
 + b)^3*d*(a + b*Sech[c + d*x]^2)^2)
```

3.156.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^2\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{4626}$$

$$\frac{\int \frac{\cosh^7(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2\left(a\cosh^2(c+dx)+b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cosh^6(c+dx)}{\left(1-\cosh^2(c+dx)\right)^2\left(a\cosh^2(c+dx)+b\right)^2}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-\frac{b^3}{a(a+b)^2\left(a\cosh^2(c+dx)+b\right)^2}+\frac{(3a+b)b^2}{a(a+b)^3\left(a\cosh^2(c+dx)+b\right)}+\frac{a+3b}{(a+b)^3\left(\cosh^2(c+dx)-1\right)}+\frac{1}{(a+b)^2\left(\cosh^2(c+dx)-1\right)^2}\right)d\cosh^2(c+$$

$$2d$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^3}{a^2(a+b)^2(a\cosh^2(c+dx)+b)}+\frac{b^2(3a+b)\log(a\cosh^2(c+dx)+b)}{a^2(a+b)^3}+\frac{1}{(a+b)^2(1-\cosh^2(c+dx))}+\frac{(a+3b)\log(1-\cosh^2(c+dx))}{(a+b)^3}}{2d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
(1/((a + b)^2*(1 - Cosh[c + d*x]^2)) + b^3/(a^2*(a + b)^2*(b + a*Cosh[c +
d*x]^2)) + ((a + 3*b)*Log[1 - Cosh[c + d*x]^2])/(a + b)^3 + (b^2*(3*a + b)
*Log[b + a*Cosh[c + d*x]^2])/(a^2*(a + b)^3))/(2*d)
```

3.156.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.156.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(104) = 208$.

Time = 23.77 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.36

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8\left(a^2+2ab+b^2\right)}-\frac{1}{8(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(4a+12b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{b^2\left(-\right.}{}}{}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8\left(a^2+2ab+b^2\right)}-\frac{1}{8(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(4a+12b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{b^2\left(-\right.}{}}{}$
risch	$\frac{x}{a^2}-\frac{2ax}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2ac}{d(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6bx}{a^3+3a^2b+3a\,b^2+b^3}-\frac{6bc}{d(a^3+3a^2b+3a\,b^2+b^3)}-\frac{}{a(a^3+3a}$

input
```
int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)-1/8/(a+b)^2/tanh(1/2*d*x+1
/2*c)^2+1/4/(a+b)^3*(4*a+12*b)*ln(tanh(1/2*d*x+1/2*c))-1/a^2*ln(tanh(1/2*d
*x+1/2*c)-1)-1/a^2*ln(1+tanh(1/2*d*x+1/2*c))+b^2/a^2/(a+b)^3*(-2*a*b*tanh(
1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1
/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2*(3*a+b)*ln(tanh(1/2*d
*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)))
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3624 vs. 2(104) = 208.

Time = 0.44 (sec) , antiderivative size = 3624, normalized size of antiderivative = 32.95

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x + c)^8 + 4*(a^4 + a^3*b - a*b^
3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^6 + 4*(
14*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^2 + a^4 + a^3*b -
 a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*sinh(d*x + c)^6
+ 8*(14*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^3 + 3*(a^4 +
 a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 4*(2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4
- (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*cosh(d*x + c)^4 + 4
*(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^4 + 2*a^4 + 6*a
^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^
3 + 4*b^4)*d*x + 15*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4 + a^3*b - a*b^3 - b^4 +
2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (2*a^4 + 6*a^
3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3
 + 4*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*d*x + 4*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^4 + 3*a^3*b + 3*a^2*b^2 + ...
```

3.156.6 Sympy [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\coth^3{(c+dx)}}{(a+b\,\text{sech}^2{(c+dx)})^2}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

3.156.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(104) = 208$.

Time = 0.22 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.49

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \frac{(3\,ab^2+b^3)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)d}$$
$$+\frac{(a+3\,b)\log\left(e^{(-dx-c)}+1\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)d}+\frac{(a+3\,b)\log\left(e^{(-dx-c)}-1\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$-\frac{2\left((a^3-b^3)e^{(-2\,dx-2\,c)}+2\,(a^3+2\,a^2b+b^3)e^{(-4\,dx-4\,c)}+(a\right.}{(a^5+2\,a^4b+a^3b^2+4\,(a^4b+2\,a^3b^2+a^2b^3)e^{(-2\,dx-2\,c)}-2\,(a^5+6\,a^4b+9\,a^3b^2+4\,a^2b^3)e^{(-4\,dx-4\,c)}+4\,(a}$$
$$+\frac{dx+c}{a^2d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c)
+ a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) + (a + 3*b)*log(e^(-d*x - c
) + 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (a + 3*b)*log(e^(-d*x - c) -
1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^3 - b^3)*e^(-2*d*x - 2*c) +
 2*(a^3 + 2*a^2*b + b^3)*e^(-4*d*x - 4*c) + (a^3 - b^3)*e^(-6*d*x - 6*c))/
((a^5 + 2*a^4*b + a^3*b^2 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-2*d*x - 2*
c) - 2*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(-4*d*x - 4*c) + 4*(a^4*b
 + 2*a^3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (a^5 + 2*a^4*b + a^3*b^2)*e^(-8
*d*x - 8*c))*d) + (d*x + c)/(a^2*d)
```

3.156.8 Giac [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\coth\left(dx+c\right)^3}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\cosh(c+d\,x)^4\,\coth(c+d\,x)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^4*coth(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.157 $\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{x}{a^2}-\frac{b^{5/2}(7a+2b)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}d} - \frac{(2a^2+6ab-b^2)\coth(c+dx)}{2a(a+b)^3d} - \frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^2-1/2*b^(5/2)*(7*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/(
a+b)^(7/2)/d-1/2*(2*a^2+6*a*b-b^2)*coth(d*x+c)/a/(a+b)^3/d-1/6*(2*a-3*b)*c
oth(d*x+c)^3/a/(a+b)^2/d-1/2*b*coth(d*x+c)^3/a/(a+b)/d/(a+b-b*tanh(d*x+c)^
2)
```

3.157.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 350 vs. 2(161) = 322.

Time = 6.35 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.17

$$\int \frac{\coth^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^4(c+dx)\left(\frac{6x(a+2b+a\cosh(2(c+dx)))}{a^2}-\frac{2(a+2b+a\cosh(2(c+dx)))\coth(c)\operatorname{csch}^2(c+dx)}{(a+b)^2d}-\right.}{}$$

input

```
Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output

```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((6*x*(a + 2*b + a*Cosh[2
*(c + d*x)]))/a^2 - (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Coth[c]*Csch[c + d*
x]^2)/((a + b)^2*d) - (3*b^3*(7*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - S
inh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sin
h[2*c]))/(a^2*(a + b)^(7/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*(2*a + 5
*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a +
 b)^3*d) + (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]^3*Sinh
[d*x])/((a + b)^2*d) + (3*b^3*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*
x]))/(a^2*(a + b)^3*d)))/(24*(a + b*Sech[c + d*x]^2)^2)
```

3.157.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 4629, 2075, 374, 25, 445, 27, 445, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^2}dx$$

$\downarrow$ 4629

$$\frac{\int \frac{\coth^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^2}d\tanh(c+dx)}{d}$$

$\downarrow$ 2075

$$\frac{\int \frac{\coth^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{d}$$

$\downarrow$ 374

$$\frac{-\frac{\int -\frac{\coth^4(c+dx)\left(5b\tanh^2(c+dx)+2a-3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\coth^4(c+dx)\left(5b\tanh^2(c+dx)+2a-3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$\downarrow$ 445

$$\frac{\frac{-\frac{\int -\frac{3\coth^2(c+dx)\left(2a^2+6ba-b^2-(2a-3b)b\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{3(a+b)}-\frac{(2a-3b)\coth^3(c+dx)}{3(a+b)}}{2a(a+b)}-\frac{b\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\coth^2(c+dx)\left(2a^2+6ba-b^2-(2a-3b)b\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{(2a-3b)\coth^3(c+dx)}{3(a+b)}}{2a(a+b)}-\frac{b\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$\downarrow$ 445

$$\frac{\frac{\frac{-\frac{\int -\frac{2a^3+8ba^2+12b^2a+b^3-b\left(2a^2+6ba-b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{a+b}}{a+b}-\frac{(2a-3b)\coth^3(c+dx)}{3(a+b)}}{2a(a+b)}-\frac{b\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)\right.}}{d}$$

$\downarrow$ 25

3.157. $\int \frac{\coth^4(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

$$\frac{\frac{\frac{\frac{\int \frac{2a^3+8ba^2+12b^2a+b^3-b\left(2a^2+6ba-b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{a+b}}{a+b}-\frac{(2a-3b)\coth^3(c+dx)}{3(a+b)}}{2a(a+b)}-\frac{b\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{\frac{2(a+b)^3\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b^3(7a+2b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{a+b}}{a+b}-\frac{(2a-3b)\coth^3(c+dx)}{3(a+b)}}{2a(a+b)}-\frac{b\,\cdots}{2a(a+b)(\cdots}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\frac{\frac{2(a+b)^3\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^3(7a+2b)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{a+b}}{a+b}-\frac{(2a-3b)\coth^3(c+dx)}{3(a+b)}}{2a(a+b)}-\frac{b\coth^3(\cdots}{2a(a+b)\left(a-b\tan\cdots\right.}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{\frac{\frac{2(a+b)^3\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^{5/2}(7a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{a+b}}{a+b}-\frac{(2a-3b)\coth^3(c+dx)}{3(a+b)}}{2a(a+b)}-\frac{b\coth^3(c+dx\cdots}{2a(a+b)\left(a-b\tanh^2(\cdots\right.}}{d}$$

input
```
Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

output
```
((-1/3*((2*a - 3*b)*Coth[c + d*x]^3)/(a + b) + (((2*(a + b)^3*ArcTanh[Tanh
[c + d*x]])/a - (b^(5/2)*(7*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x]])/Sqrt[
a + b]])/(a*Sqrt[a + b]))/(a + b) - ((2*a^2 + 6*a*b - b^2)*Coth[c + d*x])/
(a + b))/(a + b))/(2*a*(a + b)) - (b*Coth[c + d*x]^3)/(2*a*(a + b)*(a + b
- b*Tanh[c + d*x]^2)))/d
```

3.157. $\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.157. $\int \frac{\coth^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.157.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(145) = 290$.

Time = 34.98 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.25

method	result
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+13b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)(a+b)}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{}{24(a+}$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+13b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)(a+b)}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{}{24(a+}$
risch	$\frac{x}{a^2}-\frac{12a^4e^{8dx+8c}+24a^3b\,e^{8dx+8c}-3a\,b^3e^{8dx+8c}-6b^4e^{8dx+8c}+12a^4e^{6dx+6c}+60a^3b\,e^{6dx+6c}+96a^2b^2e^{6dx+6c}+6a\,b^3e^{6}}{3}$

input
```
int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8/(a^2+2*a*b+b^2)/(a+b)*(1/3*tanh(1/2*d*x+1/2*c)^3*a+1/3*tanh(1/2*
d*x+1/2*c)^3*b+5*tanh(1/2*d*x+1/2*c)*a+13*b*tanh(1/2*d*x+1/2*c))+1/a^2*ln(
1+tanh(1/2*d*x+1/2*c))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/24/(a+b)^2/tanh(1
/2*d*x+1/2*c)^3-1/8*(5*a+13*b)/(a+b)^3/tanh(1/2*d*x+1/2*c)+2*b^3/a^2/(a+b)
^3*((-1/2*tanh(1/2*d*x+1/2*c)^3*a-1/2*tanh(1/2*d*x+1/2*c)*a)/(tanh(1/2*d*x
+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)+1/2*(7*a+2*b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/
2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*
b^(1/2)+(a+b)^(1/2)))))
```

3.157.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4786 vs. $2(148) = 296$.

Time = 0.40 (sec) , antiderivative size = 9849, normalized size of antiderivative = 61.17

$$\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.157.6 Sympy [F]

$$\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \int \frac{\coth^4{(c+dx)}}{\left(a+b\,\text{sech}^2{(c+dx)}\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2961 vs. 2(148) = 296.

Time = 0.57 (sec) , antiderivative size = 2961, normalized size of antiderivative = 18.39

$$\int \frac{\coth^4(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x +
 2*c) + a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) - 1/2*b*log(a*e^(4*d*
x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*d) - 1/4*(a^2*b + 3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e
^(-4*d*x - 4*c) + a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) + 1/2*b*log
(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d) + 1/2*(a + 2*b)*log(e^(2*d*x + 2*c) - 1)/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d) + b*log(e^(2*d*x + 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d) - 1/2*(a + 2*b)*log(e^(-2*d*x - 2*c) - 1)/((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d) - b*log(e^(-2*d*x - 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d) - 1/64*(3*a^3*b + 38*a^2*b^2 + 56*a*b^3 + 16*b^4)*log((a*e^(2*d*x
+ 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqr
t((a + b)*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*b)*d) +
 1/16*(3*a*b + 8*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b)
)/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/64*(3*a^3*b + 38*a^2*b^2 + 56*a*b^3 + 1
6*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*sqrt((a + b)*b)*d) - 1/16*(3*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a
 + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a +...
```

3.157.8 Giac [F]

$$\int \frac{\coth^4(c+dx)}{(a+b\text{sech}^2(c+dx))^2}\,dx = \int \frac{\coth(dx+c)^4}{(b\,\text{sech}(dx+c)^2+a)^2}\,dx$$

input
```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx=\int \frac{\cosh(c+d\,x)^4\coth(c+d\,x)^4}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

input
```
int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)
```

output
```
int((cosh(c + d*x)^4*coth(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.158 $\int \frac{\tanh^6(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx = \frac{x}{a^3} - \frac{\sqrt{a+b}\,(3a^2-4ab+8b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d} - \frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^3-1/8*(3*a^2-4*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+
b)^(1/2)/a^3/b^(5/2)/d-1/4*(a+b)*tanh(d*x+c)^3/a/b/d/(a+b-b*tanh(d*x+c)^2)
^2+1/8*(3*a-4*b)*(a+b)*tanh(d*x+c)/a^2/b^2/d/(a+b-b*tanh(d*x+c)^2)
```

3.158.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 754, normalized size of antiderivative = 5.09

$$\int \frac{\tanh^6(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx$$

$$= \frac{(3a^3 - a^2b + 4ab^2 + 8b^3)(a + 2b + a\cosh(2c + 2dx))^3\text{sech}^6(c + dx)\left(\frac{i\arctan\left(\text{sech}(dx)\left(-\frac{i\cosh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4}}\right.\right.}{}\right.}{}$$

$$+ \frac{(a + 2b + a\cosh(2c + 2dx))\text{sech}(2c)\text{sech}^6(c + dx)\,(24a^2b^2dx\cosh(2c) + 64ab^3dx\cosh(2c) + 64b^4dx\cos}{}$$

input
```
Integrate[Tanh[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((3*a^3 - a^2*b + 4*a*b^2 + 8*b^3)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[
c + d*x]^6*(((I/64)*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sq
rt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cos
h[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x
])]*Cosh[2*c])/(a^3*b^2*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - (
(I/64)*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c
] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Si
nh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c]
)/(a^3*b^2*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/(a + b*Sech[c
+ d*x]^2)^3 + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Sech[2*c]*Sech[c + d*x]^6*(
24*a^2*b^2*d*x*Cosh[2*c] + 64*a*b^3*d*x*Cosh[2*c] + 64*b^4*d*x*Cosh[2*c] +
 16*a^2*b^2*d*x*Cosh[2*d*x] + 32*a*b^3*d*x*Cosh[2*d*x] + 16*a^2*b^2*d*x*Co
sh[4*c + 2*d*x] + 32*a*b^3*d*x*Cosh[4*c + 2*d*x] + 4*a^2*b^2*d*x*Cosh[2*c
+ 4*d*x] + 4*a^2*b^2*d*x*Cosh[6*c + 4*d*x] - 9*a^4*Sinh[2*c] - 15*a^3*b*Si
nh[2*c] + 18*a^2*b^2*Sinh[2*c] + 72*a*b^3*Sinh[2*c] + 48*b^4*Sinh[2*c] + 9
*a^4*Sinh[2*d*x] + 13*a^3*b*Sinh[2*d*x] - 28*a^2*b^2*Sinh[2*d*x] - 32*a*b^
3*Sinh[2*d*x] - 3*a^4*Sinh[4*c + 2*d*x] + a^3*b*Sinh[4*c + 2*d*x] + 20*a^2
*b^2*Sinh[4*c + 2*d*x] + 16*a*b^3*Sinh[4*c + 2*d*x] + 3*a^4*Sinh[2*c + 4*d
*x] - 3*a^3*b*Sinh[2*c + 4*d*x] - 6*a^2*b^2*Sinh[2*c + 4*d*x]))/(128*a^3*b
^2*d*(a + b*Sech[c + d*x]^2)^3)
```

3.158. $\int \frac{\tanh^6(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx$

3.158.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 25, 4629, 25, 2075, 372, 440, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ic+idx)^6}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ic+idx)^6}{\left(b\sec(ic+idx)^2+a\right)^3}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\tanh^6(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\tanh^6(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^6(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$-\frac{\frac{(a+b)\tanh^3(c+dx)}{4ab\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\int \frac{\tanh^2(c+dx)\left(3(a+b)-(3a-b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4ab}}{d}$$

$$\Big\downarrow \text{440}$$

$$-\frac{\frac{(a+b)\tanh^3(c+dx)}{4ab\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(3a-4b)(a+b)\tanh(c+dx)}{2ab\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\int \frac{(3a-4b)(a+b)-\left(3a^2-ba+4b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2ab}}{4ab}}{d}$$

$$\Big\downarrow 397$$

$$-\frac{\frac{(a+b)\tanh^3(c+dx)}{4ab\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(3a-4b)(a+b)\tanh(c+dx)}{2ab\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{(a+b)\left(3a^2-4ab+8b^2\right)\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}-\frac{8b^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}}{2ab}}{4ab}}{d}$$

$$\Big\downarrow 219$$

$$-\frac{\frac{(a+b)\tanh^3(c+dx)}{4ab\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(3a-4b)(a+b)\tanh(c+dx)}{2ab\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{(a+b)\left(3a^2-4ab+8b^2\right)\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}-\frac{8b^2\text{arctanh}(\tanh(c+dx))}{a}}{2ab}}{4ab}}{d}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{(a+b)\tanh^3(c+dx)}{4ab\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{(3a-4b)(a+b)\tanh(c+dx)}{2ab\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}-\frac{8b^2\text{arctanh}(\tanh(c+dx))}{a}}{2ab}}{4ab}}{d}$$

input
```
Int[Tanh[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-((((a + b)*Tanh[c + d*x]^3)/(4*a*b*(a + b - b*Tanh[c + d*x]^2)^2) - (-1/2
*((-8*b^2*ArcTanh[Tanh[c + d*x]])/a + (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)
*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[b]))/(a*b) + ((3*a
- 4*b)*(a + b)*Tanh[c + d*x])/(2*a*b*(a + b - b*Tanh[c + d*x]^2)))/(4*a*b)
)/d)
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.158. $\int \frac{\tanh^6(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.158. $\int \frac{\tanh^6(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.158.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(134) = 268$.

Time = 191.71 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.63

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{2\left(\frac{a\left(3a^3+2a^2b-5a\,b^2-4b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8b^2}+\frac{\left(9a^3-14a^2b-19a\,b^2+4b^3\right)a\tanh\left(\right.}{8b^2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{2\left(\frac{a\left(3a^3+2a^2b-5a\,b^2-4b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8b^2}+\frac{\left(9a^3-14a^2b-19a\,b^2+4b^3\right)a\tanh\left(\right.}{8b^2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$
risch	$\frac{x}{a^3}-\frac{3a^4e^{6dx+6c}-a^3b\,e^{6dx+6c}-20a^2b^2e^{6dx+6c}-16a\,b^3e^{6dx+6c}+9a^4e^{4dx+4c}+15a^3b\,e^{4dx+4c}-18a^2b^2e^{4dx+4c}-72a\,b^3}{4a^3d\,b^2\left(a\,e^{4dx+4c}+2\,e^{2dx+2}\right.}$

input
```
int(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^3*ln(1+tanh(1/2*d*x+1/2*c))+2/a^
3*((1/8*a*(3*a^3+2*a^2*b-5*a*b^2-4*b^3)/b^2*tanh(1/2*d*x+1/2*c)^7+1/8*(9*a
^3-14*a^2*b-19*a*b^2+4*b^3)*a/b^2*tanh(1/2*d*x+1/2*c)^5+1/8*(9*a^3-14*a^2*
b-19*a*b^2+4*b^3)*a/b^2*tanh(1/2*d*x+1/2*c)^3+1/8*a*(3*a^3+2*a^2*b-5*a*b^2
-4*b^3)/b^2*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2
*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/8*(3*
a^3-a^2*b+4*a*b^2+8*b^3)/b^2*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh
(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(
a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1
/2)+(a+b)^(1/2)))))
```

3.158.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2604 vs. 2(140) = 280.

Time = 0.34 (sec) , antiderivative size = 5463, normalized size of antiderivative = 36.91

$$\int \frac{\tanh^6(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.158.6 Sympy [F]

$$\int \frac{\tanh^6(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\tanh^6(c+dx)}{(a+b\,\text{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(tanh(d*x+c)**6/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(tanh(c + d*x)**6/(a + b*sech(c + d*x)**2)**3, x)
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3239 vs. 2(140) = 280.

Time = 1.02 (sec) , antiderivative size = 3239, normalized size of antiderivative = 21.89

$$\int \frac{\tanh^6(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-45/1024*(a + 2*b)*a*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))
/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 +
b^4)*sqrt((a + b)*b)*d) - 9/512*a^2*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*s
qrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b
^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 45/1024*(a + 2*b)*a*log((a*e^(-2*
d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 9/512*
a^2*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a +
b)*b)*d) - 1/1024*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4
 + 256*b^5)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*
d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5*b^2 + 2*a^4*b^3 + a^3*b^4
)*sqrt((a + b)*b)*d) + 1/1024*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3
 + 640*a*b^4 + 256*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)
*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*sqrt((a + b)*b)*d) + 5/256*(3*a^2 + 8*a*b + 8*b^2)*log((a*
e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a +
2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) -
1/256*(3*a^6 - 12*a^5*b - 204*a^4*b^2 - 384*a^3*b^3 - 192*a^2*b^4 + (3*a^6
 - 10*a^5*b - 560*a^4*b^2 - 2080*a^3*b^3 - 2560*a^2*b^4 - 1024*a*b^5)*e...
```

3.158.8 Giac [F]

$$\int \frac{\tanh^6(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\tanh(dx+c)^6}{\left(b\,\text{sech}\,(dx+c)^2+a\right)^3}\,dx$$

input
```
integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\left(\cosh(c+d\,x)^2-1\right)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(tanh(c + d*x)^6/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^2 - 1)^3/(b + a*cosh(c + d*x)^2)^3, x)
```

3.159 $\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = -\frac{(a+b)^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{a+b}{a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^3d}$$

output
```
-1/4*(a+b)^2/a^3/d/(b+a*cosh(d*x+c)^2)^2+(a+b)/a^3/d/(b+a*cosh(d*x+c)^2)+1
/2*ln(b+a*cosh(d*x+c)^2)/a^3/d
```

3.159.2 Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{2(a^2+4ab+3b^2)+(a+2b)^2\log(a+2b+a\cosh(2(c+dx)))+a^2\cosh^2(2(c+dx))\log(a+2b+a\cosh(2}{2a^3d(a+2b+a\cosh(2(c+}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]]
+ a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*
(c + d*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*
a^3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

3.159.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)^5}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)^5}{\left(b\sec(ic+idx)^2+a\right)^3}dx$$

$$\Big\downarrow \text{4626}$$

$$\frac{\int \frac{\cosh(c+dx)\left(1-\cosh^2(c+dx)\right)^2}{\left(a\cosh^2(c+dx)+b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^2}{\left(a\cosh^2(c+dx)+b\right)^3}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(a+b)^2}{a^2\left(a\cosh^2(c+dx)+b\right)^3}-\frac{2(a+b)}{a^2\left(a\cosh^2(c+dx)+b\right)^2}+\frac{1}{a^2\left(a\cosh^2(c+dx)+b\right)}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(a+b)^2}{2a^3\left(a\cosh^2(c+dx)+b\right)^2}+\frac{2(a+b)}{a^3\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{a^3}}{2d}$$

input
```
Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/2*(a + b)^2/(a^3*(b + a*Cosh[c + d*x]^2)^2) + (2*(a + b))/(a^3*(b + a*
Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/a^3)/(2*d)
```

3.159.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.159. $\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.159.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(73) = 146$.

Time = 145.71 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.39

method	result
risch	$-\frac{x}{a^3} - \frac{2c}{a^3 d} + \frac{4\,e^{2dx+2c}\left(a^2 e^{4dx+4c}+ab\,e^{4dx+4c}+a^2 e^{2dx+2c}+4ab\,e^{2dx+2c}+3\,e^{2dx+2c}b^2+a^2+ab\right)}{a^3 d\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^2} + \frac{\ln\left(e^{4dx+4c}+\right.}{}$
derivativedivides	$\frac{\frac{\frac{\left(-2a^2-2ab\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6-4(2a-b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a+b)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\right.}{2}}{a^3}}{d}$
default	$\frac{\frac{\frac{\left(-2a^2-2ab\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6-4(2a-b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a+b)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\right.}{2}}{a^3}}{d}$

input
```
int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-x/a^3-2/a^3/d*c+4/a^3*exp(2*d*x+2*c)*(a^2*exp(4*d*x+4*c)+a*b*exp(4*d*x+4*
c)+a^2*exp(2*d*x+2*c)+4*a*b*exp(2*d*x+2*c)+3*exp(2*d*x+2*c)*b^2+a^2+a*b)/d
/(a*exp(4*d*x+4*c)+2*exp(2*d*x+2*c)*a+4*b*exp(2*d*x+2*c)+a)^2+1/2/a^3/d*ln
(exp(4*d*x+4*c)+2*(a+2*b)/a*exp(2*d*x+2*c)+1)
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(73) = 146$.

Time = 0.29 (sec) , antiderivative size = 1741, normalized size of antiderivative = 22.61

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
 + 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x
+ c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a^2 - a*b)*sin
h(d*x + c)^6 + 16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a^2
- a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2
*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3
*a^2 + 8*a*b + 8*b^2)*d*x + 30*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x +
c)^2 - 2*a^2 - 8*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7*a^2*d*x*
cosh(d*x + c)^5 + 10*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3
*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^3 + 8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 8*(7*a^2*d*x
*cosh(d*x + c)^6 + 15*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^4 + (a
^2 + 2*a*b)*d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*
cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^
2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*co
sh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 +
8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 +
30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4
+ 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 ...
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(73) = 146.

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.68

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{4\left((a^2+ab)e^{(-2\,dx-2\,c)}+(a^2+4\,ab+3\,b^2)e^{(-4\,dx-4\,c)}+(a^2+ab)e^{(-6\,dx-6\,c)}\right)}{(a^5e^{(-8\,dx-8\,c)}+a^5+4\,(a^5+2\,a^4b)e^{(-2\,dx-2\,c)}+2\,(3\,a^5+8\,a^4b+8\,a^3b^2)e^{(-4\,dx-4\,c)}+4\,(a^5+2\,a^4b)e^{(-6\,dx-6}} + \frac{dx+c}{a^3d} + \frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^3d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
4*((a^2 + a*b)*e^(-2*d*x - 2*c) + (a^2 + 4*a*b + 3*b^2)*e^(-4*d*x - 4*c) +
 (a^2 + a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a
^4*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c)
+ 4*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(
a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

3.159.8 Giac [F]

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\tanh\left(dx+c\right)^5}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\cosh(c+d\,x)^6\,\tanh(c+d\,x)^5}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*tanh(c + d*x)^5)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.160 $\int \frac{\tanh^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

3.160.1 Optimal result

Integrand size = 23, antiderivative size = 139

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx = \frac{x}{a^3}+\frac{(a^2-4ab-8b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}\,d} - \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^3+1/8*(a^2-4*a*b-8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^3/b
^(3/2)/d/(a+b)^(1/2)-1/4*(a+b)*tanh(d*x+c)/a/b/d/(a+b-b*tanh(d*x+c)^2)^2+1
/8*(a-4*b)*tanh(d*x+c)/a^2/b/d/(a+b-b*tanh(d*x+c)^2)
```

3.160.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1317 vs. 2(139) = 278.

Time = 15.01 (sec) , antiderivative size = 1317, normalized size of antiderivative = 9.47

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))^3\operatorname{sech}^6(c+dx)\left(\frac{6a(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{4(3a^2+8ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+}{\sqrt{a+b}}\right.}{(a+b)^{5/2}}\right.}{}$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((6*a*(a + 2*b)*ArcTanh
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + (4*(3*a^2 + 8*a*b +
 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (4*a
*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[
2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) - (2*Sqrt[b]*(
3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cosh[2*(c
 + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
 + (Sqrt[b]*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4
+ 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x]
 - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cos
h[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (Sech[2
*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh[2*c] + 512*a*b^2*(
a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Cosh[2*(c + 2*d*x)] +
 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*Cosh[2*(c + 2*d*x)]
 + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*Cosh[4*c + 2*d*x]
+ 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh[4*c + 2*d*x] +
128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*c + 4*d*x] + 12
8*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b*Sinh[2*c] + 6
84*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*Sinh[2*c] + 4
608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] - 14*a^5*b...
```

3.160. $\int \frac{\tanh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.160.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 4629, 2075, 372, 402, 25, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(ic+idx)^4}{(a+b\sec(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))(a+b(1-\tanh^2(c+dx)))^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{-\left((a-3b)\tanh^2(c+dx)\right)+a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4ab}-\frac{(a+b)\tanh(c+dx)}{4ab(a-b\tanh^2(c+dx)+b)^2}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{(a-4b)\tanh(c+dx)}{2a\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\int -\frac{(a+b)\left(-\left((a-4b)\tanh^2(c+dx)\right)+a+4b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}}{4ab}-\frac{(a+b)\tanh(c+dx)}{4ab(a-b\tanh^2(c+dx)+b)^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{(a+b)\left(-\left((a-4b)\tanh^2(c+dx)\right)+a+4b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{(a-4b)\tanh(c+dx)}{2a\left(a-b\tanh^2(c+dx)+b\right)}}{4ab}-\frac{(a+b)\tanh(c+dx)}{4ab(a-b\tanh^2(c+dx)+b)^2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{-\left((a-4b)\tanh^2(c+dx)\right)+a+4b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a}+\frac{(a-4b)\tanh(c+dx)}{2a\left(a-b\tanh^2(c+dx)+b\right)}}{4ab}-\frac{(a+b)\tanh(c+dx)}{4ab\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$\big\downarrow$ 397

$$\frac{\frac{\frac{\frac{\left(a^2-4ab-8b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}+\frac{8b\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}}{2a}+\frac{(a-4b)\tanh(c+dx)}{2a\left(a-b\tanh^2(c+dx)+b\right)}}{4ab}-\frac{(a+b)\tanh(c+dx)}{4ab\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$\big\downarrow$ 219

$$\frac{\frac{\frac{\frac{\left(a^2-4ab-8b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}+\frac{8b\,\text{arctanh}(\tanh(c+dx))}{a}}{2a}+\frac{(a-4b)\tanh(c+dx)}{2a\left(a-b\tanh^2(c+dx)+b\right)}}{4ab}-\frac{(a+b)\tanh(c+dx)}{4ab\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$\big\downarrow$ 221

$$\frac{\frac{\frac{\frac{\left(a^2-4ab-8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\sqrt{a+b}}+\frac{8b\,\text{arctanh}(\tanh(c+dx))}{a}}{2a}+\frac{(a-4b)\tanh(c+dx)}{2a\left(a-b\tanh^2(c+dx)+b\right)}}{4ab}-\frac{(a+b)\tanh(c+dx)}{4ab\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/4*((a + b)*Tanh[c + d*x])/(a*b*(a + b - b*Tanh[c + d*x]^2)^2) + (((8*b
*ArcTanh[Tanh[c + d*x]])/a + ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[
c + d*x])/Sqrt[a + b]])/(a*Sqrt[b]*Sqrt[a + b]))/(2*a) + ((a - 4*b)*Tanh[c
 + d*x])/(2*a*(a + b - b*Tanh[c + d*x]^2)))/(4*a*b))/d
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.160. $\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.160.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(125) = 250$.

Time = 64.28 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.47

method	result
derivativedivides	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{2\left(-\frac{a\left(a^2+5ab+4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8b}-\frac{\left(3a^2+19ab-4b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8b}-\frac{\left(3a^2\right.}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\right.\right.}$
default	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{2\left(-\frac{a\left(a^2+5ab+4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8b}-\frac{\left(3a^2+19ab-4b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8b}-\frac{\left(3a^2\right.}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\right.\right.}$
risch	$\frac{x}{a^3}+\frac{a^3e^{6dx+6c}+12a^2b\,e^{6dx+6c}+16a\,b^2e^{6dx+6c}+3a^3e^{4dx+4c}+26a^2b\,e^{4dx+4c}+56a\,b^2e^{4dx+4c}+48\,e^{4dx+4c}b^3+3a^3e^{2dx}}{4a^3d\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^2b}$

input
```
int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^3*ln(1+tanh(1/2*d*x+1/2*c))+2/a^
3*((-1/8*a*(a^2+5*a*b+4*b^2)/b*tanh(1/2*d*x+1/2*c)^7-1/8*(3*a^2+19*a*b-4*b
^2)*a/b*tanh(1/2*d*x+1/2*c)^5-1/8*(3*a^2+19*a*b-4*b^2)*a/b*tanh(1/2*d*x+1/
2*c)^3-1/8*a*(a^2+5*a*b+4*b^2)/b*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)
^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)^2+1/8*(a^2-4*a*b-8*b^2)/b*(1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-1/4/b
^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2
*c)*b^(1/2)+(a+b)^(1/2)))))
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3111 vs. $2(131) = 262$.

Time = 0.34 (sec) , antiderivative size = 6464, normalized size of antiderivative = 46.50

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.160.6 Sympy [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\tanh^4(c+dx)}{(a+b\,\text{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

3.160.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2201 vs. $2(131) = 262$.

Time = 0.63 (sec) , antiderivative size = 2201, normalized size of antiderivative = 15.83

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/256*(a^4 - 20*a^3*b - 120*a^2*b^2 - 160*a*b^3 - 64*b^4)*log((a*e^(2*d*x
+ 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqr
t((a + b)*b)))/((a^5*b + 2*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)*d) + 1/64*(a
 - 2*b)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x
+ 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2*a*b^2 + b^3)*sqrt((a +
b)*b)*d) - 1/256*(a^4 - 20*a^3*b - 120*a^2*b^2 - 160*a*b^3 - 64*b^4)*log((
a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
+ 2*b + 2*sqrt((a + b)*b)))/((a^5*b + 2*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)
*d) - 3/128*(a + 4*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b
))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2*a*b^2 +
 b^3)*sqrt((a + b)*b)*d) - 1/64*(a - 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*
b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))
/((a^2*b + 2*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/64*(a^5 + 38*a^4*b + 88*a
^3*b^2 + 48*a^2*b^3 + (a^5 + 76*a^4*b + 392*a^3*b^2 + 576*a^2*b^3 + 256*a*
b^4)*e^(6*d*x + 6*c) + (3*a^5 + 186*a^4*b + 1024*a^3*b^2 + 2240*a^2*b^3 +
2176*a*b^4 + 768*b^5)*e^(4*d*x + 4*c) + (3*a^5 + 148*a^4*b + 648*a^3*b^2 +
 896*a^2*b^3 + 384*a*b^4)*e^(2*d*x + 2*c))/((a^7*b + 2*a^6*b^2 + a^5*b^3 +
 (a^7*b + 2*a^6*b^2 + a^5*b^3)*e^(8*d*x + 8*c) + 4*(a^7*b + 4*a^6*b^2 + 5*
a^5*b^3 + 2*a^4*b^4)*e^(6*d*x + 6*c) + 2*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^
3 + 24*a^4*b^4 + 8*a^3*b^5)*e^(4*d*x + 4*c) + 4*(a^7*b + 4*a^6*b^2 + 5*...
```

3.160.8 Giac [F]

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\tanh\left(dx+c\right)^4}{\left(b\operatorname{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^6\,\tanh(c+d\,x)^4}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*tanh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.161 $\int \frac{\tanh^3(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

3.161.1 Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx=-\frac{b(a+b)}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{a+2b}{2a^3d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^3d}$$

output
```
-1/4*b*(a+b)/a^3/d/(b+a*cosh(d*x+c)^2)^2+1/2*(a+2*b)/a^3/d/(b+a*cosh(d*x+c
)^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^3/d
```

3.161.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$$
$$=\frac{2(a^2+3ab+3b^2)+(a+2b)^2\log(a+2b+a\cosh(2(c+dx)))+a^2\cosh^2(2(c+dx))\log(a+2b+a\cosh(2}{2a^3d(a+2b+a\cosh(2(c+dx}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]]
+ a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*(a + 2*
b)*Cosh[2*(c + d*x)]*(1 + Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a
 + 2*b + a*Cosh[2*(c + d*x)])^2)
```

3.161.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\tan(ic+idx)^3}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\tan(ic+idx)^3}{\left(b\sec(ic+idx)^2+a\right)^3}dx$$

$$\Big\downarrow \text{ 4626}$$

$$-\frac{\int \frac{\cosh^3(c+dx)\left(1-\cosh^2(c+dx)\right)}{\left(a\cosh^2(c+dx)+b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{\int \frac{\cosh^2(c+dx)\left(1-\cosh^2(c+dx)\right)}{\left(a\cosh^2(c+dx)+b\right)^3}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{\int \left(-\frac{b(a+b)}{a^2\left(a\cosh^2(c+dx)+b\right)^3}-\frac{1}{a^2\left(a\cosh^2(c+dx)+b\right)}+\frac{a+2b}{a^2\left(a\cosh^2(c+dx)+b\right)^2}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{b(a+b)}{2a^3\left(a\cosh^2(c+dx)+b\right)^2}-\frac{a+2b}{a^3\left(a\cosh^2(c+dx)+b\right)}-\frac{\log\left(a\cosh^2(c+dx)+b\right)}{a^3}}{2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/2*((b*(a + b))/(2*a^3*(b + a*Cosh[c + d*x]^2)^2) - (a + 2*b)/(a^3*(b +
a*Cosh[c + d*x]^2)) - Log[b + a*Cosh[c + d*x]^2]/a^3)/d
```

3.161.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.161. $\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.161.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(75) = 150.

Time = 46.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.31

method	result
risch	$-\frac{x}{a^3}-\frac{2c}{a^3 d}+\frac{2\,\mathrm{e}^{2dx+2c}\left(a^2\mathrm{e}^{4dx+4c}+2ab\,\mathrm{e}^{4dx+4c}+2a^2\mathrm{e}^{2dx+2c}+6ab\,\mathrm{e}^{2dx+2c}+6\,\mathrm{e}^{2dx+2c}b^2+a^2+2ab\right)}{a^3 d\left(a\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+4b\,\mathrm{e}^{2dx+2c}+a\right)^2}+\frac{\ln\left(\mathrm{e}^{4dx+4c}\right.}{}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\frac{\left(-2a^2-2ab\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6-\frac{4a\left(a^2+ab-b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a+b}-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a+b)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\right.\right.}{}}{a^3}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\frac{\left(-2a^2-2ab\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6-\frac{4a\left(a^2+ab-b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a+b}-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2(a+b)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\right.\right.}{}}{a^3}}{d}$

input
```
int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-x/a^3-2/a^3/d*c+2/a^3*exp(2*d*x+2*c)*(a^2*exp(4*d*x+4*c)+2*a*b*exp(4*d*x+
4*c)+2*a^2*exp(2*d*x+2*c)+6*a*b*exp(2*d*x+2*c)+6*exp(2*d*x+2*c)*b^2+a^2+2*
a*b)/d/(a*exp(4*d*x+4*c)+2*exp(2*d*x+2*c)*a+4*b*exp(2*d*x+2*c)+a)^2+1/2/a^
3/d*ln(exp(4*d*x+4*c)+2*(a+2*b)/a*exp(2*d*x+2*c)+1)
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1753 vs. 2(75) = 150.

Time = 0.29 (sec) , antiderivative size = 1753, normalized size of antiderivative = 21.64

$$\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
 + 2*a^2*d*x*sinh(d*x + c)^8 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(
d*x + c)^6 + 4*(14*a^2*d*x*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b)*d*x - a^2 - 2
*a*b)*sinh(d*x + c)^6 + 8*(14*a^2*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 + 2*a*b)
*d*x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8
*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*
x + c)^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 15*(2*(a^2 + 2*a*b)*d*x - a^2 - 2
*a*b)*cosh(d*x + c)^2 - 2*a^2 - 6*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x
 + 16*(7*a^2*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*c
osh(d*x + c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cos
h(d*x + c))*sinh(d*x + c)^3 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d
*x + c)^2 + 4*(14*a^2*d*x*cosh(d*x + c)^6 + 15*(2*(a^2 + 2*a*b)*d*x - a^2
- 2*a*b)*cosh(d*x + c)^4 + 2*(a^2 + 2*a*b)*d*x + 6*((3*a^2 + 8*a*b + 8*b^2
)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c
)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sin
h(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2
+ a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b
)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)
^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2
+ 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 +...
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.161.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(75) = 150.

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.58

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \frac{2\left((a^2+2\,ab)e^{(-2\,dx-2\,c)}+2\,(a^2+3\,ab+3\,b^2)e^{(-4\,dx-4\,c)}+(a^2+2\,ab)e^{(-6\,dx-6\,c)}\right)}{\left(a^5e^{(-8\,dx-8\,c)}+a^5+4\,(a^5+2\,a^4b)e^{(-2\,dx-2\,c)}+2\,(3\,a^5+8\,a^4b+8\,a^3b^2)e^{(-4\,dx-4\,c)}+4\,(a^5+2\,a^4b)e^{(-6\,dx-6\,c)}\right)d} + \frac{dx+c}{a^3d}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^3d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
2*((a^2 + 2*a*b)*e^(-2*d*x - 2*c) + 2*(a^2 + 3*a*b + 3*b^2)*e^(-4*d*x - 4*
c) + (a^2 + 2*a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5
 + 2*a^4*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x -
 4*c) + 4*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*l
og(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

3.161.8 Giac [F]

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\tanh(dx+c)^3}{(b\,\text{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)^6\tanh(c+d\,x)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*tanh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.162 $\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 139

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{x}{a^3} - \frac{(3a^2+12ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}(a+b)^{3/2}d} - \frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(3a+4b)\tanh(c+dx)}{8a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^3-1/8*(3*a^2+12*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^
3/(a+b)^(3/2)/d/b^(1/2)-1/4*tanh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*(3
*a+4*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

3.162.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1317 vs. $2(139) = 278$.

Time = 11.78 (sec) , antiderivative size = 1317, normalized size of antiderivative = 9.47

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))^3\text{sech}^6(c+dx)\left(\frac{6a(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{4(3a^2+8ab+8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+\ldots}{\sqrt{a+b}}\right.}{(a+b)^{5/2}}\right.}{\ldots}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((6*a*(a + 2*b)*ArcTanh
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (4*(3*a^2 + 8*a*b +
 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + (4*a
*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[
2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) - (2*Sqrt[b]*(
3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cosh[2*(c
 + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
 + (Sqrt[b]*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4
+ 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x]
 - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cos
h[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (Sech[2
*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh[2*c] + 512*a*b^2*(
a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Cosh[2*(c + 2*d*x)] +
 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*Cosh[2*(c + 2*d*x)]
 + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*Cosh[4*c + 2*d*x]
+ 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh[4*c + 2*d*x] +
128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*c + 4*d*x] + 12
8*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b*Sinh[2*c] + 6
84*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*Sinh[2*c] + 4
608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] - 14*a^5*b...
```

3.162. $\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.162.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 25, 4629, 25, 2075, 373, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ic+idx)^2}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ic+idx)^2}{\left(b\sec(ic+idx)^2+a\right)^3}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{373}$$

$$-\frac{\frac{\tanh(c+dx)}{4a\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\int \frac{3\tanh^2(c+dx)+1}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4a}}{d}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{\tanh(c+dx)}{4a\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{-\frac{\int -\frac{(3a+4b)\tanh^2(c+dx)+5a+4b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{(3a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a}}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\tanh(c+dx)}{4a\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\int \frac{(3a+4b)\tanh^2(c+dx)+5a+4b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{(3a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a}}{d}$$

$$\Big\downarrow \; 397$$

$$-\frac{\frac{\tanh(c+dx)}{4a\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{8(a+b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{(3a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a}}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\tanh(c+dx)}{4a\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{8(a+b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{(3a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a}}{d}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\tanh(c+dx)}{4a\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{8(a+b)\text{arctanh}(\tanh(c+dx))}{a}-\frac{\left(3a^2+12ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\sqrt{a+b}}}{2a(a+b)}-\frac{(3a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a}}{d}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-((Tanh[c + d*x]/(4*a*(a + b - b*Tanh[c + d*x]^2)^2) - (((8*(a + b)*ArcTan
h[Tanh[c + d*x]])/a - ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c +
d*x])/Sqrt[a + b]])/(a*Sqrt[b]*Sqrt[a + b]))/(2*a*(a + b)) - ((3*a + 4*b)*
Tanh[c + d*x])/(2*a*(a + b)*(a + b - b*Tanh[c + d*x]^2)))/(4*a))/d)
```

3.162. $\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.162. $\int \frac{\tanh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.162.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(125) = 250$.

Time = 41.62 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.41

method	result
derivativedivides	$\frac{2\left(\left(-\frac{5}{8}a^2-\frac{1}{2}ab\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7-\frac{a\left(15a^2+15ab-4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)}-\frac{a\left(15a^2+15ab-4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a+b)}+\left(-\frac{5}{8}a^2-\frac{1}{2}ab\right)\tanh\left(\cdots\right)\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$
default	$\frac{2\left(\left(-\frac{5}{8}a^2-\frac{1}{2}ab\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7-\frac{a\left(15a^2+15ab-4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)}-\frac{a\left(15a^2+15ab-4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a+b)}+\left(-\frac{5}{8}a^2-\frac{1}{2}ab\right)\tanh\left(\cdots\right)\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}$
risch	$\frac{x}{a^3}+\frac{5a^3e^{6dx+6c}+20a^2b\,e^{6dx+6c}+16a\,b^2e^{6dx+6c}+15a^3e^{4dx+4c}+58a^2b\,e^{4dx+4c}+88a\,b^2e^{4dx+4c}+48\,e^{4dx+4c}b^3+15a^3e\cdots}{4a^3d(a+b)\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^2}$

input
```
int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^3*(((-5/8*a^2-1/2*a*b)*tanh(1/2*d*x+1/2*c)^7-1/8*a*(15*a^2+15*a*b
-4*b^2)/(a+b)*tanh(1/2*d*x+1/2*c)^5-1/8*a*(15*a^2+15*a*b-4*b^2)/(a+b)*tanh
(1/2*d*x+1/2*c)^3+(-5/8*a^2-1/2*a*b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/
2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+
1/2*c)^2*b+a+b)^2+1/8*(3*a^2+12*a*b+8*b^2)/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/2)
*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^
(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh
(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a
^3*ln(1+tanh(1/2*d*x+1/2*c)))
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3459 vs. $2(131) = 262$.

Time = 0.36 (sec) , antiderivative size = 7158, normalized size of antiderivative = 51.50

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.162.6 Sympy [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\tanh^2(c+dx)}{(a+b\,\text{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

3.162.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. $2(131) = 262$.

Time = 0.43 (sec) , antiderivative size = 1255, normalized size of antiderivative = 9.03

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/64*(3*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*log((a*e^(2*d*x + 2*c) + a +
2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b))
)/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) + 1/64*(3*a^3 + 30*a^2*b +
 40*a*b^2 + 16*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))
/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3
*b^2)*sqrt((a + b)*b)*d) + 1/16*(5*a^4 + 20*a^3*b + 12*a^2*b^2 + (5*a^4 +
66*a^3*b + 128*a^2*b^2 + 64*a*b^3)*e^(6*d*x + 6*c) + (15*a^4 + 164*a^3*b +
 460*a^2*b^2 + 512*a*b^3 + 192*b^4)*e^(4*d*x + 4*c) + (15*a^4 + 118*a^3*b
+ 208*a^2*b^2 + 96*a*b^3)*e^(2*d*x + 2*c))/((a^7 + 2*a^6*b + a^5*b^2 + (a^
7 + 2*a^6*b + a^5*b^2)*e^(8*d*x + 8*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*
a^4*b^3)*e^(6*d*x + 6*c) + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 +
 8*a^3*b^4)*e^(4*d*x + 4*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^
(2*d*x + 2*c))*d) - 1/16*(5*a^4 + 20*a^3*b + 12*a^2*b^2 + (15*a^4 + 118*a^
3*b + 208*a^2*b^2 + 96*a*b^3)*e^(-2*d*x - 2*c) + (15*a^4 + 164*a^3*b + 460
*a^2*b^2 + 512*a*b^3 + 192*b^4)*e^(-4*d*x - 4*c) + (5*a^4 + 66*a^3*b + 128
*a^2*b^2 + 64*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7
+ 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*b
+ 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*b
 + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(
-8*d*x - 8*c))*d) - 1/8*(5*a^3 + 2*a^2*b + (15*a^3 + 32*a^2*b + 8*a*b^2...
```

3.162.8 Giac [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\mathrm{sech}^2(c+dx))^3}\,dx = \int \frac{\tanh\left(dx+c\right)^2}{\left(b\,\mathrm{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx=\int \frac{\cosh(c+d\,x)^4\,\left(\cosh(c+d\,x)^2-1\right)}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^4*(cosh(c + d*x)^2 - 1))/(b + a*cosh(c + d*x)^2)^3, x)
```

3.163 $\int \frac{\tanh(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

3.163.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\tanh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = -\frac{b^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b}{a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^3d}$$

output
```
-1/4*b^2/a^3/d/(b+a*cosh(d*x+c)^2)^2+b/a^3/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+
a*cosh(d*x+c)^2)/a^3/d
```

3.163.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \frac{\tanh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{2b(2a+3b)+(a+2b)^2\log(a+2b+a\cosh(2(c+dx)))+a^2\cosh^2(2(c+dx))\log(a+2b+a\cosh(2(c+ }{2a^3d(a+2b+a\cosh(2(c+dx)))}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a^2*Co
sh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*(c + d*x
)]*(2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a + 2*b
 + a*Cosh[2*(c + d*x)])^2)
```

3.163.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)}{\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)}{\left(b\sec(ic+idx)^2+a\right)^3}dx$$

$$\Big\downarrow \text{4626}$$

$$\frac{\int \frac{\cosh^5(c+dx)}{\left(a\cosh^2(c+dx)+b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{\cosh^4(c+dx)}{\left(a\cosh^2(c+dx)+b\right)^3}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{b^2}{a^2\left(a\cosh^2(c+dx)+b\right)^3}-\frac{2b}{a^2\left(a\cosh^2(c+dx)+b\right)^2}+\frac{1}{a^2\left(a\cosh^2(c+dx)+b\right)}\right)d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2}{2a^3\left(a\cosh^2(c+dx)+b\right)^2}+\frac{2b}{a^3\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{a^3}}{2d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/2*b^2/(a^3*(b + a*Cosh[c + d*x]^2)^2) + (2*b)/(a^3*(b + a*Cosh[c + d*x
]^2)) + Log[b + a*Cosh[c + d*x]^2]/a^3)/(2*d)
```

3.163.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.163. $\int \frac{\tanh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

3.163.4 Maple [A] (verified)

Time = 43.92 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\frac{1}{4da\left(a+b\,\mathrm{sech}(dx+c)^2\right)^2}+\frac{\ln\left(a+b\,\mathrm{sech}(dx+c)^2\right)}{2d\,a^3}-\frac{1}{2d\,a^2\left(a+b\,\mathrm{sech}(dx+c)^2\right)}-\frac{\ln(\mathrm{sech}(dx+c))}{d\,a^3}$	82
default	$-\frac{1}{4da\left(a+b\,\mathrm{sech}(dx+c)^2\right)^2}+\frac{\ln\left(a+b\,\mathrm{sech}(dx+c)^2\right)}{2d\,a^3}-\frac{1}{2d\,a^2\left(a+b\,\mathrm{sech}(dx+c)^2\right)}-\frac{\ln(\mathrm{sech}(dx+c))}{d\,a^3}$	82
risch	$-\frac{x}{a^3}-\frac{2c}{a^3d}+\frac{4\left(a\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+3b\,\mathrm{e}^{2dx+2c}+a\right)\mathrm{e}^{2dx+2c}b}{a^3\left(a\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+4b\,\mathrm{e}^{2dx+2c}+a\right)^2d}+\frac{\ln\left(\mathrm{e}^{4dx+4c}+\frac{2(a+2b)\mathrm{e}^{2dx+2c}}{a}+1\right)}{2a^3d}$	150

input
```
int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a/(a+b*sech(d*x+c)^2)^2+1/2/d/a^3*ln(a+b*sech(d*x+c)^2)-1/2/d/a^2/(
a+b*sech(d*x+c)^2)-1/d/a^3*ln(sech(d*x+c))
```

3.163.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1666 vs. 2(69) = 138.

Time = 0.28 (sec) , antiderivative size = 1666, normalized size of antiderivative = 22.82

$$\int \frac{\tanh(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
 + 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^6
 + 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a*b)*sinh(d*x + c)^6
 + 16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x +
c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh
(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x
+ 30*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2 - 4*a*b - 6*b^2)*sinh(d*x +
 c)^4 + 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b)*d*x
- a*b)*cosh(d*x + c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cos
h(d*x + c))*sinh(d*x + c)^3 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2
+ 8*(7*a^2*d*x*cosh(d*x + c)^6 + 15*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c
)^4 + (a^2 + 2*a*b)*d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*
cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh
(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x
 + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a
^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^
2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7
*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b +
 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^...
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.163.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(69) = 138$.

Time = 0.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.64

$$\int \frac{\tanh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$$
$$= \frac{4\left(abe^{(-2\,dx-2\,c)}+abe^{(-6\,dx-6\,c)}+(2\,ab+3\,b^2)e^{(-4\,dx-4\,c)}\right)}{(a^5e^{(-8\,dx-8\,c)}+a^5+4\,(a^5+2\,a^4b)e^{(-2\,dx-2\,c)}+2\,(3\,a^5+8\,a^4b+8\,a^3b^2)e^{(-4\,dx-4\,c)}+4\,(a^5+2\,a^4b)e^{(-6\,dx-}}$$
$$+\frac{dx+c}{a^3d}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^3d}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
4*(a*b*e^(-2*d*x - 2*c) + a*b*e^(-6*d*x - 6*c) + (2*a*b + 3*b^2)*e^(-4*d*x
 - 4*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4*b)*e^(-2*d*x - 2*c)
 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4*(a^5 + 2*a^4*b)*e^
(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2
*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

3.163.8 Giac [F]

$$\int \frac{\tanh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\tanh\left(dx+c\right)}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.163.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int \frac{\tanh(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^3}\,dx = \frac{\ln\left(\cosh(c+d\,x)^2\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)\right)}{2\,a^3\,d} - \frac{b^2}{4\,a^3\,d\cosh\left(c+d\,x\right)^4\,{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)}^2} + \frac{b}{a^3\,d\cosh\left(c+d\,x\right)^2\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)}$$

input
```
int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
log(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2))/(2*a^3*d) - b^2/(4*a^3*d*cosh
(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2) + b/(a^3*d*cosh(c + d*x)^2*(a + b/c
osh(c + d*x)^2))
```

3.164 $\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.164.1 Optimal result

Integrand size = 14, antiderivative size = 146

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx=\frac{x}{a^3}-\frac{\sqrt{b}(15a^2+20ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d}-\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{b(7a+4b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b
^(1/2)/a^3/(a+b)^(5/2)/d-1/4*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
^2-1/8*b*(7*a+4*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

3.164.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(146) = 292.

Time = 8.31 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.06

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^6(c+dx)\left(8x(a+2b+a\cosh(2(c+dx)))^2-\frac{b(15a^2+20ab+8b^2)\text{arctanh}\left(\text{se}\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^(-3),x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(8*x*(a + 2*b + a*Cosh[2*
(c + d*x)])^2 - (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c]
 - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sq
rt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c]
 - Sinh[2*c]))/((a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - (4*b^2*Se
ch[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/((a + b)*d) + (b*(a + 2*b +
 a*Cosh[2*(c + d*x)])*Sech[2*c]*((9*a^2 + 28*a*b + 16*b^2)*Sinh[2*c] - 3*a
*(3*a + 2*b)*Sinh[2*d*x]))/((a + b)^2*d)))/(64*a^3*(a + b*Sech[c + d*x]^2)
^3)
```

3.164.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4616, 316, 25, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sec(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 316$$

$$\frac{-\frac{\int -\frac{3b\tanh^2(c+dx)+4a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b\tanh(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{3b\tanh^2(c+dx)+4a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b\tanh(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{-\frac{-\frac{\int -\frac{8a^2+9ba+4b^2+b(7a+4b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(7a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a(a+b)}-\frac{b\tanh(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int \frac{8a^2+9ba+4b^2+b(7a+4b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(7a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a(a+b)}-\frac{b\tanh(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a(a+b)}-\frac{b\tanh(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\text{arctanh}(\tanh(c+dx))}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a(a+b)}-\frac{b\tanh(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \ 221$$

3.164. $\int \frac{1}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

$$\frac{\frac{\frac{\frac{8(a+b)^2\text{arctanh}(\tanh(c+dx))}{a}-\frac{\sqrt{b}(15a^2+20ab+8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(7a+4b)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a(a+b)}-\frac{b\tanh(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^(-3),x]
```

output
```
(-1/4*(b*Tanh[c + d*x])/(a*(a + b)*(a + b - b*Tanh[c + d*x]^2)^2) + (((8*(
a + b)^2*ArcTanh[Tanh[c + d*x]])/a - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*Ar
cTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b))
 - (b*(7*a + 4*b)*Tanh[c + d*x])/(2*a*(a + b)*(a + b - b*Tanh[c + d*x]^2))
)/(4*a*(a + b)))/d
```

3.164.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.164.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(132) = 264$.

Time = 1.31 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.41

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{2b\left(\frac{-\frac{a(9a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)}-\frac{a\left(27a^2+11ab-4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2}-\frac{a\left(27a^2+11a\cdots\right.}{\cdots}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\cdots\right.}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{2b\left(\frac{-\frac{a(9a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8(a+b)}-\frac{a\left(27a^2+11ab-4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8(a+b)^2}-\frac{a\left(27a^2+11a\cdots\right.}{\cdots}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\cdots\right.}\right.}{\cdots}}{\cdots}$
risch	$\frac{x}{a^3}+\frac{b\left(9a^3e^{6dx+6c}+28a^2b\,e^{6dx+6c}+16a\,b^2e^{6dx+6c}+27a^3e^{4dx+4c}+90a^2b\,e^{4dx+4c}+120a\,b^2e^{4dx+4c}+48\,e^{4dx+4c}b^3+27\cdots\right.}{4a^3d(a+b)^2\left(a\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a\right)^2}$

input
```
int(1/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^3*ln(1+tanh(1/2*d*x+1/2*c))-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+2/a^3
*b*((-1/8*a*(9*a+4*b)/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/8*a*(27*a^2+11*a*b-4*b
^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-1/8*a*(27*a^2+11*a*b-4*b^2)/(a+b)^2*tanh
(1/2*d*x+1/2*c)^3-1/8*a*(9*a+4*b)/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x
+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2+1/8*(15*a^2+20*a*b+8*b^2)/(a^2+2*a*b+b^2)*(-1/4/b^(1/
2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*
b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1
/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))))
```

3.164.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3131 vs. $2(138) = 276$.

Time = 0.34 (sec) , antiderivative size = 6538, normalized size of antiderivative = 44.78

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.164.6 Sympy [F]

$$\int \frac{1}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{1}{\left(a+b\,\text{sech}^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**(-3), x)
```

3.164.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(138) = 276$.

Time = 0.31 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.75

$$\int \frac{1}{(a+b\text{sech}^2(c+dx))^3}\,dx = \frac{(15\,a^2b+20\,ab^2+8\,b^3)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}d} - \frac{9\,a^3b+6\,a^2b^2+(27\,a^3b+68\,a^2b^2+32\,ab^3)e^{(-2\,dx-2\,c)}+3\,(9\,a^3b+30\,a^2b^2}{4\,(a^7+2\,a^6b+a^5b^2+4\,(a^7+4\,a^6b+5\,a^5b^2+2\,a^4b^3)e^{(-2\,dx-2\,c)}+2\,(3\,a^7+14\,a^6b+27\,a^5b^2+24\,a^4b^3+} + \frac{dx+c}{a^3d}$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/16*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*s
qrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5
+ 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) - 1/4*(9*a^3*b + 6*a^2*b^2 + (27*a
^3*b + 68*a^2*b^2 + 32*a*b^3)*e^(-2*d*x - 2*c) + 3*(9*a^3*b + 30*a^2*b^2 +
 40*a*b^3 + 16*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b + 28*a^2*b^2 + 16*a*b^3)*e
^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2
+ 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*
b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b
^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(-8*d*x - 8*c))*d) + (d
*x + c)/(a^3*d)
```

3.164.8 Giac [F]

$$\int \frac{1}{\left(a+b\mathrm{sech}^2(c+dx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\mathrm{sech}^2(c+dx)\right)^3}\,dx=\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^3}\,dx$$

input
```
int(1/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int(1/(a + b/cosh(c + d*x)^2)^3, x)
```

3.165 $\int \frac{\coth(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

3.165.1 Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{\coth(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx = -\frac{b^3}{4a^3(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{b(3a^2+3ab+b^2)\log\left(b+a\cosh^2(c+dx)\right)}{2a^3(a+b)^3d} + \frac{\log(\sinh(c+dx))}{(a+b)^3d}$$

output
```
-1/4*b^3/a^3/(a+b)/d/(b+a*cosh(d*x+c)^2)^2+1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/d
/(b+a*cosh(d*x+c)^2)+1/2*b*(3*a^2+3*a*b+b^2)*ln(b+a*cosh(d*x+c)^2)/a^3/(a+
b)^3/d+ln(sinh(d*x+c))/(a+b)^3/d
```

3.165.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.19

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$$
$$= \frac{(a+2b+a\cosh(2(c+dx)))^3\text{sech}^6(c+dx)\left(4\log(\sinh(c+dx))+\frac{2b(3a^2+3ab+b^2)\log\left(a+b+a\sinh^2(c+dx)\right)}{a^3}-\frac{\cdots}{a^3(a\cdots}\right.}{32(a+b)^3d\left(a+b\text{sech}^2(c+dx)\right)^3}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(4*Log[Sinh[c + d*x]] +
 (2*b*(3*a^2 + 3*a*b + b^2)*Log[a + b + a*Sinh[c + d*x]^2])/a^3 - (b^3*(a
+ b)^2)/(a^3*(a + b + a*Sinh[c + d*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/
(a^3*(a + b + a*Sinh[c + d*x]^2))))/(32*(a + b)^3*d*(a + b*Sech[c + d*x]^2
)^3)
```

3.165.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{i}{\tan(ic+idx)\left(a+b\sec(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \; 26$$
$$i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^3\tan(ic+idx)}dx$$
$$\Big\downarrow \; 4626$$

$$-\frac{\int \frac{\cosh^7(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cosh^6(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(a\cosh^2(c+dx)+b\right)^3}d\cosh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(-\frac{b^3}{a^2(a+b)\left(a\cosh^2(c+dx)+b\right)^3}+\frac{(3a+2b)b^2}{a^2(a+b)^2\left(a\cosh^2(c+dx)+b\right)^2}-\frac{(3a^2+3ba+b^2)b}{a^2(a+b)^3\left(a\cosh^2(c+dx)+b\right)}-\frac{1}{(a+b)^3\left(\cosh^2(c+dx)-1\right)}\right)d\cosh}{2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^3}{2a^3(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{b^2(3a+2b)}{a^3(a+b)^2\left(a\cosh^2(c+dx)+b\right)}-\frac{b(3a^2+3ab+b^2)\log\left(a\cosh^2(c+dx)+b\right)}{a^3(a+b)^3}-\frac{\log\left(1-\cosh^2(c+dx)\right)}{(a+b)^3}}{2d}$$

input
```
Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/2*(b^3/(2*a^3*(a + b)*(b + a*Cosh[c + d*x]^2)^2) - (b^2*(3*a + 2*b))/(a
^3*(a + b)^2*(b + a*Cosh[c + d*x]^2)) - Log[1 - Cosh[c + d*x]^2]/(a + b)^3
 - (b*(3*a^2 + 3*a*b + b^2)*Log[b + a*Cosh[c + d*x]^2])/(a^3*(a + b)^3))/d
```

3.165.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

3.165. $\int \frac{\coth(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.165.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(124) = 248$.

Time = 64.56 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.25

method	result
derivativedivides	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a+b)^3}+\frac{b\left(\frac{\left(-6a^3b-8a^2b^2-2a\,b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6-4\left(3a^2-ab-b^2\right)a}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\right.}\right.}{}$
default	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a+b)^3}+\frac{b\left(\frac{\left(-6a^3b-8a^2b^2-2a\,b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6-4\left(3a^2-ab-b^2\right)a}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4b+2\tanh\right.}\right.}{}$
risch	$\frac{x}{a^3}-\frac{2x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6bx}{a(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6bc}{ad(a^3+3a^2b+3a\,b^2+b^3)}-\frac{}{a^2(}$

input
```
int(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/a^3*ln(1+tanh(1/2*d*x+1/2*c))+1/(a
+b)^3*ln(tanh(1/2*d*x+1/2*c))+b/a^3/(a+b)^3*(((-6*a^3*b-8*a^2*b^2-2*a*b^3)
*tanh(1/2*d*x+1/2*c)^6-4*(3*a^2-a*b-b^2)*a*b*tanh(1/2*d*x+1/2*c)^4-2*(3*a^
2+4*a*b+b^2)*a*b*tanh(1/2*d*x+1/2*c)^2)/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*
d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+
1/2*(3*a^2+3*a*b+b^2)*ln(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2
*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)))
```

3.165.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4132 vs. $2(124) = 248$.

Time = 0.50 (sec) , antiderivative size = 4132, normalized size of antiderivative = 31.78

$$\int \frac{\coth(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^
5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 +
5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*
d*x)*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 14*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 - 2*(a^5 + 5*a^4*b + 9*a^3*
b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(14*(a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b
^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^5 - 4*(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5
 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x +
c)^4 + 4*(35*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 6
*a^3*b^2 - 20*a^2*b^3 - 20*a*b^4 - 6*b^5 + (3*a^5 + 17*a^4*b + 41*a^3*b^2
+ 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x - 15*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4
 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^
2)*sinh(d*x + c)^4 + 16*(7*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(
d*x + c)^5 - 5*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3
*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 - (6*a^3*b^2 + 20*a^2*b^3
 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b
^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^5 + 3*a^4*b + 3*...
```

3.165.6 Sympy [F]

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\coth\left(c+dx\right)}{\left(a+b\,\text{sech}^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

3.165.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(124) = 248$.

Time = 0.23 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.22

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$$
$$= \frac{\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)d}$$
$$+ \frac{2\,\left(\left(3\,a^2b^2+2\,ab^3\right)e^{(-2\,dx-2\,c)}+2\,\left(3\,a^2b^2+7\,ab^3+3\right.\right.}{\left(a^7+2\,a^6b+a^5b^2+4\,\left(a^7+4\,a^6b+5\,a^5b^2+2\,a^4b^3\right)e^{(-2\,dx-2\,c)}+2\,\left(3\,a^7+14\,a^6b+27\,a^5b^2+24\,a^4b^3+8\right.\right.}$$
$$+ \frac{\log\left(e^{(-dx-c)}+1\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d} + \frac{dx+c}{a^3d}$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d
*x - 4*c) + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 2*((3*a^2*b^2 +
 2*a*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4
*c) + (3*a^2*b^2 + 2*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 +
4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14
*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 +
4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b
^2)*e^(-8*d*x - 8*c))*d) + log(e^(-d*x - c) + 1)/((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d) + log(e^(-d*x - c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (
d*x + c)/(a^3*d)
```

3.165.8 Giac [F]

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\coth(dx+c)}{(b\,\text{sech}(dx+c)^2+a)^3}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)^6\coth(c+d\,x)}{(a\cosh(c+d\,x)^2+b)^3}\,dx$$

input
```
int(coth(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*coth(c + d*x))/(b + a*cosh(c + d*x)^2)^3, x)
```

3.166 $\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.166.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{x}{a^3} - \frac{b^{3/2}(35a^2+28ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}d} - \frac{(8a^2-11ab-4b^2)\coth(c+dx)}{8a^2(a+b)^3d} - \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(9a+4b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^3-1/8*b^(3/2)*(35*a^2+28*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^
(1/2))/a^3/(a+b)^(7/2)/d-1/8*(8*a^2-11*a*b-4*b^2)*coth(d*x+c)/a^2/(a+b)^3/
d-1/4*b*coth(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(9*a+4*b)*coth
(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

3.166.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.75 (sec) , antiderivative size = 2083, normalized size of antiderivative = 11.45

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]
^6*(((I/64)*b^2*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b
*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*
c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*
Cosh[2*c])/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I/64)*b
^2*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] -
b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4
*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a
^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*Sech
[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]*S
ech[2*c]*Sech[c + d*x]^6*(8*a^5*d*x*Cosh[d*x] + 56*a^4*b*d*x*Cosh[d*x] + 1
84*a^3*b^2*d*x*Cosh[d*x] + 296*a^2*b^3*d*x*Cosh[d*x] + 224*a*b^4*d*x*Cosh[
d*x] + 64*b^5*d*x*Cosh[d*x] - 12*a^5*d*x*Cosh[3*d*x] - 68*a^4*b*d*x*Cosh[3
*d*x] - 132*a^3*b^2*d*x*Cosh[3*d*x] - 108*a^2*b^3*d*x*Cosh[3*d*x] - 32*a*b
^4*d*x*Cosh[3*d*x] - 8*a^5*d*x*Cosh[2*c - d*x] - 56*a^4*b*d*x*Cosh[2*c - d
*x] - 184*a^3*b^2*d*x*Cosh[2*c - d*x] - 296*a^2*b^3*d*x*Cosh[2*c - d*x] -
224*a*b^4*d*x*Cosh[2*c - d*x] - 64*b^5*d*x*Cosh[2*c - d*x] - 8*a^5*d*x*Cos
h[2*c + d*x] - 56*a^4*b*d*x*Cosh[2*c + d*x] - 184*a^3*b^2*d*x*Cosh[2*c + d
*x] - 296*a^2*b^3*d*x*Cosh[2*c + d*x] - 224*a*b^4*d*x*Cosh[2*c + d*x] - 64
*b^5*d*x*Cosh[2*c + d*x] + 8*a^5*d*x*Cosh[4*c + d*x] + 56*a^4*b*d*x*Cos...
```

3.166.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.17, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 25, 4629, 25, 2075, 374, 25, 441, 25, 445, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ic+idx)^2\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^3\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\frac{\int -\frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{374}$$

$$-\frac{\frac{\int -\frac{\coth^2(c+dx)\left(5b\tanh^2(c+dx)+4a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4a(a+b)}+\frac{b\coth(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{b\coth(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\int \frac{\coth^2(c+dx)\left(5b\tanh^2(c+dx)+4a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{441}$$

$$-\frac{\frac{b\coth(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{-\frac{\int -\frac{\coth^2(c+dx)\left(8a^2-11ba-4b^2+3b(9a+4b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(9a+4b)\coth(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{25}$$

3.166. $\int \frac{\coth^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$

$$-\frac{\frac{b\coth(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\int \frac{\coth^2(c+dx)\left(8a^2-11ba-4b^2+3b(9a+4b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(9a+4b)\coth(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

$\downarrow$ 445

$$-\frac{\frac{b\coth(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{-\frac{\int -\frac{8a^3+32ba^2+13b^2a+4b^3-b\left(8a^2-11ba-4b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{a+b}}{2a(a+b)}-\frac{b(}{2a(a+}}{4a(a+b)}}{d}$$

$\downarrow$ 25

$$-\frac{\frac{b\coth(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\int \frac{8a^3+32ba^2+13b^2a+4b^3-b\left(8a^2-11ba-4b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{a+b}}{2a(a+b)}-\frac{b(9a-}{2a(a+b)(}}{4a(a+b)}}{d}$$

$\downarrow$ 397

$$-\frac{\frac{b\coth(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\frac{8(a+b)^3\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b^2\left(35a^2+28ab+8b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{\left(8a^2-11ab-4\right.}{a}}{2a(a+b)}}{4a(a+b)}}{d}$$

$\downarrow$ 219

$$-\frac{\frac{b\coth(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\frac{8(a+b)^3\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^2\left(35a^2+28ab+8b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{\left(8a^2-11ab-4b^2\right)\text{cot}}{a+b}}{2a(a+b)}}{4a(a+b)}}{d}$$

$\downarrow$ 221

$$-\frac{\frac{b\coth(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\frac{8(a+b)^3\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{\left(8a^2-11ab-4b^2\right)\coth(c+}{a+b}}{2a(a+b)}}{4a(a+b)}}{d}$$

3.166. $\displaystyle\int \frac{\coth^2(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

input
```
Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-(((b*Coth[c + d*x])/(4*a*(a + b)*(a + b - b*Tanh[c + d*x]^2)^2) - ((((8*(
a + b)^3*ArcTanh[Tanh[c + d*x]])/a - (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*Ar
cTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b) - ((8
*a^2 - 11*a*b - 4*b^2)*Coth[c + d*x])/(a + b))/(2*a*(a + b)) - (b*(9*a + 4
*b)*Coth[c + d*x])/(2*a*(a + b)*(a + b - b*Tanh[c + d*x]^2)))/(4*a*(a + b)
))/d)
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.166.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(166) = 332$.

Time = 136.06 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.16

method	result
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}-\frac{1}{2(a+b)^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^2\left(\frac{\left(-\frac{13}{8}a^3-\frac{17}{8}a^2b-\frac{1}{2}a\,b^2\right.}{}\right.}{}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}-\frac{1}{2(a+b)^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^2\left(\frac{\left(-\frac{13}{8}a^3-\frac{17}{8}a^2b-\frac{1}{2}a\,b^2\right.}{}\right.}{}$
risch	$\frac{x}{a^3}-\frac{8a^5e^{8dx+8c}-13a^3b^2e^{8dx+8c}-36a^2b^3e^{8dx+8c}-16a\,b^4e^{8dx+8c}+32a^5e^{6dx+6c}+64a^4b\,e^{6dx+6c}-26a^3b^2e^{6dx+6c}-86}{}$

input
```
int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)+1/a^3*ln(1+tanh(1/
2*d*x+1/2*c))-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/(a+b)^3/tanh(1/2*d*x+1/2
*c)+2*b^2/(a+b)^3/a^3*(((-13/8*a^3-17/8*a^2*b-1/2*a*b^2)*tanh(1/2*d*x+1/2*
c)^7-1/8*(39*a^2+7*a*b-4*b^2)*a*tanh(1/2*d*x+1/2*c)^5-1/8*(39*a^2+7*a*b-4*
b^2)*a*tanh(1/2*d*x+1/2*c)^3+(-13/8*a^3-17/8*a^2*b-1/2*a*b^2)*tanh(1/2*d*x
+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1
/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/8*(35*a^2+28*a*b+8*b^2)*(-1/4
/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1
/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2
*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))))
```

3.166.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5665 vs. $2(172) = 344$.

Time = 0.39 (sec) , antiderivative size = 11606, normalized size of antiderivative = 63.77

$$\int \frac{\coth^2(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.166.6 Sympy [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\coth^2(c+dx)}{(a+b\,\text{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

3.166.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1971 vs. 2(172) = 344.

Time = 0.46 (sec) , antiderivative size = 1971, normalized size of antiderivative = 10.83

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x
 + 2*c) + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) - 1/4*(3*a^2*b + 3*
a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((
a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) - 1/64*(15*a^3*b + 70*a^2*b^2 + 56
*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*
e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 3*a^5*b + 3*a^4*b^
2 + a^3*b^3)*sqrt((a + b)*b)*d) + 1/64*(15*a^3*b + 70*a^2*b^2 + 56*a*b^3 +
 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d
*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^
3*b^3)*sqrt((a + b)*b)*d) - 15/32*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*
sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/16*(8*a^5 + 9*a^4*b + 2
8*a^3*b^2 + 12*a^2*b^3 + (8*a^5 - 9*a^4*b - 98*a^3*b^2 - 160*a^2*b^3 - 64*
a*b^4)*e^(8*d*x + 8*c) + 2*(16*a^5 + 23*a^4*b - 77*a^3*b^2 - 246*a^2*b^3 -
 288*a*b^4 - 96*b^5)*e^(6*d*x + 6*c) + 2*(24*a^5 + 64*a^4*b + 99*a^3*b^2 +
 190*a^2*b^3 + 272*a*b^4 + 96*b^5)*e^(4*d*x + 4*c) + 2*(16*a^5 + 41*a^4*b
+ 77*a^3*b^2 + 130*a^2*b^3 + 48*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 3*a^7*b +
3*a^6*b^2 + a^5*b^3 - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(10*d*x + 10
*c) - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^(8*d*x +
8*c) - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*...
```

3.166.8 Giac [F]

$$\int \frac{\coth^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx=\int \frac{\coth\left(dx+c\right)^2}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^6\coth(c+d\,x)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*coth(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.167 $\int \frac{\coth^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

3.167.1 Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{\coth^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = -\frac{b^4}{4a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b^3(2a+b)}{a^3(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{csch}^2(c+dx)}{2(a+b)^3d} + \frac{b^2(6a^2+4ab+b^2)\log\left(b+a\cosh^2(c+dx)\right)}{2a^3(a+b)^4d} + \frac{(a+4b)\log(\sinh(c+dx))}{(a+b)^4d}$$

output
```
-1/4*b^4/a^3/(a+b)^2/d/(b+a*cosh(d*x+c)^2)^2+b^3*(2*a+b)/a^3/(a+b)^3/d/(b+
a*cosh(d*x+c)^2)-1/2*csch(d*x+c)^2/(a+b)^3/d+1/2*b^2*(6*a^2+4*a*b+b^2)*ln(
b+a*cosh(d*x+c)^2)/a^3/(a+b)^4/d+(a+4*b)*ln(sinh(d*x+c))/(a+b)^4/d
```

3.167.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.13

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx =$$
$$-\frac{(a+2b+a\cosh(2(c+dx)))^3\text{sech}^6(c+dx)\left(2(a+b)\text{csch}^2(c+dx)-4(a+4b)\log(\sinh(c+dx))-\frac{2b^2}{}\right.}{32(a+b)^4d\left(a+b\text{sech}^2(c+dx\right.}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
-1/32*((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(2*(a + b)*Csch[c
 + d*x]^2 - 4*(a + 4*b)*Log[Sinh[c + d*x]] - (2*b^2*(6*a^2 + 4*a*b + b^2)*
Log[a + b + a*Sinh[c + d*x]^2])/a^3 + (b^4*(a + b)^2)/(a^3*(a + b + a*Sinh
[c + d*x]^2)^2) - (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b + a*Sinh[c + d*x]^
2))))/((a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

3.167.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i}{\tan(ic+idx)^3\left(a+b\sec(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \frac{1}{\left(b\sec(ic+idx)^2+a\right)^3\tan(ic+idx)^3}dx$$
$$\Big\downarrow \text{ 4626}$$

$$\frac{\int \frac{\cosh^9(c+dx)}{(1-\cosh^2(c+dx))^2(a\cosh^2(c+dx)+b)^3}d\cosh(c+dx)}{d}$$

$\downarrow$ 354

$$\frac{\int \frac{\cosh^8(c+dx)}{(1-\cosh^2(c+dx))^2(a\cosh^2(c+dx)+b)^3}d\cosh^2(c+dx)}{2d}$$

$\downarrow$ 99

$$\frac{\int \left(\frac{b^4}{a^2(a+b)^2(a\cosh^2(c+dx)+b)^3} - \frac{2(2a+b)b^3}{a^2(a+b)^3(a\cosh^2(c+dx)+b)^2} + \frac{(6a^2+4ba+b^2)b^2}{a^2(a+b)^4(a\cosh^2(c+dx)+b)} + \frac{a+4b}{(a+b)^4(\cosh^2(c+dx)-1)} + \frac{}{(a+b)^3(c}\right.}{2d}$$

$\downarrow$ 2009

$$\frac{-\frac{b^4}{2a^3(a+b)^2(a\cosh^2(c+dx)+b)^2} + \frac{2b^3(2a+b)}{a^3(a+b)^3(a\cosh^2(c+dx)+b)} + \frac{b^2(6a^2+4ab+b^2)\log(a\cosh^2(c+dx)+b)}{a^3(a+b)^4} + \frac{1}{(a+b)^3(1-\cosh^2(c+dx))} + \frac{(}{}}{2d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(1/((a + b)^3*(1 - Cosh[c + d*x]^2)) - b^4/(2*a^3*(a + b)^2*(b + a*Cosh[c
+ d*x]^2)^2) + (2*b^3*(2*a + b))/(a^3*(a + b)^3*(b + a*Cosh[c + d*x]^2)) +
 ((a + 4*b)*Log[1 - Cosh[c + d*x]^2])/(a + b)^4 + (b^2*(6*a^2 + 4*a*b + b^
2)*Log[b + a*Cosh[c + d*x]^2])/(a^3*(a + b)^4))/(2*d)
```

3.167.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

3.167. $\int \frac{\coth^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.167.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(146) = 292$.

Time = 191.06 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.33

method	result
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{1}{8(a+b)^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(4a+16b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^4}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{b^2\left(\frac{\left(-8a^3b-10a^2b^2-\cdots\right.}{\left(\mathrm{ta}\cdots\right.}\right.}{\cdots}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{1}{8(a+b)^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(4a+16b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^4}-\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{b^2\left(\frac{\left(-8a^3b-10a^2b^2-\cdots\right.}{\left(\mathrm{ta}\cdots\right.}\right.}{\cdots}$
risch	Expression too large to display

input
```
int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*tanh(1/2*d*x+1/2*c)^2/(a^3+3*a^2*b+3*a*b^2+b^3)-1/8/(a+b)^3/tanh
(1/2*d*x+1/2*c)^2+1/4/(a+b)^4*(4*a+16*b)*ln(tanh(1/2*d*x+1/2*c))-1/a^3*ln(
1+tanh(1/2*d*x+1/2*c))+b^2/(a+b)^4/a^3*(((-8*a^3*b-10*a^2*b^2-2*a*b^3)*tan
h(1/2*d*x+1/2*c)^6-4*(4*a^2-2*a*b-b^2)*a*b*tanh(1/2*d*x+1/2*c)^4-2*(4*a^2+
5*a*b+b^2)*a*b*tanh(1/2*d*x+1/2*c)^2)/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*
x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/
2*(6*a^2+4*a*b+b^2)*ln(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*t
anh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b))-1/a^3*ln(tanh(1/2*d
*x+1/2*c)-1))
```

3.167.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10255 vs. $2(146) = 292$.

Time = 0.80 (sec) , antiderivative size = 10255, normalized size of antiderivative = 67.47

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.167.6 Sympy [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \int \frac{\coth^3(c+dx)}{(a+b\,\text{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

3.167.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(146) = 292$.

Time = 0.27 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.55

$$\int \frac{\coth^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$
$$= \frac{(6\,a^2b^2+4\,ab^3+b^4)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)d}$$
$$+\frac{(a+4\,b)\log\left(e^{(-dx-c)}+1\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d}+\frac{(a+4\,b)\log\left(e^{(-dx-c)}-1\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d}$$
$$-\frac{2\left((a^5-4\,a^2b^3-2\,ab^4\right.}{(a^8+3\,a^7b+3\,a^6b^2+a^5b^3+2\,(a^8+7\,a^7b+15\,a^6b^2+13\,a^5b^3+4\,a^4b^4)e^{(-2\,dx-2\,c)}-(a^8+3\,a^7b-13\,a^6}$$
$$+\frac{dx+c}{a^3d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4
*d*x - 4*c) + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) + (
a + 4*b)*log(e^(-d*x - c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
)*d) + (a + 4*b)*log(e^(-d*x - c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*d) - 2*((a^5 - 4*a^2*b^3 - 2*a*b^4)*e^(-2*d*x - 2*c) + 2*(2*a^5
+ 4*a^4*b - 7*a*b^4 - 3*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3
*b^2 + 4*a^2*b^3 + 16*a*b^4 + 6*b^5)*e^(-6*d*x - 6*c) + 2*(2*a^5 + 4*a^4*b
 - 7*a*b^4 - 3*b^5)*e^(-8*d*x - 8*c) + (a^5 - 4*a^2*b^3 - 2*a*b^4)*e^(-10*
d*x - 10*c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^8 + 7*a^7*b + 15
*a^6*b^2 + 13*a^5*b^3 + 4*a^4*b^4)*e^(-2*d*x - 2*c) - (a^8 + 3*a^7*b - 13*
a^6*b^2 - 47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)*e^(-4*d*x - 4*c) - 4*(a^8
+ 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(-6*d*x -
6*c) - (a^8 + 3*a^7*b - 13*a^6*b^2 - 47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)
*e^(-8*d*x - 8*c) + 2*(a^8 + 7*a^7*b + 15*a^6*b^2 + 13*a^5*b^3 + 4*a^4*b^4
)*e^(-10*d*x - 10*c) + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(-12*d*x -
12*c))*d) + (d*x + c)/(a^3*d)
```

3.167.8 Giac [F]

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\coth\left(dx+c\right)^3}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^6\coth(c+d\,x)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*coth(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.168 $\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

3.168.1 Optimal result

Integrand size = 23, antiderivative size = 232

$$\begin{aligned}\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx &= \frac{x}{a^3}-\frac{b^{5/2}(63a^2+36ab+8b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}d}\\ &\quad-\frac{(8a^3+32a^2b-15ab^2-4b^3)\coth(c+dx)}{8a^2(a+b)^4d}\\ &\quad-\frac{(8a^2-39ab-12b^2)\coth^3(c+dx)}{24a^2(a+b)^3d}\\ &\quad-\frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}\\ &\quad-\frac{b(11a+4b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}\end{aligned}$$

output
```
x/a^3-1/8*b^(5/2)*(63*a^2+36*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^
(1/2))/a^3/(a+b)^(9/2)/d-1/8*(8*a^3+32*a^2*b-15*a*b^2-4*b^3)*coth(d*x+c)/a
^2/(a+b)^4/d-1/24*(8*a^2-39*a*b-12*b^2)*coth(d*x+c)^3/a^2/(a+b)^3/d-1/4*b*
coth(d*x+c)^3/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(11*a+4*b)*coth(d*x+
c)^3/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

3.168.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.50 (sec) , antiderivative size = 3334, normalized size of antiderivative = 14.37

$$\int \frac{\coth^4(c+dx)}{(a+b\text{sech}^2(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]
^6*(((I/64)*b^3*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b
*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*
c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*
Cosh[2*c])/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I/64)*b
^3*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] -
b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4
*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a
^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^4*(a + b*Sech
[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]^3
*Sech[2*c]*Sech[c + d*x]^6*(-36*a^6*d*x*Cosh[d*x] - 336*a^5*b*d*x*Cosh[d*x
] - 1560*a^4*b^2*d*x*Cosh[d*x] - 3600*a^3*b^3*d*x*Cosh[d*x] - 4260*a^2*b^4
*d*x*Cosh[d*x] - 2496*a*b^5*d*x*Cosh[d*x] - 576*b^6*d*x*Cosh[d*x] + 36*a^6
*d*x*Cosh[3*d*x] + 240*a^5*b*d*x*Cosh[3*d*x] + 408*a^4*b^2*d*x*Cosh[3*d*x]
 - 48*a^3*b^3*d*x*Cosh[3*d*x] - 732*a^2*b^4*d*x*Cosh[3*d*x] - 672*a*b^5*d*
x*Cosh[3*d*x] - 192*b^6*d*x*Cosh[3*d*x] + 36*a^6*d*x*Cosh[2*c - d*x] + 336
*a^5*b*d*x*Cosh[2*c - d*x] + 1560*a^4*b^2*d*x*Cosh[2*c - d*x] + 3600*a^3*b
^3*d*x*Cosh[2*c - d*x] + 4260*a^2*b^4*d*x*Cosh[2*c - d*x] + 2496*a*b^5*d*x
*Cosh[2*c - d*x] + 576*b^6*d*x*Cosh[2*c - d*x] + 36*a^6*d*x*Cosh[2*c + d*x
] + 336*a^5*b*d*x*Cosh[2*c + d*x] + 1560*a^4*b^2*d*x*Cosh[2*c + d*x] + ...
```

3.168.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.13, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4629, 2075, 374, 25, 441, 25, 445, 27, 445, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\tan(ic+idx)^4\left(a+b\sec(ic+idx)^2\right)^3}dx$$

$\Big\downarrow$ 4629

$$\frac{\int \frac{\coth^4(c+dx)}{(1-\tanh^2(c+dx))\left(a+b\left(1-\tanh^2(c+dx)\right)\right)^3}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 2075

$$\frac{\int \frac{\coth^4(c+dx)}{(1-\tanh^2(c+dx))\left(-b\tanh^2(c+dx)+a+b\right)^3}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 374

$$\frac{-\frac{\int -\frac{\coth^4(c+dx)\left(7b\tanh^2(c+dx)+4a-3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b\coth^3(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\coth^4(c+dx)\left(7b\tanh^2(c+dx)+4a-3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b\coth^3(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$\Big\downarrow$ 441

$$\frac{\frac{-\frac{\int -\frac{\coth^4(c+dx)\left(8a^2-39ba-12b^2+5b(11a+4b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(11a+4b)\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a(a+b)}-\frac{b\coth^3(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{\coth^4(c+dx)\left(8a^2-39ba-12b^2+5b(11a+4b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(11a+4b)\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}}{4a(a+b)}-\frac{b\coth^3(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{d}$$

$\Big\downarrow$ 445

3.168. $\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}dx$

$$\frac{\frac{-\frac{-\frac{\int -\frac{3\coth^2(c+dx)\left(8a^3+32ba^2-15b^2a-4b^3-b\left(8a^2-39ba-12b^2\right)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{3(a+b)}-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{3(a+b)}}{2a(a+b)}-\frac{b(11a+4b)\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\cdots\right.}\cdots}{4a(a+b)}\cdots}{d}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{\int \frac{\coth^2(c+dx)\left(8a^3+32ba^2-15b^2a-4b^3-b\left(8a^2-39ba-12b^2\right)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{3(a+b)}}{2a(a+b)}-\frac{b(11a+4b)\coth^3(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\cdots}{4a(a+b)}\cdots}{d}$$

$\downarrow$ 445

$$\frac{\frac{\frac{\frac{-\frac{\int -\frac{8a^4+40ba^3+80b^2a^2+17b^3a+4b^4-b\left(8a^3+32ba^2-15b^2a-4b^3\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\coth(c+dx)}{a+b}}{a+b}-\frac{\left(8a^2-39ab-12b^2\right)\coth\cdots}{3(a+b)}\cdots}{2a(a+b)}\cdots}{4a(a+b)}\cdots}{d}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\frac{\frac{\int \frac{8a^4+40ba^3+80b^2a^2+17b^3a+4b^4-b\left(8a^3+32ba^2-15b^2a-4b^3\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\coth(c+dx)}{a+b}}{a+b}-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c\cdots}{3(a+b)}\cdots}{2a(a+b)}\cdots}{4a(a+b)}\cdots}{d}$$

$\downarrow$ 397

$$\frac{\frac{\frac{\frac{\frac{\frac{8(a+b)^4\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b^3\left(63a^2+36ab+8b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\coth(c+dx)}{a+b}}{a+b}-\frac{\left(8a^2-39ab-12\cdots\right.}{3(a\cdots}\cdots}{2a(a+b)}\cdots}{4a(a+b)}\cdots}{d}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\frac{\frac{\frac{8(a+b)^4\text{arctanh}(\tanh(c+dx))}{a}-\frac{b^3\left(63a^2+36ab+8b^2\right)\int \frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\coth(c+dx)}{a+b}}{a+b}-\frac{\left(8a^2-39ab-12b^2\right)\coth\cdots}{3(a+b)}\cdots}{2a(a+b)}\cdots}{4a(a+b)}\cdots}{d}$$

3.168. $\int \frac{\coth^4(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{\frac{\frac{8(a+b)^4\operatorname{arctanh}(\tanh(c+dx))}{a}-\frac{b^{5/2}(63a^2+36ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{(8a^3+32a^2b-15ab^2-4b^3)\coth(c+dx)}{a+b}}{a+b}}{2a(a+b)}-\frac{(8a^2-39ab-12b^2)\coth^3(c+\cdots}{3(a+b)}\cdots}{4a(a+b)}\cdots}{d}$$

input
```
Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

output
```
(-1/4*(b*Coth[c + d*x]^3)/(a*(a + b)*(a + b - b*Tanh[c + d*x]^2)^2) + ((-1
/3*((8*a^2 - 39*a*b - 12*b^2)*Coth[c + d*x]^3)/(a + b) + (((8*(a + b)^4*Ar
cTanh[Tanh[c + d*x]])/a - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTanh[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b) - ((8*a^3 + 32*a
^2*b - 15*a*b^2 - 4*b^3)*Coth[c + d*x])/(a + b))/(a + b))/(2*a*(a + b)) -
(b*(11*a + 4*b)*Coth[c + d*x]^3)/(2*a*(a + b)*(a + b - b*Tanh[c + d*x]^2))
)/(4*a*(a + b)))/d
```

3.168.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.168. $\int \frac{\coth^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.168. $\int \frac{\coth^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.168.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(214) = 428.

Time = 268.04 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+17b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^3+3a^2b+3ab^2+b^3\right)(a+b)}-\frac{1}{24(a+b)^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{}{8(a\cdots}$
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}+5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+17b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^3+3a^2b+3ab^2+b^3\right)(a+b)}-\frac{1}{24(a+b)^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{}{8(a\cdots}$
risch	$\frac{x}{a^3}-\frac{480a^5b\,e^{2dx+2c}+48a^6e^{12dx+12c}+144a^6e^{10dx+10c}+2073a^3b^3e^{8dx+8c}+642a^2b^4e^{8dx+8c}+1416ab^5e^{8dx+8c}-384a^5\cdots}{}$

input
```
int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^3*ln(1+tanh(1/2*d*x+1/2*c))-1/8/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)*(
1/3*tanh(1/2*d*x+1/2*c)^3*a+1/3*tanh(1/2*d*x+1/2*c)^3*b+5*tanh(1/2*d*x+1/2
*c)*a+17*b*tanh(1/2*d*x+1/2*c))-1/24/(a+b)^3/tanh(1/2*d*x+1/2*c)^3-1/8*(5*
a+17*b)/(a+b)^4/tanh(1/2*d*x+1/2*c)-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+2*b^3/
(a+b)^4/a^3*(((-17/8*a^3-21/8*a^2*b-1/2*a*b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(
51*a^2+3*a*b-4*b^2)*a*tanh(1/2*d*x+1/2*c)^5-1/8*(51*a^2+3*a*b-4*b^2)*a*tan
h(1/2*d*x+1/2*c)^3+(-17/8*a^3-21/8*a^2*b-1/2*a*b^2)*tanh(1/2*d*x+1/2*c))/(
tanh(1/2*d*x+1/2*c)^4*a+tanh(1/2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/8*(63*a^2+36*a*b+8*b^2)*(-1/4/b^(1/2)/(
a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1
/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c
)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))))
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11993 vs. $2(220) = 440$.

Time = 0.56 (sec) , antiderivative size = 24263, normalized size of antiderivative = 104.58

$$\int \frac{\coth^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.168.6 Sympy [F]

$$\int \frac{\coth^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx = \int \frac{\coth^4(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

input
```
integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

3.168.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4920 vs. 2(220) = 440.

Time = 0.85 (sec) , antiderivative size = 4920, normalized size of antiderivative = 21.21

$$\int \frac{\coth^4(c+dx)}{(a+b\text{sech}^2(c+dx))^3} \, dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3 + 2*b^4)*log(a*e^(4*d*x + 4*c) + 2*(a
+ 2*b)*e^(2*d*x + 2*c) + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*
b^4)*d) - 3/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/8*(3*a^3*b + 12*a^2*b^2
 + 8*a*b^3 + 2*b^4)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c)
+ a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) + 3/4*b*log(2*(
a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*d) + 1/4*(2*a + 5*b)*log(e^(2*d*x + 2*c) - 1)/((a^4
 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + 3/2*b*log(e^(2*d*x + 2*c) - 1
)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/4*(2*a + 5*b)*log(e^
(-2*d*x - 2*c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/2*
b*log(e^(-2*d*x - 2*c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d
) - 1/256*(15*a^4*b + 260*a^3*b^2 + 504*a^2*b^3 + 288*a*b^4 + 64*b^5)*log(
(a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a +
 2*b + 2*sqrt((a + b)*b)))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4)*sqrt((a + b)*b)*d) + 5/64*(3*a*b + 10*b^2)*log((a*e^(2*d*x + 2*c) + a
+ 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b
)))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 1/25
6*(15*a^4*b + 260*a^3*b^2 + 504*a^2*b^3 + 288*a*b^4 + 64*b^5)*log((a*e^(-2
*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*...
```

3.168.8 Giac [F]

$$\int \frac{\coth^4(c+dx)}{(a+b\text{sech}^2(c+dx))^3} \, dx = \int \frac{\coth(dx+c)^4}{(b\,\text{sech}(dx+c)^2+a)^3} \, dx$$

input
```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^6\,\coth(c+d\,x)^4}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

input
```
int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)
```

output
```
int((cosh(c + d*x)^6*coth(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.169 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^4}\,dx$

3.169.1 Optimal result

Integrand size = 14, antiderivative size = 207

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^4}\,dx = \frac{x}{a^4}-\frac{\sqrt{b}(35a^3+70a^2b+56ab^2+16b^3)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} - \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(19a^2+22ab+8b^2)\tanh(c+dx)}{16a^3(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)}$$

output
```
x/a^4-1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)*arctanh(b^(1/2)*tanh(d*x+c)/(
a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(7/2)/d-1/6*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*t
anh(d*x+c)^2)^3-1/24*b*(11*a+6*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*
x+c)^2)^2-1/16*b*(19*a^2+22*a*b+8*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b-b*ta
nh(d*x+c)^2)
```

3.169.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.74 (sec) , antiderivative size = 1405, normalized size of antiderivative = 6.79

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^4}\,dx$$

$$= \frac{(35a^3+70a^2b+56ab^2+16b^3)\,(a+2b+a\cosh(2c+2dx))^4\text{sech}^8(c+dx)\left(\frac{ib\arctan\left(\text{sech}(dx)\left(-\frac{i\cosh(2}{2\sqrt{a+b}\sqrt{b\cosh(4c}}\right.\right.}{}\right.}{}$$

$$+\frac{(a+2b+a\cosh(2c+2dx))\text{sech}(2c)\text{sech}^8(c+dx)\,(480a^6dx\cosh(2c)+3168a^5bdx\cosh(2c)+8928a^4b^2}{}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^(-4),x]
```

output
```
((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cosh[2*c + 2*d*x])^4
*Sech[c + d*x]^8*(((I/256)*b*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[
a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*S
qrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[
2*c + d*x])]*Cosh[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]
) - ((I/256)*b*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*
Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c
] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*S
inh[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3
*(a + b*Sech[c + d*x]^2)^4) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Sech[2*c]*S
ech[c + d*x]^8*(480*a^6*d*x*Cosh[2*c] + 3168*a^5*b*d*x*Cosh[2*c] + 8928*a^
4*b^2*d*x*Cosh[2*c] + 14112*a^3*b^3*d*x*Cosh[2*c] + 13248*a^2*b^4*d*x*Cosh
[2*c] + 6912*a*b^5*d*x*Cosh[2*c] + 1536*b^6*d*x*Cosh[2*c] + 360*a^6*d*x*Co
sh[2*d*x] + 2232*a^5*b*d*x*Cosh[2*d*x] + 5688*a^4*b^2*d*x*Cosh[2*d*x] + 72
72*a^3*b^3*d*x*Cosh[2*d*x] + 4608*a^2*b^4*d*x*Cosh[2*d*x] + 1152*a*b^5*d*x
*Cosh[2*d*x] + 360*a^6*d*x*Cosh[4*c + 2*d*x] + 2232*a^5*b*d*x*Cosh[4*c + 2
*d*x] + 5688*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 7272*a^3*b^3*d*x*Cosh[4*c + 2
*d*x] + 4608*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1152*a*b^5*d*x*Cosh[4*c + 2*d
*x] + 144*a^6*d*x*Cosh[2*c + 4*d*x] + 720*a^5*b*d*x*Cosh[2*c + 4*d*x] + 12
96*a^4*b^2*d*x*Cosh[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cosh[2*c + 4*d*x] +...
```

3.169.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3042, 4616, 316, 25, 402, 27, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{sech}^2(c+dx))^4}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(a+b\sec(ic+idx)^2)^4}dx$$

$$\Big\downarrow \ 4616$$

$$\frac{\int \frac{1}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^4}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 316$$

$$\frac{-\frac{\int -\frac{5b\tanh^2(c+dx)+6a+b}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^3}d\tanh(c+dx)}{6a(a+b)}-\frac{b\tanh(c+dx)}{6a(a+b)(a-b\tanh^2(c+dx)+b)^3}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{5b\tanh^2(c+dx)+6a+b}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^3}d\tanh(c+dx)}{6a(a+b)}-\frac{b\tanh(c+dx)}{6a(a+b)(a-b\tanh^2(c+dx)+b)^3}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{-\frac{\int -\frac{3(8a^2+5ba+2b^2+b(11a+6b)\tanh^2(c+dx))}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b(11a+6b)\tanh(c+dx)}{4a(a+b)(a-b\tanh^2(c+dx)+b)^2}}{6a(a+b)}-\frac{b\tanh(c+dx)}{6a(a+b)(a-b\tanh^2(c+dx)+b)^3}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\int \frac{8a^2+5ba+2b^2+b(11a+6b)\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(-b\tanh^2(c+dx)+a+b)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b(11a+6b)\tanh(c+dx)}{4a(a+b)(a-b\tanh^2(c+dx)+b)^2}}{6a(a+b)}-\frac{b\tanh(c+dx)}{6a(a+b)(a-b\tanh^2(c+dx)+b)^3}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{3\left(-\frac{\int -\frac{16a^3+29ba^2+26b^2a+8b^3+b\left(19a^2+22ba+8b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)\tanh(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{6a(a+b)}-\frac{\cdots}{6a(\cdots}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\frac{3\left(\frac{\int \frac{16a^3+29ba^2+26b^2a+8b^3+b\left(19a^2+22ba+8b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)\tanh(c+dx)}{4a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}}{6a(a+b)}-\frac{\cdots}{6a(a+\cdots}}{d}$$

$\downarrow$ 397

$$\frac{\frac{\frac{3\left(\frac{\frac{16(a+b)^3\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{b\left(35a^3+70a^2b+56ab^2+16b^3\right)\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(1\cdots}{4a(a+b\cdots}}{6a(a+b)}\cdots}{d}$$

$\downarrow$ 219

$$\frac{\frac{\frac{3\left(\frac{\frac{16(a+b)^3\text{arctanh}(\tanh(c+dx))}{a}-\frac{b\left(35a^3+70a^2b+56ab^2+16b^3\right)\int\frac{1}{-b\tanh^2(c+dx)+a+b}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b\cdots}{4a(a+b)\left(a-b\,\mathrm{t}\cdots\right.}}{6a(a+b)}\cdots}{d}$$

$\downarrow$ 221

$$\frac{\frac{\frac{3\left(\frac{\frac{16(a+b)^3\text{arctanh}(\tanh(c+dx))}{a}-\frac{\sqrt{b}\left(35a^3+70a^2b+56ab^2+16b^3\right)\text{arctanh}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tanh(c+dx)}{2a(a+b)\left(a-b\tanh^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)\tanh(\cdots}{4a(a+b)\left(a-b\tanh^2(\cdots\right.}}{6a(a+b)}\cdots}{d}$$

input

```
Int[(a + b*Sech[c + d*x]^2)^(-4),x]
```

3.169. $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^4}\,dx$

output
```
(-1/6*(b*Tanh[c + d*x])/(a*(a + b)*(a + b - b*Tanh[c + d*x]^2)^3) + (-1/4*
(b*(11*a + 6*b)*Tanh[c + d*x])/(a*(a + b)*(a + b - b*Tanh[c + d*x]^2)^2) +
 (3*(((16*(a + b)^3*ArcTanh[Tanh[c + d*x]])/a - (Sqrt[b]*(35*a^3 + 70*a^2*
b + 56*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sq
rt[a + b]))/(2*a*(a + b)) - (b*(19*a^2 + 22*a*b + 8*b^2)*Tanh[c + d*x])/(2
*a*(a + b)*(a + b - b*Tanh[c + d*x]^2))))/(4*a*(a + b)))/(6*a*(a + b)))/d
```

3.169.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.169. $\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^4}\,dx$

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.169.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(191) = 382$.

Time = 3.26 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.63

method	result
derivativedivides	$\frac{2b\left(\frac{-\frac{a(29a^2+26ab+8b^2)\tanh(\frac{dx}{2}+\frac{c}{2})^{11}}{16(a+b)}-\frac{(435a^3+281a^2b-66ab^2-72b^3)a\tanh(\frac{dx}{2}+\frac{c}{2})^9}{48(a^2+2ab+b^2)}-\frac{a(145a^4+148a^3b+37a^2b^2+2ab^3+8\ldots}{8(a+b)(a^2+2ab+b\ldots}}{(\tanh(\frac{dx}{2}+\frac{c}{2})^4a+\tanh\ldots}\right.}{\ldots}$
default	$\frac{2b\left(\frac{-\frac{a(29a^2+26ab+8b^2)\tanh(\frac{dx}{2}+\frac{c}{2})^{11}}{16(a+b)}-\frac{(435a^3+281a^2b-66ab^2-72b^3)a\tanh(\frac{dx}{2}+\frac{c}{2})^9}{48(a^2+2ab+b^2)}-\frac{a(145a^4+148a^3b+37a^2b^2+2ab^3+8\ldots}{8(a+b)(a^2+2ab+b\ldots}}{(\tanh(\frac{dx}{2}+\frac{c}{2})^4a+\tanh\ldots}\right.}{\ldots}$
risch	$\frac{x}{a^4}+\frac{b(435a^5e^{8dx+8c}+1408b^5e^{6dx+6c}+870a^5e^{4dx+4c}+87a^5+4292a^4b\,e^{6dx+6c}+8792a^3b^2e^{6dx+6c}+9936a^2b^3e^{6dx+6c}\ldots}{\ldots}$

input
```
int(1/(a+b*sech(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/a^4*((-1/16*a*(29*a^2+26*a*b+8*b^2)/(a+b)*tanh(1/2*d*x+1/2*c)^11-
1/48*(435*a^3+281*a^2*b-66*a*b^2-72*b^3)*a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^9-1/8*a*(145*a^4+148*a^3*b+37*a^2*b^2+2*a*b^3+8*b^4)/(a+b)/(a^2+2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*a*(145*a^4+148*a^3*b+37*a^2*b^2+2*a*b^3+8*
b^4)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5-1/48*(435*a^3+281*a^2
*b-66*a*b^2-72*b^3)*a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/16*a*(29*a^2
+26*a*b+8*b^2)/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+tanh(1/
2*d*x+1/2*c)^4*b+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^
3+1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)/(a^3+3*a^2*b+3*a*b^2+b^3)*(-1/4/b
^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2
*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))-1/a^4*ln(tanh(1/2
*d*x+1/2*c)-1)+1/a^4*ln(1+tanh(1/2*d*x+1/2*c)))
```

3.169.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8503 vs. $2(200) = 400$.

Time = 0.45 (sec) , antiderivative size = 17283, normalized size of antiderivative = 83.49

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sech(d*x+c)**2)**4,x)
```

output
```
Timed out
```

3.169.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. 2(200) = 400.

Time = 0.34 (sec) , antiderivative size = 718, normalized size of antiderivative = 3.47

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^4}\,dx$$
$$= \frac{(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{32\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{(a+b)b}d} - \frac{87\,a^5b+116\,a^4b^2+44\,a^3b^3+3\,(145\,a^5b+458\,a^4b^2+416\,a^3b^3+128\,a^2b^4)e^{(-2\,d}}{24\,(a^{10}+3\,a^9b+3\,a^8b^2+a^7b^3+6\,(a^{10}+5\,a^9b+9\,a^8b^2+7\,a^7b^3+2\,a^6b^4)e^{(-2\,dx-2\,c)}+3\,(5\,a^{10}+31\,a^9b} + \frac{dx+c}{a^4d}$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
1/32*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) +
 a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a +
b)*b)))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt((a + b)*b)*d) - 1/24*(
87*a^5*b + 116*a^4*b^2 + 44*a^3*b^3 + 3*(145*a^5*b + 458*a^4*b^2 + 416*a^3
*b^3 + 128*a^2*b^4)*e^(-2*d*x - 2*c) + 6*(145*a^5*b + 632*a^4*b^2 + 1072*a
^3*b^3 + 768*a^2*b^4 + 208*a*b^5)*e^(-4*d*x - 4*c) + 2*(435*a^5*b + 2146*a
^4*b^2 + 4396*a^3*b^3 + 4968*a^2*b^4 + 2912*a*b^5 + 704*b^6)*e^(-6*d*x - 6
*c) + 3*(145*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1024*a^2*b^4 + 288*a*b^5
)*e^(-8*d*x - 8*c) + 3*(29*a^5*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^4)
*e^(-10*d*x - 10*c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3 + 6*(a^10 + 5*
a^9*b + 9*a^8*b^2 + 7*a^7*b^3 + 2*a^6*b^4)*e^(-2*d*x - 2*c) + 3*(5*a^10 +
31*a^9*b + 79*a^8*b^2 + 101*a^7*b^3 + 64*a^6*b^4 + 16*a^5*b^5)*e^(-4*d*x -
 4*c) + 4*(5*a^10 + 33*a^9*b + 93*a^8*b^2 + 147*a^7*b^3 + 138*a^6*b^4 + 72
*a^5*b^5 + 16*a^4*b^6)*e^(-6*d*x - 6*c) + 3*(5*a^10 + 31*a^9*b + 79*a^8*b^
2 + 101*a^7*b^3 + 64*a^6*b^4 + 16*a^5*b^5)*e^(-8*d*x - 8*c) + 6*(a^10 + 5*
a^9*b + 9*a^8*b^2 + 7*a^7*b^3 + 2*a^6*b^4)*e^(-10*d*x - 10*c) + (a^10 + 3*
a^9*b + 3*a^8*b^2 + a^7*b^3)*e^(-12*d*x - 12*c))*d) + (d*x + c)/(a^4*d)
```

3.169.8 Giac [F]

$$\int \frac{1}{\left(a+b\mathrm{sech}^2(c+dx)\right)^4}\,dx=\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)^2+a\right)^4}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\mathrm{sech}^2(c+dx)\right)^4}\,dx=\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^4}\,dx$$

input
```
int(1/(a + b/cosh(c + d*x)^2)^4,x)
```

output
```
int(1/(a + b/cosh(c + d*x)^2)^4, x)
```

3.170 $\int (1-\mathrm{sech}^2(x))^{3/2}\,dx$

3.170.1 Optimal result

Integrand size = 12, antiderivative size = 29

$$\int (1-\mathrm{sech}^2(x))^{3/2}\,dx = \coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)} - \frac{1}{2}\coth(x)\tanh^2(x)^{3/2}$$

output
```
coth(x)*ln(cosh(x))*(tanh(x)^2)^(1/2)-1/2*coth(x)*(tanh(x)^2)^(3/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int (1-\mathrm{sech}^2(x))^{3/2}\,dx = \frac{1}{2}(-1+2\coth^2(x)\log(\cosh(x)))\tanh(x)\sqrt{\tanh^2(x)}$$

input
```
Integrate[(1 - Sech[x]^2)^(3/2),x]
```

output
```
((-1 + 2*Coth[x]^2*Log[Cosh[x]])*Tanh[x]*Sqrt[Tanh[x]^2])/2
```

3.170.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 4609, 3042, 4141, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\text{sech}^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sec(ix)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \tanh^2(x)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\tan(ix)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4141}$$

$$\sqrt{\tanh^2(x)}\coth(x)\int \tanh^3(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tanh^2(x)}\coth(x)\int i\tan(ix)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\sqrt{\tanh^2(x)}\coth(x)\int \tan(ix)^3dx$$

$$\Big\downarrow \text{3954}$$

$$i\sqrt{\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-\int i\tanh(x)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\sqrt{\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-i\int \tanh(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\sqrt{\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-i\int -i\tan(ix)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\sqrt{\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-\int \tan(ix)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$i\sqrt{\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-i\log(\cosh(x))\right)$$

input
```
Int[(1 - Sech[x]^2)^(3/2),x]
```

output
```
I*Coth[x]*Sqrt[Tanh[x]^2]*((-I)*Log[Cosh[x]] + (I/2)*Tanh[x]^2)
```

3.170.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.170. $\int \left(1-\text{sech}^2(x)\right)^{3/2}\,dx$

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.170.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$\operatorname{csgn}\left(\tanh\left(x\right)\right)\left(-\frac{\tanh\left(x\right)^{2}}{2}-\frac{\ln\left(\tanh\left(x\right)-1\right)}{2}-\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}\right)$	26
risch	$\frac{\sqrt{\frac{\left(e^{2x}-1\right)^{2}}{\left(1+e^{2x}\right)^{2}}}\,\left(e^{4x}\ln\left(1+e^{2x}\right)-e^{4x}x+2\,e^{2x}\ln\left(1+e^{2x}\right)-2\,e^{2x}x+2\,e^{2x}+\ln\left(1+e^{2x}\right)-x\right)}{\left(e^{2x}-1\right)\left(1+e^{2x}\right)}$	93

input
```
int((1-sech(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(tanh(x))*(-1/2*tanh(x)^2-1/2*ln(tanh(x)-1)-1/2*ln(1+tanh(x)))
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(23) = 46.

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 6.31

$$\int \left(1-\operatorname{sech}^2(x)\right)^{3/2}\,dx =$$

$$-\frac{x\cosh\left(x\right)^{4}+4\,x\cosh\left(x\right)\sinh\left(x\right)^{3}+x\sinh\left(x\right)^{4}+2\left(x-1\right)\cosh\left(x\right)^{2}+2\left(3\,x\cosh\left(x\right)^{2}+x-1\right)\sinh\left(x\right)^{2}}{\cosh\left(x\right)^{4}+4\,\mathrm{co}}$$

input
```
integrate((1-sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 + 2*(x - 1)*cosh(x)^2
+ 2*(3*x*cosh(x)^2 + x - 1)*sinh(x)^2 - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 +
cosh(x))*sinh(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 4*(x*cosh(x)^3
+ (x - 1)*cosh(x))*sinh(x) + x)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)
^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))
*sinh(x) + 1)
```

3.170.6 Sympy [F]

$$\int \left(1-\operatorname{sech}^2(x)\right)^{3/2} dx = \int \left(1-\operatorname{sech}^2(x)\right)^{\frac{3}{2}} dx$$

input
```
integrate((1-sech(x)**2)**(3/2),x)
```

output
```
Integral((1 - sech(x)**2)**(3/2), x)
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \left(1-\operatorname{sech}^2(x)\right)^{3/2} dx = -x - \frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1} - \log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate((1-sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-x - 2*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) - log(e^(-2*x) + 1)
```

3.170.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(23) = 46.

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.48

$$\int \left(1-\operatorname{sech}^2(x)\right)^{3/2}\,dx = -x\operatorname{sgn}\left(e^{(4x)}-1\right)+\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(e^{(4x)}-1\right) - \frac{3\,e^{(4x)}\operatorname{sgn}\left(e^{(4x)}-1\right)+2\,e^{(2x)}\operatorname{sgn}\left(e^{(4x)}-1\right)+3\operatorname{sgn}\left(e^{(4x)}-1\right)}{2\left(e^{(2x)}+1\right)^2}$$

input
```
integrate((1-sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1) - 1/2*(3*e^(4*x)*s
gn(e^(4*x) - 1) + 2*e^(2*x)*sgn(e^(4*x) - 1) + 3*sgn(e^(4*x) - 1))/(e^(2*x
) + 1)^2
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \left(1-\operatorname{sech}^2(x)\right)^{3/2}\,dx = \int \left(1-\frac{1}{\cosh(x)^2}\right)^{3/2}\,dx$$

input
```
int((1 - 1/cosh(x)^2)^(3/2),x)
```

output
```
int((1 - 1/cosh(x)^2)^(3/2), x)
```

3.171 $\int \sqrt{1-\operatorname{sech}^2(x)}\,dx$

3.171.1 Optimal result

Integrand size = 12, antiderivative size = 14

$$\int \sqrt{1-\operatorname{sech}^2(x)}\,dx=\coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}$$

output
```
coth(x)*ln(cosh(x))*(tanh(x)^2)^(1/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{1-\operatorname{sech}^2(x)}\,dx=\coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}$$

input
```
Integrate[Sqrt[1 - Sech[x]^2],x]
```

output
```
Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]
```

3.171.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 4609, 3042, 4141, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\operatorname{sech}^2(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sqrt{1-\sec(ix)^2}dx$$
$$\Big\downarrow \; 4609$$
$$\int \sqrt{\tanh^2(x)}dx$$
$$\Big\downarrow \; 3042$$
$$\int \sqrt{-\tan(ix)^2}dx$$
$$\Big\downarrow \; 4141$$
$$\sqrt{\tanh^2(x)}\coth(x)\int \tanh(x)dx$$
$$\Big\downarrow \; 3042$$
$$\sqrt{\tanh^2(x)}\coth(x)\int -i\tan(ix)dx$$
$$\Big\downarrow \; 26$$
$$-i\sqrt{\tanh^2(x)}\coth(x)\int \tan(ix)dx$$
$$\Big\downarrow \; 3956$$
$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))$$

input
```
Int[Sqrt[1 - Sech[x]^2],x]
```

output
```
Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]
```

3.171.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.171.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\operatorname{csgn}(\tanh(x))(\ln(\tanh(x)-1)+\ln(1+\tanh(x)))}{2}$	17
risch	$-\frac{(1+e^{2x})\sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,x}{e^{2x}-1}+\frac{(1+e^{2x})\sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,\ln(1+e^{2x})}{e^{2x}-1}$	79

input
```
int((1-sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(tanh(x))*(ln(tanh(x)-1)+ln(1+tanh(x)))
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \sqrt{1-\operatorname{sech}^2(x)}\,dx=-x+\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

input
```
integrate((1-sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

3.171.6 Sympy [F]

$$\int \sqrt{1-\operatorname{sech}^2(x)}\,dx=\int \sqrt{1-\operatorname{sech}^2\left(x\right)}\,dx$$

input
```
integrate((1-sech(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(1 - sech(x)**2), x)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \sqrt{1-\operatorname{sech}^2(x)}\,dx=-x-\log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate((1-sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-x - log(e^(-2*x) + 1)
```

3.171.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \sqrt{1-\operatorname{sech}^2(x)}\,dx = -x\operatorname{sgn}\left(e^{(4x)}-1\right)+\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(e^{(4x)}-1\right)$$

input
```
integrate((1-sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-\operatorname{sech}^2(x)}\,dx = \int \sqrt{1-\frac{1}{\cosh(x)^2}}\,dx$$

input
```
int((1 - 1/cosh(x)^2)^(1/2),x)
```

output
```
int((1 - 1/cosh(x)^2)^(1/2), x)
```

3.172 $\int \frac{1}{\sqrt{1-\text{sech}^2(x)}}\,dx$

3.172.1 Optimal result

Integrand size = 12, antiderivative size = 14

$$\int \frac{1}{\sqrt{1-\text{sech}^2(x)}}\,dx = \frac{\log(\sinh(x))\tanh(x)}{\sqrt{\tanh^2(x)}}$$

output
```
ln(sinh(x))*tanh(x)/(tanh(x)^2)^(1/2)
```

3.172.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{1-\text{sech}^2(x)}}\,dx = \frac{(\log(\cosh(x))+\log(\tanh(x)))\tanh(x)}{\sqrt{\tanh^2(x)}}$$

input
```
Integrate[1/Sqrt[1 - Sech[x]^2],x]
```

output
```
((Log[Cosh[x]] + Log[Tanh[x]])*Tanh[x])/Sqrt[Tanh[x]^2]
```

3.172.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4609, 3042, 4141, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-\sec(ix)^2}}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \frac{1}{\sqrt{\tanh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tanh(x)\int \coth(x)dx}{\sqrt{\tanh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\int -i\tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{\tanh^2(x)}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\tanh(x)\int \tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{\tanh^2(x)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

input
```
Int[1/Sqrt[1 - Sech[x]^2],x]
```

output
```
(Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]
```

3.172.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.172.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 5.64

method	result	size
risch	$-\frac{\left(\mathrm{e}^{2x}-1\right)x}{\sqrt{\frac{\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)}+\frac{\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{2x}-1\right)}{\sqrt{\frac{\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)}$	79

input
```
int(1/(1-sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*x+1/((e
xp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*ln(exp(2*x)-1
)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}}\,dx=-x+\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

3.172.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}}\,dx=\int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}}\,dx$$

input
```
integrate(1/(1-sech(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(1 - sech(x)**2), x)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}}\,dx=-x-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

input
```
integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-x - log(e^(-x) + 1) - log(e^(-x) - 1)
```

3.172.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}}\,dx = -\frac{x}{\operatorname{sgn}\left(e^{(4x)}-1\right)} + \frac{\log\left(\left|e^{(2x)}-1\right|\right)}{\operatorname{sgn}\left(e^{(4x)}-1\right)}$$

input
```
integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-x/sgn(e^(4*x) - 1) + log(abs(e^(2*x) - 1))/sgn(e^(4*x) - 1)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\frac{1}{\cosh(x)^2}}}\,dx$$

input
```
int(1/(1 - 1/cosh(x)^2)^(1/2),x)
```

output
```
int(1/(1 - 1/cosh(x)^2)^(1/2), x)
```

3.173 $\int \left(-1+\text{sech}^2(x)\right)^{3/2}\,dx$

3.173.1 Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \left(-1+\text{sech}^2(x)\right)^{3/2}\,dx=-\coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}+\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}$$

output
```
-coth(x)*ln(cosh(x))*(-tanh(x)^2)^(1/2)+1/2*(-tanh(x)^2)^(1/2)*tanh(x)
```

3.173.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \left(-1+\text{sech}^2(x)\right)^{3/2}\,dx=\frac{1}{2}\left(1-2\coth^2(x)\log(\cosh(x))\right)\tanh(x)\sqrt{-\tanh^2(x)}$$

input
```
Integrate[(-1 + Sech[x]^2)^(3/2),x]
```

output
```
((1 - 2*Coth[x]^2*Log[Cosh[x]])*Tanh[x]*Sqrt[-Tanh[x]^2])/2
```

3.173.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {3042, 4609, 3042, 4141, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\operatorname{sech}^2(x)-1\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(-1+\sec(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 4609}$$

$$\int \left(-\tanh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\tan(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 4141}$$

$$\sqrt{-\tanh^2(x)}(-\coth(x)) \int \tanh^3(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{-\tanh^2(x)}(-\coth(x)) \int i\tan(ix)^3dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\sqrt{-\tanh^2(x)}\coth(x) \int \tan(ix)^3dx$$

$$\Big\downarrow \text{ 3954}$$

$$-i\sqrt{-\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-\int i\tanh(x)dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\sqrt{-\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-i\int \tanh(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\sqrt{-\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-i\int -i\tan(ix)dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\sqrt{-\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-\int \tan(ix)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$-i\sqrt{-\tanh^2(x)}\coth(x)\left(\frac{1}{2}i\tanh^2(x)-i\log(\cosh(x))\right)$$

input
```
Int[(-1 + Sech[x]^2)^(3/2),x]
```

output
```
(-I)*Coth[x]*Sqrt[-Tanh[x]^2]*((-I)*Log[Cosh[x]] + (I/2)*Tanh[x]^2)
```

3.173.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.173. $\int \left(-1+\text{sech}^2(x)\right)^{3/2} dx$

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.173.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(28) = 56$.

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.79

method	result	size
risch	$-\frac{\sqrt{-\frac{\left(e^{2x}-1\right)^{2}}{\left(1+e^{2x}\right)^{2}}}\,\left(e^{4x} \ln\left(1+e^{2x}\right)-e^{4x} x+2 e^{2x} \ln\left(1+e^{2x}\right)-2 e^{2x} x+2 e^{2x}+\ln\left(1+e^{2x}\right)-x\right)}{\left(e^{2x}-1\right)\left(1+e^{2x}\right)}$	95

input
```
int((-1+sech(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*(exp(4*x)*ln(1+exp(2*x))-exp(4*x)*
x+2*exp(2*x)*ln(1+exp(2*x))-2*exp(2*x)*x+2*exp(2*x)+ln(1+exp(2*x))-x)/(exp
(2*x)-1)/(1+exp(2*x))
```

3.173.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 404, normalized size of antiderivative = 11.88

$$\int (-1 +\operatorname{sech}^2(x))^{3/2}\,dx = \frac{\left(x\cosh(x)^4 + \left(xe^{(2x)} + x\right)\sinh(x)^4 + 4\left(x\cosh(x)e^{(2x)} + x\cosh(x)\right)\sinh(x)^3 + 2(x \cdots\right.}{}$$

input
```
integrate((-1+sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
(x*cosh(x)^4 + (x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(2*x) + x*cosh(x
))*sinh(x)^3 + 2*(x - 1)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x
 - 1)*e^(2*x) + x - 1)*sinh(x)^2 + (x*cosh(x)^4 + 2*(x - 1)*cosh(x)^2 + x)
*e^(2*x) - ((e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(2*x) + cos
h(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^
2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (
cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*log(2*cosh(x)/(cosh(x
) - sinh(x))) + 4*(x*cosh(x)^3 + (x - 1)*cosh(x) + (x*cosh(x)^3 + (x - 1)*
cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(-(e^(4*x) - 2*e^(2*x) + 1)/(e^(4*x) +
2*e^(2*x) + 1))/((e^(2*x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(cosh(x)*e^(2*x)
- cosh(x))*sinh(x)^3 - 2*(3*cosh(x)^2 - (3*cosh(x)^2 + 1)*e^(2*x) + 1)*sin
h(x)^2 - 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) - 4*(cosh(x)^
3 - (cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) - 1)
```

3.173.6 Sympy [F]

$$\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2} dx = \int \left(\operatorname{sech}^2(x)-1\right)^{\frac{3}{2}} dx$$

input
```
integrate((-1+sech(x)**2)**(3/2),x)
```

output
```
Integral((sech(x)**2 - 1)**(3/2), x)
```

3.173.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2} dx = i\,x + \frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1} + i\,\log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate((-1+sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
I*x + 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + I*log(e^(-2*x) + 1)
```

3.173.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.44

$$\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2} dx = -i\, x\operatorname{sgn}\left(-e^{(4\,x)}+1\right) + i\,\log\left(e^{(2\,x)}+1\right)\operatorname{sgn}\left(-e^{(4\,x)}+1\right) - \frac{i\left(3\,e^{(4\,x)}\operatorname{sgn}\left(-e^{(4\,x)}+1\right)+2\,e^{(2\,x)}\operatorname{sgn}\left(-e^{(4\,x)}+1\right)+3\operatorname{sgn}\left(-e^{(4\,x)}+1\right)\right)}{2\left(e^{(2\,x)}+1\right)^2}$$

input
```
integrate((-1+sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-I*x*sgn(-e^(4*x) + 1) + I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1) - 1/2*I*(3*e
^(4*x)*sgn(-e^(4*x) + 1) + 2*e^(2*x)*sgn(-e^(4*x) + 1) + 3*sgn(-e^(4*x) +
1))/(e^(2*x) + 1)^2
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2} dx = \int \left(\frac{1}{\cosh(x)^2}-1\right)^{3/2} dx$$

input
```
int((1/cosh(x)^2 - 1)^(3/2),x)
```

output
```
int((1/cosh(x)^2 - 1)^(3/2), x)
```

3.174 $\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx$

3.174.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx = \coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}$$

output
```
coth(x)*ln(cosh(x))*(-tanh(x)^2)^(1/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx = \coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}$$

input
```
Integrate[Sqrt[-1 + Sech[x]^2],x]
```

output
```
Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]
```

3.174.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 4609, 3042, 4141, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\operatorname{sech}^2(x)-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-1+\sec(ix)^2}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \sqrt{-\tanh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(ix)^2}dx$$

$$\Big\downarrow \text{4141}$$

$$\sqrt{-\tanh^2(x)}\coth(x)\int \tanh(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{-\tanh^2(x)}\coth(x)\int -i\tan(ix)dx$$

$$\Big\downarrow \text{26}$$

$$-i\sqrt{-\tanh^2(x)}\coth(x)\int \tan(ix)dx$$

$$\Big\downarrow \text{3956}$$

$$\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

input
```
Int[Sqrt[-1 + Sech[x]^2],x]
```

output
```
Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]
```

3.174.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.174.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

method	result	size
default	$-\frac{\sqrt{-\tanh(x)^{2}}\,(\ln(\tanh(x)-1)+\ln(1+\tanh(x)))}{2\tanh(x)}$	26
risch	$-\frac{(1+e^{2x})\sqrt{-\frac{(e^{2x}-1)^{2}}{(1+e^{2x})^{2}}}\,x}{e^{2x}-1}+\frac{(1+e^{2x})\sqrt{-\frac{(e^{2x}-1)^{2}}{(1+e^{2x})^{2}}}\,\ln(1+e^{2x})}{e^{2x}-1}$	81

input
```
int((-1+sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-tanh(x)^2)^(1/2)*(ln(tanh(x)-1)+ln(1+tanh(x)))/tanh(x)
```

3.174.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.38

$$\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx = -\frac{\left(xe^{(2x)} - \left(e^{(2x)}+1\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+x\right)\sqrt{-\frac{e^{(4x)}-2e^{(2x)}+1}{e^{(4x)}+2e^{(2x)}+1}}}{e^{(2x)}-1}$$

input
```
integrate((-1+sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(x*e^(2*x) - (e^(2*x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + x)*sqrt(-
(e^(4*x) - 2*e^(2*x) + 1)/(e^(4*x) + 2*e^(2*x) + 1))/(e^(2*x) - 1)
```

3.174.6 Sympy [F]

$$\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx = \int \sqrt{\operatorname{sech}^2(x)-1}\,dx$$

input
```
integrate((-1+sech(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(sech(x)**2 - 1), x)
```

3.174.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx = -i\,x - i\,\log\left(e^{(-2x)}+1\right)$$

input
```
integrate((-1+sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-I*x - I*log(e^(-2*x) + 1)
```

3.174.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx = i\,x\operatorname{sgn}\left(-e^{(4\,x)}+1\right) - i\,\log\left(e^{(2\,x)}+1\right)\operatorname{sgn}\left(-e^{(4\,x)}+1\right)$$

input
```
integrate((-1+sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
I*x*sgn(-e^(4*x) + 1) - I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx = \int \sqrt{\frac{1}{\cosh(x)^2}-1}\,dx$$

input
```
int((1/cosh(x)^2 - 1)^(1/2),x)
```

output
```
int((1/cosh(x)^2 - 1)^(1/2), x)
```

3.175 $\int \frac{1}{\sqrt{-1+\text{sech}^2(x)}}\,dx$

3.175.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{-1+\text{sech}^2(x)}}\,dx=\frac{\log(\sinh(x))\tanh(x)}{\sqrt{-\tanh^2(x)}}$$

output
```
ln(sinh(x))*tanh(x)/(-tanh(x)^2)^(1/2)
```

3.175.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{-1+\text{sech}^2(x)}}\,dx=\frac{(\log(\cosh(x))+\log(\tanh(x)))\tanh(x)}{\sqrt{-\tanh^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 + Sech[x]^2],x]
```

output
```
((Log[Cosh[x]] + Log[Tanh[x]])*Tanh[x])/Sqrt[-Tanh[x]^2]
```

3.175.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 4609, 3042, 4141, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(x)-1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-1+\sec(ix)^2}}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \frac{1}{\sqrt{-\tanh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tanh(x)\int \coth(x)dx}{\sqrt{-\tanh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\int -i\tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{-\tanh^2(x)}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\tanh(x)\int \tan\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{-\tanh^2(x)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

input
```
Int[1/Sqrt[-1 + Sech[x]^2],x]
```

output
```
(Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]
```

3.175.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.175.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

method	result	size
default	$\frac{\tanh(x)(2\ln(\tanh(x))-\ln(1+\tanh(x))-\ln(\tanh(x)-1))}{2\sqrt{-\tanh(x)^2}}$	33
risch	$-\frac{(e^{2x}-1)x}{\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,(1+e^{2x})}+\frac{(e^{2x}-1)\ln(e^{2x}-1)}{\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,(1+e^{2x})}$	81

input
```
int(1/(-1+sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*tanh(x)*(2*ln(tanh(x))-ln(1+tanh(x))-ln(tanh(x)-1))/(-tanh(x)^2)^(1/2)
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

$$\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}}\,dx = \frac{\left(xe^{(2x)}-\left(e^{(2x)}+1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)+x\right)\sqrt{-\frac{e^{(4x)}-2e^{(2x)}+1}{e^{(4x)}+2e^{(2x)}+1}}}{e^{(2x)}-1}$$

input
```
integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
(x*e^(2*x) - (e^(2*x) + 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) + x)*sqrt(-(
e^(4*x) - 2*e^(2*x) + 1)/(e^(4*x) + 2*e^(2*x) + 1))/(e^(2*x) - 1)
```

3.175.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}^2(x)-1}}\,dx$$

input
```
integrate(1/(-1+sech(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(sech(x)**2 - 1), x)
```

3.175.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}}\,dx = i\,x + i\,\log\left(e^{(-x)}+1\right) + i\,\log\left(e^{(-x)}-1\right)$$

input
```
integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
I*x + I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

3.175.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}}\,dx = -\frac{i\,x}{\operatorname{sgn}\left(-e^{(4\,x)}+1\right)} + \frac{i\,\log\left(\left|e^{(2\,x)}-1\right|\right)}{\operatorname{sgn}\left(-e^{(4\,x)}+1\right)}$$

input
```
integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-I*x/sgn(-e^(4*x) + 1) + I*log(abs(e^(2*x) - 1))/sgn(-e^(4*x) + 1)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cosh(x)^2}-1}}\,dx$$

input
```
int(1/(1/cosh(x)^2 - 1)^(1/2),x)
```

output
```
int(1/(1/cosh(x)^2 - 1)^(1/2), x)
```

3.176 $\int \sqrt{a+b\text{sech}^2(x)} \tanh^5(x) \, dx$

3.176.1 Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \sqrt{a+b\text{sech}^2(x)} \tanh^5(x) \, dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\text{sech}^2(x)} + \frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2}$$

output
```
1/3*(a+2*b)*(a+b*sech(x)^2)^(3/2)/b^2-1/5*(a+b*sech(x)^2)^(5/2)/b^2+arctan
h((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*sech(x)^2)^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int \sqrt{a+b\text{sech}^2(x)} \tanh^5(x) \, dx$$

$$= \frac{-15b^2\left(a+b\text{sech}^2(x)\right) + 5a\left(a+b\text{sech}^2(x)\right)^2 + 10b\left(a+b\text{sech}^2(x)\right)^2 - 3\left(a+b\text{sech}^2(x)\right)^3 + 15ab^2\text{arctanh}\left(\right.}{15b^2\sqrt{a+b\text{sech}^2(x)}}$$

input
```
Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]
```

output
```
(-15*b^2*(a + b*Sech[x]^2) + 5*a*(a + b*Sech[x]^2)^2 + 10*b*(a + b*Sech[x]
^2)^2 - 3*(a + b*Sech[x]^2)^3 + 15*a*b^2*ArcTanh[Sqrt[1 + (b*Sech[x]^2)/a]
]*Sqrt[1 + (b*Sech[x]^2)/a])/(15*b^2*Sqrt[a + b*Sech[x]^2])
```

3.176.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)^5\sqrt{a+b\sec(ix)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{b\sec(ix)^2+a}\tan(ix)^5dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \cosh(x)\left(1-\text{sech}^2(x)\right)^2\sqrt{b\text{sech}^2(x)+a}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \cosh(x)\left(1-\text{sech}^2(x)\right)^2\sqrt{b\text{sech}^2(x)+a}d\text{sech}^2(x)$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{2}\int \left(\frac{\left(b\text{sech}^2(x)+a\right)^{3/2}}{b}+\cosh(x)\sqrt{b\text{sech}^2(x)+a}+\frac{(-a-2b)\sqrt{b\text{sech}^2(x)+a}}{b}\right)d\text{sech}^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\frac{2\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2}+\frac{2(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}-2\sqrt{a+b\text{sech}^2(x)}\right)$$

input
```
Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - 2*Sqrt[a + b*Sech[x]^2
] + (2*(a + 2*b)*(a + b*Sech[x]^2)^(3/2))/(3*b^2) - (2*(a + b*Sech[x]^2)^(
5/2))/(5*b^2))/2
```

3.176.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.176. $\int \sqrt{a+b\text{sech}^2(x)}\tanh^5(x)\,dx$

3.176.4 Maple [F]

$$\int \sqrt{a+\operatorname{sech}(x)^2 b}\,\tanh(x)^5\,dx$$

input
```
int((a+sech(x)^2*b)^(1/2)*tanh(x)^5,x)
```

output
```
int((a+sech(x)^2*b)^(1/2)*tanh(x)^5,x)
```

3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1929 vs. 2(67) = 134.

Time = 0.50 (sec) , antiderivative size = 4594, normalized size of antiderivative = 55.35

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh^5(x)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")
```

output
```
[1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*
b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 4
0*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^
2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70
*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2
*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b
^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3
+ 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x
)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*c
osh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a
^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a
^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*co
sh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)
*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*
a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^
3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(...
```

3.176.6 Sympy [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^5(x)\,dx=\int \sqrt{a+b\,\text{sech}^2\,(x)}\tanh^5\,(x)\,dx$$

input
```
integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**5,x)
```

output
```
Integral(sqrt(a + b*sech(x)**2)*tanh(x)**5, x)
```

3.176.7 Maxima [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^5(x)\,dx=\int \sqrt{b\,\text{sech}\,(x)^2+a}\tanh\,(x)^5\,dx$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^5, x)
```

3.176.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^5(x)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^5(x)\,dx=\int \tanh(x)^5\,\sqrt{a+\frac{b}{\cosh\left(x\right)^2}}\,dx$$

input
```
int(tanh(x)^5*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^5*(a + b/cosh(x)^2)^(1/2), x)
```

3.177 $\int \sqrt{a+b\text{sech}^2(x)} \tanh^4(x) \, dx$

3.177.1 Optimal result

Integrand size = 17, antiderivative size = 125

$$\int \sqrt{a+b\text{sech}^2(x)} \tanh^4(x) \, dx = -\frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)}$$

output
```
-1/8*(a^2+6*a*b-3*b^2)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(
3/2)+arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)+1/8*(a-3*b)*
(a+b-b*tanh(x)^2)^(1/2)*tanh(x)/b-1/4*(a+b-b*tanh(x)^2)^(1/2)*tanh(x)^3
```

3.177.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.54

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^4(x)\,dx =$$
$$-\frac{\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)-8\sqrt{2}\sqrt{a}b^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sin}{\sqrt{a+2b+a\,c}}\right.\right.}{8b^{3/2}\sqrt{a+}}$$

input
```
Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4,x]
```

output
```
-1/8*(Cosh[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*(a^2 + 6*a*b - 3*b^2)*ArcTan[
(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*Sqrt[a]
*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] -
(a - 5*b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] - 2*b^(3/2)*
Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(b^(3/2)*Sqrt[a + 2*b + a*
Cosh[2*x]])
```

3.177.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 4629, 2075, 380, 444, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(ix)^4\sqrt{a+b\sec(ix)^2}dx$$
$$\Big\downarrow \text{4629}$$
$$\int \frac{\tanh^4(x)\sqrt{a+b\left(1-\tanh^2(x)\right)}}{1-\tanh^2(x)}d\tanh(x)$$
$$\Big\downarrow \text{2075}$$

$$\int \frac{\tanh^4(x)\sqrt{a-b\tanh^2(x)+b}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 380$$

$$\frac{1}{4}\int \frac{\tanh^2(x)\left((a-3b)\tanh^2(x)+3(a+b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \ 444$$

$$\frac{1}{4}\left(\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\int \frac{(a-3b)(a+b)-(a^2+6ba-3b^2)\tanh^2(x)}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{2b}\right)-\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \ 398$$

$$\frac{1}{4}\left(\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\left(a^2+6ab-3b^2\right)\int \frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-8ab\int \frac{1}{(1-\tanh^2(x))\sqrt{-b}}}{2b}\right.$$
$$\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \ 224$$

$$\frac{1}{4}\left(\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\left(a^2+6ab-3b^2\right)\int \frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-8ab\int \frac{}{(1-\tanh^2}}{2b}\right.$$
$$\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{4}\left(\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}-8ab\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}}{2b}\right.$$
$$\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \ 291$$

3.177. $\int \sqrt{a+b\text{sech}^2(x)}\tanh^4(x)\,dx$

$$\frac{1}{4}\left(\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}-8ab\int\frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}}{2b}\right)$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}-8\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2b}\right)$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

input
```
Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4,x]
```

output
```
-1/4*(Tanh[x]^3*Sqrt[a + b - b*Tanh[x]^2]) + (-1/2*(((a^2 + 6*a*b - 3*b^2)
*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[b] - 8*Sqrt[a]*
b*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/b + ((a - 3*b)*Tan
h[x]*Sqrt[a + b - b*Tanh[x]^2])/(2*b))/4
```

3.177.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.177. $\int \sqrt{a+b\text{sech}^2(x)}\tanh^4(x)\,dx$

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.177.4 Maple [F]

$$\int \sqrt{a+\operatorname{sech}(x)^{2} b}\, \tanh(x)^{4} dx$$

input
```
int((a+sech(x)^2*b)^(1/2)*tanh(x)^4,x)
```

output
```
int((a+sech(x)^2*b)^(1/2)*tanh(x)^4,x)
```

3.177.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(103) = 206$.

Time = 0.52 (sec) , antiderivative size = 8852, normalized size of antiderivative = 70.82

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^4(x)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.177.6 Sympy [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^4(x)\, dx = \int \sqrt{a+b\operatorname{sech}^{2}(x)}\tanh^{4}(x)\, dx$$

input
```
integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**4,x)
```

output
```
Integral(sqrt(a + b*sech(x)**2)*tanh(x)**4, x)
```

3.177.7 Maxima [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^4(x)\,dx=\int \sqrt{b\,\text{sech}\,(x)^2+a}\tanh\,(x)^4\,dx$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^4, x)
```

3.177.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^4(x)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^4(x)\,dx=\int \tanh(x)^4\sqrt{a+\frac{b}{\cosh\,(x)^2}}\,dx$$

input
```
int(tanh(x)^4*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^4*(a + b/cosh(x)^2)^(1/2), x)
```

3.178 $\int \sqrt{a+b\text{sech}^2(x)}\tanh^3(x)\,dx$

3.178.1 Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^3(x)\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\text{sech}^2(x)} + \frac{\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b}$$

output
```
1/3*(a+b*sech(x)^2)^(3/2)/b+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)
-(a+b*sech(x)^2)^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^3(x)\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) + \frac{\sqrt{a+b\text{sech}^2(x)}\left(a-3b+b\text{sech}^2(x)\right)}{3b}$$

input
```
Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]
```

output
```
Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] + (Sqrt[a + b*Sech[x]^2]*(a
 - 3*b + b*Sech[x]^2))/(3*b)
```

3.178.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 26, 4627, 25, 354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\tan(ix)^3\sqrt{a+b\sec(ix)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \sqrt{b\sec(ix)^2+a}\tan(ix)^3dx$$

$$\Big\downarrow \text{ 4627}$$

$$\int -\cosh(x)\left(1-\text{sech}^2(x)\right)\sqrt{a+b\text{sech}^2(x)}d\text{sech}(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \cosh(x)\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}d\text{sech}(x)$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{1}{2}\int \cosh(x)\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}d\text{sech}^2(x)$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{2}\left(\frac{2\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b}-\int \cosh(x)\sqrt{b\text{sech}^2(x)+a}d\text{sech}^2(x)\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(-a\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)+\frac{2\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b}-2\sqrt{a+b\text{sech}^2(x)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(-\frac{2a\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}+\frac{2\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b}-2\sqrt{a+b\text{sech}^2(x)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)+\frac{2\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b}-2\sqrt{a+b\text{sech}^2(x)}\right)$$

input
```
Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - 2*Sqrt[a + b*Sech[x]^2
] + (2*(a + b*Sech[x]^2)^(3/2))/(3*b))/2
```

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.178.4 Maple [F]

$$\int \sqrt{a+\operatorname{sech}(x)^{2} b}\,\tanh(x)^{3}\,dx$$

input
```
int((a+sech(x)^2*b)^(1/2)*tanh(x)^3,x)
```

output
```
int((a+sech(x)^2*b)^(1/2)*tanh(x)^3,x)
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 829 vs. 2(47) = 94.

Time = 0.34 (sec) , antiderivative size = 2394, normalized size of antiderivative = 40.58

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^3(x)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

output
```
[1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^
4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh
(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 +
6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(((a^3
 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^
7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*
a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*
a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*
b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 +
4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 +
b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2
*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 1
5*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3
 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)
*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)
*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*co
sh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^
3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^...
```

3.178.6 Sympy [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^3(x)\,dx = \int \sqrt{a+b\,\text{sech}^2(x)}\tanh^3(x)\,dx$$

input
```
integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**3,x)
```

output
```
Integral(sqrt(a + b*sech(x)**2)*tanh(x)**3, x)
```

3.178.7 Maxima [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^3(x)\,dx=\int \sqrt{b\,\text{sech}\,(x)^2+a}\tanh\,(x)^3\;dx$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^3, x)
```

3.178.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^3(x)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^3(x)\,dx=\int \tanh(x)^3\sqrt{a+\frac{b}{\cosh\,(x)^2}}\,dx$$

input
```
int(tanh(x)^3*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^3*(a + b/cosh(x)^2)^(1/2), x)
```

3.179 $\int \sqrt{a+b\text{sech}^2(x)}\tanh^2(x)\,dx$

3.179.1 Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^2(x)\,dx = -\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2\sqrt{b}} + \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-1/2*(a-b)*arctan(
b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)-1/2*(a+b-b*tanh(x)^2)^(1/
2)*tanh(x)
```

3.179.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.72

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^2(x)\,dx = -\frac{\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}(a-b)\arctan\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right) - 2\sqrt{2}\sqrt{a}\sqrt{b}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right) + \right.}{2\sqrt{b}\sqrt{a+2b+a\cosh(2x)}}$$

input
```
Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]
```

output
```
-1/2*(Cosh[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*(a - b)*ArcTan[(Sqrt[2]*Sqrt[
b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 2*Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcTa
nh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + Sqrt[b]*Sqrt[a
 + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x]))/(Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x
]])
```

3.179.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 25, 4629, 25, 2075, 380, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(ix)^2\left(-\sqrt{a+b\sec(ix)^2}\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \sqrt{b\sec(ix)^2+a}\tan(ix)^2dx$$

$$\Big\downarrow \ 4629$$

$$-\int -\frac{\tanh^2(x)\sqrt{a+b\left(1-\tanh^2(x)\right)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 25$$

$$\int \frac{\tanh^2(x)\sqrt{a+b\left(1-\tanh^2(x)\right)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 2075$$

$$\int \frac{\tanh^2(x)\sqrt{a-b\tanh^2(x)+b}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 380$$

$$\frac{1}{2}\int \frac{(a-b)\tanh^2(x)+a+b}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(2a\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-(a-b)\int \frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)\right)-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(2a\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-(a-b)\int \frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(2a\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(2a\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

3.179. $\int \sqrt{a+b\text{sech}^2(x)}\tanh^2(x)\,dx$

input
```
Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]
```

output
```
(-(((a - b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[b])
+ 2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/2 - (Tan
h[x]*Sqrt[a + b - b*Tanh[x]^2])/2
```

3.179.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.179.4 Maple [F]

$$\int \sqrt{a+\operatorname{sech}\left(x\right)^{2} b}\,\tanh\left(x\right)^{2}dx$$

input
```
int((a+sech(x)^2*b)^(1/2)*tanh(x)^2,x)
```

output
```
int((a+sech(x)^2*b)^(1/2)*tanh(x)^2,x)
```

3.179.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(69) = 138$.

Time = 0.39 (sec) , antiderivative size = 4316, normalized size of antiderivative = 49.61

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh^2(x)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")
```

output
```
[1/4*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 +
 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b
)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)
^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh
(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
+ 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a
*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*
(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a
^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3
)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(
x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 -
3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3
 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4
- 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6
*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2
)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*s
inh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*si
nh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a - b)*cosh(x)^4 + 4*(a...
```

3.179.6 Sympy [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh^2(x)\,dx = \int \sqrt{a+b\,\text{sech}^2\,(x)}\tanh^2\,(x)\,dx$$

input
```
integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**2,x)
```

output
```
Integral(sqrt(a + b*sech(x)**2)*tanh(x)**2, x)
```

3.179.7 Maxima [F]

$$\int \sqrt{a+b\mathrm{sech}^2(x)}\tanh^2(x)\,dx = \int \sqrt{b\,\mathrm{sech}\,(x)^2+a}\tanh\,(x)^2\,dx$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^2, x)
```

3.179.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\mathrm{sech}^2(x)}\tanh^2(x)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\mathrm{sech}^2(x)}\tanh^2(x)\,dx = \int \tanh(x)^2\sqrt{a+\frac{b}{\cosh\,(x)^2}}\,dx$$

input
```
int(tanh(x)^2*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^2*(a + b/cosh(x)^2)^(1/2), x)
```

3.180 $\int \sqrt{a+b\text{sech}^2(x)}\tanh(x)\,dx$

3.180.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh(x)\,dx=\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\text{sech}^2(x)}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*sech(x)^2)^(1/2)
```

3.180.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(40) = 80.

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh(x)\,dx = -\frac{\left(a+2b+a\cosh(2x)-\sqrt{2}\sqrt{a}\sqrt{b}\text{arcsinh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{b}}\right)\cosh(x)\sqrt{\frac{a+2b+a\cosh(2x)}{b}}\right)\sqrt{a+b\text{sech}^2(x)}}{a+2b+a\cosh(2x)}$$

input
```
Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x],x]
```

output
```
-(((a + 2*b + a*Cosh[2*x] - Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[a]*Cosh[
x])/Sqrt[b]]*Cosh[x]*Sqrt[(a + 2*b + a*Cosh[2*x])/b])*Sqrt[a + b*Sech[x]^2
])/(a + 2*b + a*Cosh[2*x]))
```

3.180.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 4627, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)\sqrt{a+b\sec(ix)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{b\sec(ix)^2+a}\tan(ix)dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \cosh(x)\sqrt{b\text{sech}^2(x)+a}d\text{sech}(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \cosh(x)\sqrt{b\text{sech}^2(x)+a}d\text{sech}^2(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)-2\sqrt{a+b\text{sech}^2(x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{2a\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}-2\sqrt{a+b\text{sech}^2(x)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-2\sqrt{a+b\text{sech}^2(x)}\right)$$

input
```
Int[Sqrt[a + b*Sech[x]^2]*Tanh[x],x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - 2*Sqrt[a + b*Sech[x]^2
])/2
```

3.180.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.180.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\sqrt{a+\operatorname{sech}\left(x\right)^{2} b}+\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+\operatorname{sech}(x)^{2} b}}{\operatorname{sech}(x)}\right)$	43
default	$-\sqrt{a+\operatorname{sech}\left(x\right)^{2} b}+\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+\operatorname{sech}(x)^{2} b}}{\operatorname{sech}(x)}\right)$	43

input
```
int((a+sech(x)^2*b)^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
-(a+sech(x)^2*b)^(1/2)+a^(1/2)*ln((2*a+2*a^(1/2)*(a+sech(x)^2*b)^(1/2))/se
ch(x))
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(32) = 64$.

Time = 0.33 (sec) , antiderivative size = 1608, normalized size of antiderivative = 40.20

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh(x)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")
```

output
```
[1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*log(((a^3 +
2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 +
 (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)
*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(
14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2
*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 +
14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*co
sh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*si
nh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(
x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 +
 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (1
5*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2
 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 +
2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt...
```

3.180.6 Sympy [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\tanh(x)\,dx = \int \sqrt{a+b\,\text{sech}^2\left(x\right)}\tanh\left(x\right)dx$$

input
```
integrate((a+b*sech(x)**2)**(1/2)*tanh(x),x)
```

output
```
Integral(sqrt(a + b*sech(x)**2)*tanh(x), x)
```

3.180.7 Maxima [F]

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh(x)\,dx = \int \sqrt{b\operatorname{sech}(x)^2+a}\tanh(x)\,dx$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*tanh(x), x)
```

3.180.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh(x)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.180.9 Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\tanh(x)\,dx = \sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)-\sqrt{a+\frac{b}{\cosh(x)^2}}$$

input
```
int(tanh(x)*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
a^(1/2)*atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2)) - (a + b/cosh(x)^2)^(1/2)
```

3.181 $\int \sqrt{a+b\operatorname{sech}^2(x)}\,dx$

3.181.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\,dx = \sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + \sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)+arctan(b^(1/2)*ta
nh(x)/(a+b-b*tanh(x)^2)^(1/2))*b^(1/2)
```

3.181.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 134 vs. 2(59) = 118.

Time = 0.61 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.27

$$\int \sqrt{a+b\operatorname{sech}^2(x)}\,dx = \frac{\sqrt{2}\cosh(x)\left(\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\sqrt{a+2b+a\cosh(2x)}+\sqrt{a}\sqrt{a+b}\operatorname{arcsinh}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sqrt{\frac{a+2b+a}{a}}\right.}{a+2b+a\cosh(2x)}$$

input
```
Integrate[Sqrt[a + b*Sech[x]^2],x]
```

output
```
(Sqrt[2]*Cosh[x]*(Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b +
a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*Sqrt[a + b]*ArcSinh[(S
qrt[a]*Sinh[x])/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)])*Sqrt[a
 + b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])
```

3.181.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 4616, 301, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sec(ix)^2}dx$$

$$\Big\downarrow \text{4616}$$

$$\int \frac{\sqrt{a-b\tanh^2(x)+b}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{301}$$

$$b\int \frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)$$

$$\Big\downarrow \text{224}$$

$$a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+ b\int \frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}$$

$$\Big\downarrow \text{216}$$

$$a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

$$\Big\downarrow \text{291}$$

$$a\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}+\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

$$\Big\downarrow \text{219}$$

$$\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

input
```
Int[Sqrt[a + b*Sech[x]^2],x]
```

output
```
Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + Sqrt[a]*ArcT
anh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
```

3.181.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.181.4 Maple [F]

$$\int \sqrt{a+\operatorname{sech}(x)^{2} b}dx$$

input
```
int((a+sech(x)^2*b)^(1/2),x)
```

output
```
int((a+sech(x)^2*b)^(1/2),x)
```

3.181.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(47) = 94$.

Time = 0.35 (sec) , antiderivative size = 2949, normalized size of antiderivative = 49.98

$$\int \sqrt{a+b\text{sech}^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh
(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*s
inh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b +
9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 -
10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3
+ a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 -
b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*si
nh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6
 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x
)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)
^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5
- 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*
b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^
5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2
*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(-b)*log(-((a - b
)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b
)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(...
```

3.181.6 Sympy [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\,dx=\int \sqrt{a+b\,\text{sech}^2\,(x)}\,dx$$

input
```
integrate((a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sech(x)**2), x)
```

3.181.7 Maxima [F]

$$\int \sqrt{a+b\text{sech}^2(x)}\,dx=\int \sqrt{b\,\text{sech}\,(x)^2+a}\,dx$$

input
```
integrate((a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a), x)
```

3.181.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\text{sech}^2(x)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{sech}^2(x)}\,dx=\int \sqrt{a+\frac{b}{\cosh\,(x)^2}}\,dx$$

input
```
int((a + b/cosh(x)^2)^(1/2),x)
```

output
```
int((a + b/cosh(x)^2)^(1/2), x)
```

3.182 $\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

3.182.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-arctanh((a+b*sech(x)^2)^(1/
2)/(a+b)^(1/2))*(a+b)^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)$$

input
```
Integrate[Coth[x]*Sqrt[a + b*Sech[x]^2],x]
```

output
```
Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b]*ArcTanh[Sqrt[
a + b*Sech[x]^2]/Sqrt[a + b]]
```

3.182.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 26, 4627, 25, 354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sqrt{a+b\sec(ix)^2}}{\tan(ix)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sqrt{b\sec(ix)^2+a}}{\tan(ix)}dx$$

$$\Big\downarrow \; 4627$$

$$\int -\frac{\cosh(x)\sqrt{a+b\text{sech}^2(x)}}{1-\text{sech}^2(x)}d\text{sech}(x)$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\cosh(x)\sqrt{b\text{sech}^2(x)+a}}{1-\text{sech}^2(x)}d\text{sech}(x)$$

$$\Big\downarrow \; 354$$

$$-\frac{1}{2}\int \frac{\cosh(x)\sqrt{b\text{sech}^2(x)+a}}{1-\text{sech}^2(x)}d\text{sech}^2(x)$$

$$\Big\downarrow \; 94$$

$$\frac{1}{2}\left(-(a+b)\int \frac{1}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)-a\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\text{sech}^4(x)}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}-\frac{2a\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)\right)$$

input
```
Int[Coth[x]*Sqrt[a + b*Sech[x]^2],x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - 2*Sqrt[a + b]*ArcTanh[
Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/2
```

3.182.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.182. $\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.182.4 Maple [F]

$$\int \coth(x)\sqrt{a+\operatorname{sech}(x)^2 b}dx$$

input
```
int(coth(x)*(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(coth(x)*(a+sech(x)^2*b)^(1/2),x)
```

3.182.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(44) = 88$.

Time = 0.34 (sec) , antiderivative size = 3597, normalized size of antiderivative = 64.23

$$\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5
*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 +
(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3
+ 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x
))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b
+ a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^
3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2
)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5
 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*
(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 +
 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^
2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b
 + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b
^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))
*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2...
```

3.182.6 Sympy [F]

$$\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \sqrt{a+b\,\text{sech}^2\,(x)}\coth\,(x)\,dx$$

input
```
integrate(coth(x)*(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sech(x)**2)*coth(x), x)
```

3.182.7 Maxima [F]

$$\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \sqrt{b\,\text{sech}\,(x)^2+a}\coth\,(x)\;dx$$

input
```
integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*coth(x), x)
```

3.182.8 Giac [F(-2)]

Exception generated.

$$\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \coth(x)\,\sqrt{a+\frac{b}{\cosh\,(x)^2}}\,dx$$

input
```
int(coth(x)*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(coth(x)*(a + b/cosh(x)^2)^(1/2), x)
```

3.183 $\int \coth^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

3.183.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \coth^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx=\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b-b\tanh^2(x)}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-coth(x)*(a+b-b*ta
nh(x)^2)^(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int \coth^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx=\left(\frac{\sqrt{2}\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\cosh(x)}{\sqrt{a+b}\sqrt{\frac{a+2b+a\cosh(2x)}{a+b}}}-\coth(x)\right)\sqrt{a+b\text{sech}^2(x)}$$

input
```
Integrate[Coth[x]^2*Sqrt[a + b*Sech[x]^2],x]
```

output
```
((Sqrt[2]*Sqrt[a]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*Cosh[x])/(Sqrt[a
+ b]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)]) - Coth[x])*Sqrt[a + b*Sech[x]^
2]
```

3.183.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 25, 4629, 25, 2075, 377, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a+b\sec(ix)^2}}{\tan(ix)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{b\sec(ix)^2+a}}{\tan(ix)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\int -\frac{\coth^2(x)\sqrt{a+b\left(1-\tanh^2(x)\right)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^2(x)\sqrt{a+b\left(1-\tanh^2(x)\right)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\coth^2(x)\sqrt{a-b\tanh^2(x)+b}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{377}$$

$$\int \frac{a}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \; 27$$

$$a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \; 291$$

$$a\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \; 219$$

$$\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

input
```
Int[Coth[x]^2*Sqrt[a + b*Sech[x]^2],x]
```

output
```
Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - Coth[x]*Sqr
t[a + b - b*Tanh[x]^2]
```

3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.183.4 Maple [F]

$$\int \coth(x)^2 \sqrt{a+\operatorname{sech}(x)^2 b}dx$$

input
```
int(coth(x)^2*(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(coth(x)^2*(a+sech(x)^2*b)^(1/2),x)
```

3.183.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(40) = 80$.

Time = 0.30 (sec) , antiderivative size = 1303, normalized size of antiderivative = 27.15

$$\int \coth^2(x)\sqrt{a + b\text{sech}^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log((a*b^2*c
osh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*c
osh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*co
sh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*c
osh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3
)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^
3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)
*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*
a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*co
sh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5
*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh
(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a
^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2
+ 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3
*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*
sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*si
nh(x)^5 + sinh(x)^6)) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sq
rt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + ...
```

3.183.6 Sympy [F]

$$\int \coth^2(x)\sqrt{a + b\text{sech}^2(x)}\,dx = \int \sqrt{a + b\,\text{sech}^2(x)}\coth^2(x)\,dx$$

input
```
integrate(coth(x)**2*(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sech(x)**2)*coth(x)**2, x)
```

3.183.7 Maxima [F]

$$\int \coth^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \sqrt{b\,\text{sech}\,(x)^2+a}\coth\,(x)^2\,dx$$

input
```
integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*coth(x)^2, x)
```

3.183.8 Giac [F(-2)]

Exception generated.

$$\int \coth^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \coth^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \coth(x)^2\,\sqrt{a+\frac{b}{\cosh\,(x)^2}}\,dx$$

input
```
int(coth(x)^2*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2*(a + b/cosh(x)^2)^(1/2), x)
```

3.184 $\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

3.184.1 Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}} - \frac{1}{2}\coth^2(x)\sqrt{a+b\text{sech}^2(x)}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-1/2*(2*a+b)*arctanh((a+b*se
ch(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-1/2*coth(x)^2*(a+b*sech(x)^2)^(1/2
)
```

3.184.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.88

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx = -\frac{\left(\sqrt{2}(2a+b)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh(x)+\sqrt{a+b}\left(\sqrt{a+2b+a\cosh(2x)}\coth^2(x)-2\sqrt{2}\sqrt{a}\,\text{co}\right.\right.}{2\sqrt{a+b}\sqrt{a+2b+a\cosh(2x)}}$$

input
```
Integrate[Coth[x]^3*Sqrt[a + b*Sech[x]^2],x]
```

output
```
-1/2*((Sqrt[2]*(2*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*
b + a*Cosh[2*x]]]*Cosh[x] + Sqrt[a + b]*(Sqrt[a + 2*b + a*Cosh[2*x]]*Coth[
x]^2 - 2*Sqrt[2]*Sqrt[a]*Cosh[x]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*
b + a*Cosh[2*x]]]))*Sqrt[a + b*Sech[x]^2])/(Sqrt[a + b]*Sqrt[a + 2*b + a*C
osh[2*x]])
```

3.184.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 26, 4627, 354, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sqrt{a+b\sec(ix)^2}}{\tan(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sqrt{b\sec(ix)^2+a}}{\tan(ix)^3}dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \frac{\cosh(x)\sqrt{b\text{sech}^2(x)+a}}{\left(1-\text{sech}^2(x)\right)^2}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)\sqrt{b\text{sech}^2(x)+a}}{\left(1-\text{sech}^2(x)\right)^2}d\text{sech}^2(x)$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{2}\left(\int -\frac{\cosh(x)\left(b\text{sech}^2(x)+2a\right)}{2\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)-\frac{\sqrt{a+b\text{sech}^2(x)}}{1-\text{sech}^2(x)}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{\cosh(x)\left(b\text{sech}^2(x)+2a\right)}{(1-\text{sech}^2(x))\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)-\frac{\sqrt{a+b\text{sech}^2(x)}}{1-\text{sech}^2(x)}\right)$$

$$\Big\downarrow 174$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-(2a+b)\int \frac{1}{(1-\text{sech}^2(x))\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)-2a\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)\right)-\frac{\sqrt{a+b\text{sech}^2(x)}}{1-\text{sech}^2(x)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{2(2a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\text{sech}^4(x)}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}-\frac{4a\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}\right)-\frac{\sqrt{a+b\text{sech}^2(x)}}{1-\text{sech}^2(x)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\frac{2(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}\right)-\frac{\sqrt{a+b\text{sech}^2(x)}}{1-\text{sech}^2(x)}\right)$$

input
```
Int[Coth[x]^3*Sqrt[a + b*Sech[x]^2],x]
```

output
```
((4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - (2*(2*a + b)*ArcTanh[
Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/2 - Sqrt[a + b*Sech[x]^2]
/(1 - Sech[x]^2))/2
```

3.184.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.184. $\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

3.184.4 Maple [F]

$$\int \coth(x)^{3} \sqrt{a+\operatorname{sech}(x)^{2} b}dx$$

input
```
int(coth(x)^3*(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(coth(x)^3*(a+sech(x)^2*b)^(1/2),x)
```

3.184.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. 2(65) = 130.

Time = 0.40 (sec) , antiderivative size = 5247, normalized size of antiderivative = 63.22

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.184.6 Sympy [F]

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \sqrt{a+b\operatorname{sech}^{2}(x)}\coth^{3}(x)\,dx$$

input
```
integrate(coth(x)**3*(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sech(x)**2)*coth(x)**3, x)
```

3.184.7 Maxima [F]

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \sqrt{b\,\text{sech}\,(x)^2+a}\,\coth\,(x)^3\;dx$$

input
```
integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*coth(x)^3, x)
```

3.184.8 Giac [F(-2)]

Exception generated.

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \coth(x)^3\,\sqrt{a+\frac{b}{\cosh\,(x)^2}}\,dx$$

input
```
int(coth(x)^3*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3*(a + b/cosh(x)^2)^(1/2), x)
```

3.185 $\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

3.185.1 Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-1/3*(3*a+2*b)*cot
h(x)*(a+b-b*tanh(x)^2)^(1/2)/(a+b)-1/3*coth(x)^3*(a+b-b*tanh(x)^2)^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.77

$$\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \frac{\sqrt{2}\cosh(x)\sqrt{a+b\text{sech}^2(x)}\sqrt{a+b+a\sinh^2(x)}\left(3\sqrt{a}(a+b)\text{arcsinh}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)-\sqrt{a+b}\text{csch}(x)\left(4a+3\right.\right.}{3(a+b)^{3/2}\sqrt{a+2b+a\cosh(2x)}\sqrt{\frac{a+b+a\sinh^2(x)}{a+b}}}$$

input
```
Integrate[Coth[x]^4*Sqrt[a + b*Sech[x]^2],x]
```

output
```
(Sqrt[2]*Cosh[x]*Sqrt[a + b*Sech[x]^2]*Sqrt[a + b + a*Sinh[x]^2]*(3*Sqrt[a
]*(a + b)*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]] - Sqrt[a + b]*Csch[x]*(4*
a + 3*b + (a + b)*Csch[x]^2)*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/(3*(a +
 b)^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]]*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)])
```

3.185.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 4629, 2075, 377, 445, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(ix)^2}}{\tan(ix)^4}dx$$

$$\Big\downarrow \text{4629}$$

$$\int \frac{\coth^4(x)\sqrt{a+b\left(1-\tanh^2(x)\right)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\coth^4(x)\sqrt{a-b\tanh^2(x)+b}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{377}$$

$$\frac{1}{3}\int \frac{\coth^2(x)\left(-2b\tanh^2(x)+3a+2b\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{3}\left(-\frac{\int -\frac{3a(a+b)}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a+b}-\frac{(3a+2b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}\right)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(3a\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{(3a+2b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}\right)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{3}\left(3a\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-\frac{(3a+2b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}\right)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(3\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{(3a+2b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}\right)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}$$

input
```
Int[Coth[x]^4*Sqrt[a + b*Sech[x]^2],x]
```

output
```
-1/3*(Coth[x]^3*Sqrt[a + b - b*Tanh[x]^2]) + (3*Sqrt[a]*ArcTanh[(Sqrt[a]*T
anh[x])/Sqrt[a + b - b*Tanh[x]^2]] - ((3*a + 2*b)*Coth[x]*Sqrt[a + b - b*T
anh[x]^2])/(a + b))/3
```

3.185. $\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

3.185.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.185. $\int \coth^4(x)\sqrt{a+b\operatorname{sech}^2(x)}\,dx$

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.185.4 Maple [F]

$$\int \coth(x)^{4} \sqrt{a+\operatorname{sech}(x)^{2} b}dx$$

input
```
int(coth(x)^4*(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(coth(x)^4*(a+sech(x)^2*b)^(1/2),x)
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(70) = 140$.

Time = 0.36 (sec) , antiderivative size = 2341, normalized size of antiderivative = 27.87

$$\int \coth^4(x)\sqrt{a+b\operatorname{sech}^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x
)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*
(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2
+ 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a
+ b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b)*s
qrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8
- 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)
^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^
2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*
b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
+ 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*c
osh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b
^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 -
3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 1
8*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^
2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2
*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2...
```

3.185.6 Sympy [F]

$$\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \sqrt{a+b\,\text{sech}^2(x)}\coth^4(x)\,dx$$

input
```
integrate(coth(x)**4*(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sech(x)**2)*coth(x)**4, x)
```

3.185.7 Maxima [F]

$$\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \sqrt{b\,\text{sech}\,(x)^2+a}\coth\,(x)^4\,dx$$

input
```
integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*coth(x)^4, x)
```

3.185.8 Giac [F(-2)]

Exception generated.

$$\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \coth(x)^4\sqrt{a+\frac{b}{\cosh\,(x)^2}}\,dx$$

input
```
int(coth(x)^4*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^4*(a + b/cosh(x)^2)^(1/2), x)
```

3.186 $\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

3.186.1 Optimal result

Integrand size = 17, antiderivative size = 125

$$\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \frac{(8a^2+12ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}} - \frac{(4a+3b)\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)}$$

output
```
-1/8*(8*a^2+12*a*b+3*b^2)*arctanh((a+b*sech(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)
^(3/2)+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-1/8*(4*a+3*b)*coth(x
)^2*(a+b*sech(x)^2)^(1/2)/(a+b)-1/4*coth(x)^4*(a+b*sech(x)^2)^(1/2)
```

3.186.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.53

$$\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx =$$

$$-\frac{\cosh(x)\left(\sqrt{2}(8a^2+12ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+\sqrt{a+b}\left(\frac{1}{2}\sqrt{a+2b+a\cosh(2x)}(-2a-b\right.\right.}{8(a+b}$$

input
```
Integrate[Coth[x]^5*Sqrt[a + b*Sech[x]^2],x]
```

output
```
-1/8*(Cosh[x]*(Sqrt[2]*(8*a^2 + 12*a*b + 3*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a +
b]*Cosh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + Sqrt[a + b]*((Sqrt[a + 2*b + a*
Cosh[2*x]]*(-2*a - b + (6*a + 5*b)*Cosh[2*x])*Coth[x]*Csch[x]^3)/2 - 8*Sqr
t[2]*Sqrt[a]*(a + b)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2
*x]]]))*Sqrt[a + b*Sech[x]^2])/((a + b)^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]])
```

3.186.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {3042, 26, 4627, 25, 354, 110, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sqrt{a+b\sec(ix)^2}}{\tan(ix)^5}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sqrt{b\sec(ix)^2+a}}{\tan(ix)^5}dx$$

$$\Big\downarrow \; 4627$$

$$\int -\frac{\cosh(x)\sqrt{a+b\text{sech}^2(x)}}{\left(1-\text{sech}^2(x)\right)^3}d\text{sech}(x)$$

$$\Big\downarrow 25$$

$$-\int \frac{\cosh(x)\sqrt{b\text{sech}^2(x)+a}}{\left(1-\text{sech}^2(x)\right)^3}d\text{sech}(x)$$

$$\Big\downarrow 354$$

$$-\frac{1}{2}\int \frac{\cosh(x)\sqrt{b\text{sech}^2(x)+a}}{\left(1-\text{sech}^2(x)\right)^3}d\text{sech}^2(x)$$

$$\Big\downarrow 110$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{\cosh(x)\left(3b\text{sech}^2(x)+4a\right)}{2\left(1-\text{sech}^2(x)\right)^2\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)-\frac{\sqrt{a+b\text{sech}^2(x)}}{2\left(1-\text{sech}^2(x)\right)^2}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{\cosh(x)\left(3b\text{sech}^2(x)+4a\right)}{\left(1-\text{sech}^2(x)\right)^2\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)-\frac{\sqrt{a+b\text{sech}^2(x)}}{2\left(1-\text{sech}^2(x)\right)^2}\right)$$

$$\Big\downarrow 168$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int -\frac{\cosh(x)\left(b(4a+3b)\text{sech}^2(x)+8a(a+b)\right)}{2\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a+b}-\frac{(4a+3b)\sqrt{a+b\text{sech}^2(x)}}{(a+b)\left(1-\text{sech}^2(x)\right)}\right)-\frac{\sqrt{a+b\text{sech}^2(x)}}{2\left(1-\text{sech}^2(x)\right)^2}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{\int \frac{\cosh(x)\left(b(4a+3b)\text{sech}^2(x)+8a(a+b)\right)}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{2(a+b)}-\frac{(4a+3b)\sqrt{a+b\text{sech}^2(x)}}{(a+b)\left(1-\text{sech}^2(x)\right)}\right)-\frac{\sqrt{a+b\text{sech}^2(x)}}{2\left(1-\text{sech}^2(x)\right)^2}\right)$$

$$\Big\downarrow 174$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{\left(8a^2+12ab+3b^2\right)\int \frac{1}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)+8a(a+b)\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{2(a+b)}-\frac{(4a+3}{(a+}\right.\right.$$

$$\Big\downarrow 73$$

3.186. $\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{\frac{2(8a^2+12ab+3b^2)\int \frac{1}{\frac{a+b}{b}-\frac{\text{sech}^4(x)}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}+\frac{16a(a+b)\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}}{2(a+b)}-\frac{(4a+3b)\sqrt{a+b\text{sech}}}{(a+b)\left(1-\text{sech}^2(\right.}\right.\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{\frac{2(8a^2+12ab+3b^2)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-16\sqrt{a}(a+b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{2(a+b)}-\frac{(4a+3b)\sqrt{a+b\text{sech}^2(}}{(a+b)\left(1-\text{sech}^2(x)\right.}\right.\right.$$

input
```
Int[Coth[x]^5*Sqrt[a + b*Sech[x]^2],x]
```

output
```
(-1/2*Sqrt[a + b*Sech[x]^2]/(1 - Sech[x]^2)^2 + (-1/2*(-16*Sqrt[a]*(a + b)
*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] + (2*(8*a^2 + 12*a*b + 3*b^2)*ArcT
anh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a + b) - ((4*a + 3*b
)*Sqrt[a + b*Sech[x]^2])/((a + b)*(1 - Sech[x]^2)))/4)/2
```

3.186.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.186. $\int \coth^5(x)\sqrt{a+b\operatorname{sech}^2(x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.186.4 Maple [F]

$$\int \coth(x)^5 \sqrt{a + \operatorname{sech}(x)^2 b} dx$$

input
```
int(coth(x)^5*(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(coth(x)^5*(a+sech(x)^2*b)^(1/2),x)
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. $2(103) = 206$.

Time = 0.60 (sec) , antiderivative size = 12548, normalized size of antiderivative = 100.38

$$\int \coth^5(x)\sqrt{a + b\operatorname{sech}^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(coth(x)**5*(a+b*sech(x)**2)**(1/2),x)
```

output
```
Timed out
```

3.186.7 Maxima [F]

$$\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \int \sqrt{b\,\text{sech}\,(x)^2+a}\coth\,(x)^5\;dx$$

input
```
integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(x)^2 + a)*coth(x)^5, x)
```

3.186.8 Giac [F(-2)]

Exception generated.

$$\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx=\int \coth(x)^5\,\sqrt{a+\frac{b}{\cosh\left(x\right)^2}}\,dx$$

input
```
int(coth(x)^5*(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^5*(a + b/cosh(x)^2)^(1/2), x)
```

3.187 $\int \left(a+b\text{sech}^2(x)\right)^{3/2} \tanh^3(x)\,dx$

3.187.1 Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} \tanh^3(x)\,dx = a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - a\sqrt{a+b\text{sech}^2(x)} - \frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b}$$

output
```
a^(3/2)*arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))-1/3*(a+b*sech(x)^2)^(3/2)+1
/5*(a+b*sech(x)^2)^(5/2)/b-a*(a+b*sech(x)^2)^(1/2)
```

3.187.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} \tanh^3(x)\,dx = \frac{a^{3/2}b\,\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - ab\sqrt{a+b\text{sech}^2(x)} - \frac{1}{3}b\left(a+b\text{sech}^2(x)\right)^{3/2} + \frac{1}{5}\cdots}{b}$$

input
```
Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3,x]
```

output
```
(a^(3/2)*b*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - a*b*Sqrt[a + b*Sech[x]
^2] - (b*(a + b*Sech[x]^2)^(3/2))/3 + (a + b*Sech[x]^2)^(5/2)/5)/b
```

3.187.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 26, 4627, 25, 354, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(x) \left(a+b\text{sech}^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ix)^3\left(a+b\sec(ix)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \left(b\sec(ix)^2+a\right)^{3/2}\tan(ix)^3dx$$

$$\Big\downarrow \text{4627}$$

$$\int -\cosh(x)\left(1-\text{sech}^2(x)\right)\left(a+b\text{sech}^2(x)\right)^{3/2}d\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \cosh(x)\left(1-\text{sech}^2(x)\right)\left(b\text{sech}^2(x)+a\right)^{3/2}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \cosh(x)\left(1-\text{sech}^2(x)\right)\left(b\text{sech}^2(x)+a\right)^{3/2}d\text{sech}^2(x)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{2\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b}-\int \cosh(x)\left(b\text{sech}^2(x)+a\right)^{3/2}d\text{sech}^2(x)\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\int \cosh(x)\sqrt{b\text{sech}^2(x)+a}d\text{sech}^2(x)+\frac{2\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b}-\frac{2}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(a\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)+2\sqrt{a+b\text{sech}^2(x)}\right)+\frac{2\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b}-\frac{2}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left(-a\left(\frac{2a\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}+2\sqrt{a+b\text{sech}^2(x)}\right)+\frac{2\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b}-\frac{2}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left(-a\left(2\sqrt{a+b\text{sech}^2(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)\right)+\frac{2\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b}-\frac{2}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}\right)$$

input
```
Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3,x]
```

output
```
((-2*(a + b*Sech[x]^2)^(3/2))/3 + (2*(a + b*Sech[x]^2)^(5/2))/(5*b) - a*(-
2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sech[x]^2]
))/2
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.187. $\int \left(a+b\text{sech}^2(x)\right)^{3/2} \tanh^3(x)\, dx$

3.187.4 Maple [F]

$$\int \left(a+\operatorname{sech}\left(x\right)^{2} b\right)^{\frac{3}{2}} \tanh\left(x\right)^{3}dx$$

input
```
int((a+sech(x)^2*b)^(3/2)*tanh(x)^3,x)
```

output
```
int((a+sech(x)^2*b)^(3/2)*tanh(x)^3,x)
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1745 vs. 2(60) = 120.

Time = 0.51 (sec) , antiderivative size = 4226, normalized size of antiderivative = 55.61

$$\int \left(a+b\operatorname{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="fricas")
```

output
```
[1/60*(15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*
a*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 4
0*(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*
b*cosh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70
*a*b*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b
*cosh(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a
*b*cosh(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3
+ 5*(9*a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x
)^2 + a*b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*c
osh(x)^5 + 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a
^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a
^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*co
sh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)
*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*
a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^
3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(...
```

3.187.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx=\int \left(a+b\,\text{sech}^2\left(x\right)\right)^{\frac{3}{2}}\tanh^3\left(x\right)dx$$

input
```
integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**3,x)
```

output
```
Integral((a + b*sech(x)**2)**(3/2)*tanh(x)**3, x)
```

3.187.7 Maxima [F]

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx=\int \left(b\,\text{sech}\left(x\right)^2+a\right)^{\frac{3}{2}}\tanh\left(x\right)^3\,dx$$

input
```
integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="maxima")
```

output
```
integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

3.187.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx=\int \tanh(x)^3\,\left(a+\frac{b}{\cosh\left(x\right)^2}\right)^{3/2}\,dx$$

input
```
int(tanh(x)^3*(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^3*(a + b/cosh(x)^2)^(3/2), x)
```

3.188 $\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx$

3.188.1 Optimal result

Integrand size = 17, antiderivative size = 125

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx = -\frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8\sqrt{b}} + a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)}$$

output
```
a^(3/2)*arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))-1/8*(3*a^2-6*a*b-
b^2)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)-1/8*(5*a+b)*(
a+b-b*tanh(x)^2)^(1/2)*tanh(x)+1/4*b*(a+b-b*tanh(x)^2)^(1/2)*tanh(x)^3
```

3.188.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.58

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx =$$

$$-\frac{\cosh^3(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\left(\sqrt{2}(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)-8\sqrt{2}a^{3/2}\sqrt{b}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\,\text{s}}{\sqrt{a+2b+a}}\right.\right.}{4\sqrt{b}(a+2b}$$

input
```
Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]
```

output
```
-1/4*(Cosh[x]^3*(a + b*Sech[x]^2)^(3/2)*(Sqrt[2]*(3*a^2 - 6*a*b - b^2)*Arc
Tan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*a^(
3/2)*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]
] + (5*a - b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] + 2*b^(3
/2)*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(Sqrt[b]*(a + 2*b + a*
Cosh[2*x])^(3/2))
```

3.188.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {3042, 25, 4629, 25, 2075, 379, 25, 444, 25, 27, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(x)\left(a+b\text{sech}^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^2\left(-\left(a+b\sec(ix)^2\right)^{3/2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(b\sec(ix)^2+a\right)^{3/2}\tan(ix)^2dx$$

$$\Big\downarrow \text{4629}$$

$$-\int -\frac{\tanh^2(x)\left(a+b(1-\tanh^2(x))\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^2(x)\left(a+b(1-\tanh^2(x))\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\tanh^2(x)\left(a-b\tanh^2(x)+b\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{379}$$

$$\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}-\frac{1}{4}\int -\frac{\tanh^2(x)\left((a+b)(4a+b)-b(5a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{\tanh^2(x)\left((a+b)(4a+b)-b(5a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{444}$$

$$\frac{1}{4}\left(-\frac{\int -\frac{b\left(\left(3a^2-6ba-b^2\right)\tanh^2(x)+(a+b)(5a+b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{2b}-\frac{1}{2}(5a+b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}\right)+$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{\int \frac{b\left(\left(3a^2-6ba-b^2\right)\tanh^2(x)+(a+b)(5a+b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{2b}-\frac{1}{2}(5a+b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}\right)+$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\left(3a^2-6ba-b^2\right)\tanh^2(x)+(a+b)(5a+b)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{1}{2}(5a+b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}\right)+$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8a^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\left(3a^2-6ab-b^2\right)\int \frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh\right.\right.$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{224}$$

3.188. $\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx$

$$\frac{1}{4}\left(\frac{1}{2}\left(8a^2\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\left(3a^2-6ab-b^2\right)\int \frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}\right)\right.$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$\downarrow$ 216

$$\frac{1}{4}\left(\frac{1}{2}\left(8a^2\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}\right)-\right.$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$\downarrow$ 291

$$\frac{1}{4}\left(\frac{1}{2}\left(8a^2\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(5a\right.$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

$\downarrow$ 219

$$\frac{1}{4}\left(\frac{1}{2}\left(8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(5a+b)\tanh(x\right.$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

input
```
Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]
```

output
```
(b*Tanh[x]^3*Sqrt[a + b - b*Tanh[x]^2])/4 + ((-(((3*a^2 - 6*a*b - b^2)*Arc
Tan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[b]) + 8*a^(3/2)*Arc
Tanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/2 - ((5*a + b)*Tanh[x]*
Sqrt[a + b - b*Tanh[x]^2])/2)/4
```

3.188. $\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx$

3.188.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.188. $\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx$

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.188.4 Maple [F]

$$\int \left(a+\operatorname{sech}\left(x\right)^{2} b\right)^{\frac{3}{2}} \tanh\left(x\right)^{2}dx$$

input
```
int((a+sech(x)^2*b)^(3/2)*tanh(x)^2,x)
```

output
```
int((a+sech(x)^2*b)^(3/2)*tanh(x)^2,x)
```

3.188.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1826 vs. 2(103) = 206.

Time = 0.52 (sec) , antiderivative size = 8582, normalized size of antiderivative = 68.66

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.188.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx=\int \left(a+b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}\tanh^2(x)\,dx$$

input
```
integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**2,x)
```

output
```
Integral((a + b*sech(x)**2)**(3/2)*tanh(x)**2, x)
```

3.188.7 Maxima [F]

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx=\int \left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}\tanh(x)^2\,dx$$

input
```
integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="maxima")
```

output
```
integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

3.188.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\operatorname{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\operatorname{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx=\int \tanh(x)^2\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}\,dx$$

input
```
int(tanh(x)^2*(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^2*(a + b/cosh(x)^2)^(3/2), x)
```

3.189 $\int \left(a+b\text{sech}^2(x)\right)^{3/2} \tanh(x)\,dx$

3.189.1 Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} \tanh(x)\,dx = a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - a\sqrt{a+b\text{sech}^2(x)} - \frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}$$

output
```
a^(3/2)*arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))-1/3*(a+b*sech(x)^2)^(3/2)-a
*(a+b*sech(x)^2)^(1/2)
```

3.189.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} \tanh(x)\,dx = -\frac{2b\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{a\cosh^2(x)}{b}\right)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3\sqrt{1+\frac{a\cosh^2(x)}{b}}\,(a+2b+a\cosh(2x))}$$

input
```
Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x],x]
```

output
```
(-2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cosh[x]^2)/b)]*(a + b*Sech[
x]^2)^(3/2))/(3*Sqrt[1 + (a*Cosh[x]^2)/b]*(a + 2*b + a*Cosh[2*x]))
```

3.189.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 26, 4627, 243, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x) \left(a+b\text{sech}^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\tan(ix) \left(a+b\sec(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \ 26$$

$$-i\int \left(b\sec(ix)^2+a\right)^{3/2} \tan(ix)dx$$

$$\Big\downarrow \ 4627$$

$$-\int \cosh(x) \left(b\text{sech}^2(x)+a\right)^{3/2} d\text{sech}(x)$$

$$\Big\downarrow \ 243$$

$$-\frac{1}{2}\int \cosh(x) \left(b\text{sech}^2(x)+a\right)^{3/2} d\text{sech}^2(x)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(-a\int \cosh(x)\sqrt{b\text{sech}^2(x)+a}d\text{sech}^2(x)-\frac{2}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(-a\left(a\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)+2\sqrt{a+b\text{sech}^2(x)}\right)-\frac{2}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(-a\left(\frac{2a\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}+2\sqrt{a+b\text{sech}^2(x)}\right)-\frac{2}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-a\left(2\sqrt{a+b\text{sech}^2(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)\right)-\frac{2}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}\right)$$

input
```
Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x],x]
```

output
```
((-2*(a + b*Sech[x]^2)^(3/2))/3 - a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a]] + 2*Sqrt[a + b*Sech[x]^2]))/2
```

3.189.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.189.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$-\frac{\left(a+\operatorname{sech}(x)^{2} b\right)^{\frac{3}{2}}}{3}+a^{\frac{3}{2}} \ln \left(\frac{2 a+2 \sqrt{a}\, \sqrt{a+\operatorname{sech}(x)^{2} b}}{\operatorname{sech}(x)}\right)-a \sqrt{a+\operatorname{sech}\left(x\right)^{2} b}$	56
default	$-\frac{\left(a+\operatorname{sech}(x)^{2} b\right)^{\frac{3}{2}}}{3}+a^{\frac{3}{2}} \ln \left(\frac{2 a+2 \sqrt{a}\, \sqrt{a+\operatorname{sech}(x)^{2} b}}{\operatorname{sech}(x)}\right)-a \sqrt{a+\operatorname{sech}\left(x\right)^{2} b}$	56

input
```
int((a+sech(x)^2*b)^(3/2)*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a+sech(x)^2*b)^(3/2)+a^(3/2)*ln((2*a+2*a^(1/2)*(a+sech(x)^2*b)^(1/2)
)/sech(x))-a*(a+sech(x)^2*b)^(1/2)
```

3.189.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(45) = 90$.

Time = 0.35 (sec) , antiderivative size = 2312, normalized size of antiderivative = 40.56

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh(x)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^
4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh
(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 +
6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3
 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^
7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*
a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*
a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*
b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 +
4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 +
b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2
*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 1
5*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3
 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)
*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)
*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*co
sh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^
3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^...
```

3.189.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh(x)\,dx = \int \left(a+b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}\tanh(x)\,dx$$

input
```
integrate((a+b*sech(x)**2)**(3/2)*tanh(x),x)
```

output
```
Integral((a + b*sech(x)**2)**(3/2)*tanh(x), x)
```

3.189.7 Maxima [F]

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh(x)\,dx=\int \left(b\,\text{sech}\,(x)^2+a\right)^{\frac{3}{2}}\tanh\,(x)\;dx$$

input
```
integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="maxima")
```

output
```
integrate((b*sech(x)^2 + a)^(3/2)*tanh(x), x)
```

3.189.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh(x)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.189.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh(x)\,dx=a^{3/2}\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}{3}-a\sqrt{a+\frac{b}{\cosh\,(x)^2}}$$

input
```
int(tanh(x)*(a + b/cosh(x)^2)^(3/2),x)
```

output
```
a^(3/2)*atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2)) - (a + b/cosh(x)^2)^(3/2)/3
 - a*(a + b/cosh(x)^2)^(1/2)
```

3.190 $\int \left(a+b\text{sech}^2(x)\right)^{3/2} \, dx$

3.190.1 Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} \, dx = \frac{1}{2}\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)}$$

output
```
a^(3/2)*arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))+1/2*(3*a+b)*arcta
n(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*b^(1/2)+1/2*b*(a+b-b*tanh(x)^2)
^(1/2)*tanh(x)
```

3.190.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

$$\int (a +b\text{sech}^2(x))^{3/2} \, dx = \frac{\left(b+a\cosh^2(x)\right)\text{sech}(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh^2 \cdots\right.}{(a+2b+a\,\text{c}\cdots}$$

input
```
Integrate[(a + b*Sech[x]^2)^(3/2),x]
```

output
```
((b + a*Cosh[x]^2)*Sech[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*Sqrt[b]*(3*a + b
)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x]^2
+ 2*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*Cosh[x]^2 + b*Sqrt[a + 2*b + a*Cosh[2*x]]*Sinh[x]))/(a + 2*b + a*
Cosh[2*x])^(3/2)
```

3.190.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4616, 318, 25, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4616}$$

$$\int \frac{\left(a-b\tanh^2(x)+b\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{318}$$

$$\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}-\frac{1}{2}\int -\frac{(a+b)(2a+b)-b(3a+b)\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{(a+b)(2a+b)-b(3a+b)\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(2a^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+b(3a+b)\int \frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)\right)+\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow 224$$

$$\frac{1}{2}\left(2a^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+b(3a+b)\int \frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a\cdots}}\right.$$
$$\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow 216$$

$$\frac{1}{2}\left(2a^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)\right)+$$
$$\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow 291$$

$$\frac{1}{2}\left(2a^2\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}+\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)\right)+$$
$$\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)\right)+$$
$$\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

input
```
Int[(a + b*Sech[x]^2)^(3/2),x]
```

output
```
(Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + 2
*a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/2 + (b*Tanh
[x]*Sqrt[a + b - b*Tanh[x]^2])/2
```

3.190. $\int \left(a+b\text{sech}^2(x)\right)^{3/2}\,dx$

3.190.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.190. $\int (a+b\text{sech}^2(x))^{3/2}\,dx$

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.190.4 Maple [F]

$$\int \left(a+\operatorname{sech}\left(x\right)^{2} b\right)^{\frac{3}{2}}d x$$

input
```
int((a+sech(x)^2*b)^(3/2),x)
```

output
```
int((a+sech(x)^2*b)^(3/2),x)
```

3.190.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(70) = 140$.

Time = 0.38 (sec) , antiderivative size = 4140, normalized size of antiderivative = 47.05

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} \, dx=\text{Too large to display}$$

input
```
integrate((a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a
)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)
^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh
(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
+ 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a
*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*
(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a
^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3
)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(
x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 -
3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3
 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4
- 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6
*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2
)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*s
inh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*si
nh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((3*a + b)*cosh(x)^4 + 4*...
```

3.190.6 Sympy [F]

$$\int \left(a+b\operatorname{sech}^2(x)\right)^{3/2}\,dx=\int \left(a+b\operatorname{sech}^2{(x)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sech(x)**2)**(3/2),x)
```

output
```
Integral((a + b*sech(x)**2)**(3/2), x)
```

3.190.7 Maxima [F]

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} dx = \int \left(b\,\text{sech}\,(x)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(x)^2 + a)^(3/2), x)
```

3.190.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2} dx = \int \left(a+\frac{b}{\cosh\,(x)^2}\right)^{3/2} dx$$

input
```
int((a + b/cosh(x)^2)^(3/2),x)
```

output
```
int((a + b/cosh(x)^2)^(3/2), x)
```

3.191 $\int \coth(x) \left(a+b\text{sech}^2(x)\right)^{3/2} dx$

3.191.1 Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \coth(x) \left(a+b\text{sech}^2(x)\right)^{3/2} dx = a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right) + b\sqrt{a+b\text{sech}^2(x)}$$

output
```
a^(3/2)*arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))-(a+b)^(3/2)*arctanh((a+b*se
ch(x)^2)^(1/2)/(a+b)^(1/2))+b*(a+b*sech(x)^2)^(1/2)
```

3.191.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 159 vs. 2(70) = 140.

Time = 0.52 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.27

$$\int \coth(x) \left(a+b\text{sech}^2(x)\right)^{3/2} dx = -\frac{2(b+a\cosh^2(x))\left(\sqrt{2}(a+b)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh(x)-\sqrt{a+b}\left(b\sqrt{a+2b+a\cosh(2x)}+\sqrt{2}\right.\right.}{\sqrt{a+b}(a+2b+a\cosh(2x))^{3/}}$$

input
```
Integrate[Coth[x]*(a + b*Sech[x]^2)^(3/2),x]
```

output
```
(-2*(b + a*Cosh[x]^2)*(Sqrt[2]*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh
[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x] - Sqrt[a + b]*(b*Sqrt[a + 2*b +
a*Cosh[2*x]] + Sqrt[2]*a^(3/2)*Cosh[x]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[
a + 2*b + a*Cosh[2*x]]]))*Sqrt[a + b*Sech[x]^2])/(Sqrt[a + b]*(a + 2*b + a
*Cosh[2*x])^(3/2))
```

3.191.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = $\{3042, 26, 4627, 25, 354, 95, 25, 174, 73, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x) \left(a+b\text{sech}^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sec(ix)^2\right)^{3/2}}{\tan(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sec(ix)^2+a\right)^{3/2}}{\tan(ix)}dx$$

$$\Big\downarrow \text{4627}$$

$$\int -\frac{\cosh(x)\left(a+b\text{sech}^2(x)\right)^{3/2}}{1-\text{sech}^2(x)}d\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cosh(x)\left(b\text{sech}^2(x)+a\right)^{3/2}}{1-\text{sech}^2(x)}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)\left(b\text{sech}^2(x)+a\right)^{3/2}}{1-\text{sech}^2(x)}d\text{sech}^2(x)$$

$$\Big\downarrow \text{95}$$

$$\frac{1}{2}\left(\int -\frac{\cosh(x)\left(a^2+b(2a+b)\text{sech}^2(x)\right)}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)+2b\sqrt{a+b\text{sech}^2(x)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(2b\sqrt{a+b\text{sech}^2(x)}-\int \frac{\cosh(x)\left(a^2+b(2a+b)\text{sech}^2(x)\right)}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(a^2\left(-\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)\right)-(a+b)^2\int \frac{1}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)+2b\sqrt{a+b\text{sech}^2}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{2a^2\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}-\frac{2(a+b)^2\int \frac{1}{\frac{a+b}{b}-\frac{\text{sech}^4(x)}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}+2b\sqrt{a+b\text{sech}^2(x)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)+2b\sqrt{a+b\text{sech}^2(x)}\right)$$

input
```
Int[Coth[x]*(a + b*Sech[x]^2)^(3/2),x]
```

output
```
(2*a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - 2*(a + b)^(3/2)*ArcTan
h[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]] + 2*b*Sqrt[a + b*Sech[x]^2])/2
```

3.191.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.191. $\int \coth(x) \left(a + b\text{sech}^2(x)\right)^{3/2} dx$

3.191.4 Maple [F]

$$\int \coth(x) \left(a + \operatorname{sech}(x)^2 b\right)^{\frac{3}{2}} dx$$

input
```
int(coth(x)*(a+sech(x)^2*b)^(3/2),x)
```

output
```
int(coth(x)*(a+sech(x)^2*b)^(3/2),x)
```

3.191.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(56) = 112$.

Time = 0.38 (sec) , antiderivative size = 4123, normalized size of antiderivative = 58.90

$$\int \coth(x) \left(a + b\text{sech}^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*log(((
a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(
x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 +
b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*
(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a
^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2
 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
+ 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6
+ 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*
a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b
^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b
^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)
*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(
x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^
2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 +
 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a...
```

3.191.6 Sympy [F(-1)]

Timed out.

$$\int \coth(x)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(coth(x)*(a+b*sech(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.191.7 Maxima [F]

$$\int \coth(x)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}\,dx=\int \left(b\,\mathrm{sech}\,(x)^2+a\right)^{\frac{3}{2}}\coth\,(x)\;dx$$

input
```
integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(x)^2 + a)^(3/2)*coth(x), x)
```

3.191.8 Giac [F(-2)]

Exception generated.

$$\int \coth(x)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{2048,[4,6]%%%}+%%%{%%%{6144,[1]%%%},[4,5]%%%}+%%%{%%%{6144
,[2]%%%},
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \coth(x)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}\,dx=\int \coth(x)\,\left(a+\frac{b}{\cosh\left(x\right)^2}\right)^{3/2}\,dx$$

input
```
int(coth(x)*(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(coth(x)*(a + b/cosh(x)^2)^(3/2), x)
```

3.192 $\int \coth^2(x) \left(a+b\text{sech}^2(x)\right)^{3/2} \, dx$

3.192.1 Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \coth^2(x) \left(a+b\text{sech}^2(x)\right)^{3/2} \, dx = -b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - (a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)}$$

output
```
-b^(3/2)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))+a^(3/2)*arctanh(a
^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))-(a+b)*coth(x)*(a+b-b*tanh(x)^2)^(1
/2)
```

3.192.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int \coth^2(x) \left(a+b\text{sech}^2(x)\right)^{3/2} \, dx = -\frac{2\left(b+a\cosh^2(x)\right)\left(\sqrt{2}b^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh(x)-\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh(x)+}{(a+2b+a\cosh(2x))^{3/2}}$$

input
```
Integrate[Coth[x]^2*(a + b*Sech[x]^2)^(3/2),x]
```

output
```
(-2*(b + a*Cosh[x]^2)*(Sqrt[2]*b^(3/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sq
rt[a + 2*b + a*Cosh[2*x]]]*Cosh[x] - Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt
[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x] + (a + b)*Sqrt[a + 2*b +
 a*Cosh[2*x]]*Coth[x])*Sqrt[a + b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])^(3/2
)
```

3.192.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 25, 4629, 25, 2075, 376, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x)\left(a+b\text{sech}^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+b\sec(ix)^2\right)^{3/2}}{\tan(ix)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sec(ix)^2+a\right)^{3/2}}{\tan(ix)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\int -\frac{\coth^2(x)\left(a+b\left(1-\tanh^2(x)\right)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^2(x)\left(a+b\left(1-\tanh^2(x)\right)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\coth^2(x)\left(a-b\tanh^2(x)+b\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{376}$$

$$\int \frac{a^2-b^2+b^2\tanh^2(x)}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-(a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{398}$$

$$a^2\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-b^2\int \frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-(a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{224}$$

$$a^2\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-b^2\int \frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-(a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{216}$$

$$a^2\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-(a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{291}$$

$$a^2\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-(a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

$$\Big\downarrow \text{219}$$

$$a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-(a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

input
```
Int[Coth[x]^2*(a + b*Sech[x]^2)^(3/2),x]
```

output
```
-(b^(3/2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]) + a^(3/2)*A
rcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (a + b)*Coth[x]*Sqrt
[a + b - b*Tanh[x]^2]
```

3.192. $\int \coth^2(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\,dx$

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.192. $\int \coth^2(x) \left(a + b\,\mathrm{sech}^2(x)\right)^{3/2} dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.192.4 Maple [F]

$$\int \coth(x)^2 \left(a+\operatorname{sech}(x)^2 b\right)^{\frac{3}{2}} dx$$

input
```
int(coth(x)^2*(a+sech(x)^2*b)^(3/2),x)
```

output
```
int(coth(x)^2*(a+sech(x)^2*b)^(3/2),x)
```

3.192.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(67) = 134$.

Time = 0.38 (sec) , antiderivative size = 3349, normalized size of antiderivative = 41.35

$$\int \coth^2(x) \left(a+b\operatorname{sech}^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log((a
*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 -
b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*
b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*
b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2
 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*co
sh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*
a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^
3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4
+ 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x)
)*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)
^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 -
 (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x
)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a
^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh
(x)*sinh(x)^5 + sinh(x)^6)) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 - b)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x...
```

3.192.6 Sympy [F(-1)]

Timed out.

$$\int \coth^2(x) \left(a + b\text{sech}^2(x)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(coth(x)**2*(a+b*sech(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.192.7 Maxima [F]

$$\int \coth^2(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\,dx=\int \left(b\,\text{sech}\,(x)^2+a\right)^{\frac{3}{2}}\coth\,(x)^2\,dx$$

input
```
integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(x)^2 + a)^(3/2)*coth(x)^2, x)
```

3.192.8 Giac [F(-2)]

Exception generated.

$$\int \coth^2(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{2048,[4,6]%%%}+%%%{%%%{6144,[1]%%%},[4,5]%%%}+%%%{%%%{6144
,[2]%%%},
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \coth^2(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\,dx=\int \coth(x)^2\left(a+\frac{b}{\cosh\,(x)^2}\right)^{3/2}\,dx$$

input
```
int(coth(x)^2*(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2*(a + b/cosh(x)^2)^(3/2), x)
```

3.193 $\int (a+b\text{sech}^2(c+dx))^{5/2}\,dx$

3.193.1 Optimal result

Integrand size = 16, antiderivative size = 170

$$\int (a+b\text{sech}^2(c+dx))^{5/2}\,dx = \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{8d} + \frac{a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{d} + \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^{3/2}}{4d}$$

output
```
a^(5/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+b-b*tanh(d*x+c)^2)^(1/2))/d+1/8*(15
*a^2+10*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/(a+b-b*tanh(d*x+c)^2)^(1/2))
*b^(1/2)/d+1/8*b*(7*a+3*b)*(a+b-b*tanh(d*x+c)^2)^(1/2)*tanh(d*x+c)/d+1/4*b
*tanh(d*x+c)*(a+b-b*tanh(d*x+c)^2)^(3/2)/d
```

3.193.2 Mathematica [A] (verified)

Time = 6.96 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.65

$$\int (a+b\text{sech}^2(c+dx))^{5/2}\,dx=\frac{\left(\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a+b+a\sinh^2(c+dx)}}\right)+8a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b+a\sinh^2(c+dx)}}\right)\right)\text{co}}{\sqrt{2}d(a+2b+a\cosh(2c+2dx))^{5/2}}$$
$$+\frac{\cosh^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^{5/2}\left(\frac{b^2\text{sech}(c)\text{sech}^4(c+dx)\sinh(dx)}{d}+\frac{3\text{sech}(c)\text{sech}^2(c+dx)(3ab\sinh(dx)+b^2\sinh(dx))}{2d}\right.}{(a+2b+a\cosh(2c+2dx))^2}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^(5/2),x]
```

output
```
((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a
+ b + a*Sinh[c + d*x]^2]] + 8*a^(5/2)*ArcTanh[(Sqrt[a]*Sinh[c + d*x])/Sqrt
[a + b + a*Sinh[c + d*x]^2]])*Cosh[c + d*x]^5*(a + b*Sech[c + d*x]^2)^(5/2
))/(Sqrt[2]*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^(5/2)) + (Cosh[c + d*x]^5*(a
 + b*Sech[c + d*x]^2)^(5/2)*((b^2*Sech[c]*Sech[c + d*x]^4*Sinh[d*x])/d + (
3*Sech[c]*Sech[c + d*x]^2*(3*a*b*Sinh[d*x] + b^2*Sinh[d*x]))/(2*d) + (3*b*
(3*a + b)*Sech[c + d*x]*Tanh[c])/(2*d) + (b^2*Sech[c + d*x]^3*Tanh[c])/d))
/(a + 2*b + a*Cosh[2*c + 2*d*x])^2
```

3.193.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 4616, 318, 403, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{sech}^2(c+dx)\right)^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\sec(ic+idx)^2\right)^{5/2}dx$$

$$\Big\downarrow \ 4616$$

$$\frac{\int \frac{(-b\tanh^2(c+dx)+a+b)^{5/2}}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$\downarrow$ 318

$$\frac{\frac{1}{4}b\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^{3/2}-\frac{1}{4}\int \frac{\sqrt{-b\tanh^2(c+dx)+a+b}(b(7a+3b)\tanh^2(c+dx)+(a+b)(b-4(a+b)))}{1-\tanh^2(c+dx)}d\tanh(c}{d}$$

$\downarrow$ 403

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{(a+b)(8a^2+7ba+3b^2)-b(15a^2+10ba+3b^2)\tanh^2(c+dx)}{(1-\tanh^2(c+dx))\sqrt{-b\tanh^2(c+dx)+a+b}}d\tanh(c+dx)+\frac{1}{2}b(7a+3b)\tanh(c+dx)\sqrt{a-b\tanh^2(c+}\right.}{d}$$

$\downarrow$ 398

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{(1-\tanh^2(c+dx))\sqrt{-b\tanh^2(c+dx)+a+b}}d\tanh(c+dx)+b\left(15a^2+10ab+3b^2\right)\int \frac{1}{\sqrt{-b\tanh^2(c+dx)+a+b}}d\tanh\right.\right.}{d}$$

$\downarrow$ 224

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{(1-\tanh^2(c+dx))\sqrt{-b\tanh^2(c+dx)+a+b}}d\tanh(c+dx)+b\left(15a^2+10ab+3b^2\right)\int \frac{1}{\frac{b\tanh^2(c+dx)}{-b\tanh^2(c+dx)+a+b}+1}d\frac{\mathrm{ta}}{\sqrt{-b\,\mathrm{ta}}}\right.\right.}{}$$

$\downarrow$ 216

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{(1-\tanh^2(c+dx))\sqrt{-b\tanh^2(c+dx)+a+b}}d\tanh(c+dx)+\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+}}\right.\right.\right.}{d}$$

$\downarrow$ 291

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{1-\frac{a\tanh^2(c+dx)}{-b\tanh^2(c+dx)+a+b}}d\frac{\tanh(c+dx)}{\sqrt{-b\tanh^2(c+dx)+a+b}}+\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)\right)+\frac{1}{2}\right.}{d}$$

$\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)+\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)\right)+\frac{1}{2}b(7a+3b)\right.}{d}$$

3.193. $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^{5/2}\,dx$

input
```
Int[(a + b*Sech[c + d*x]^2)^(5/2),x]
```

output
```
((b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2)^(3/2))/4 + ((Sqrt[b]*(15*a^2
 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b - b*Tanh[c +
d*x]^2]] + 8*a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + b - b*Tanh[c
 + d*x]^2]])/2 + (b*(7*a + 3*b)*Tanh[c + d*x]*Sqrt[a + b - b*Tanh[c + d*x]
^2])/2)/4)/d
```

3.193.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.193.4 Maple [F]

$$\int \left(a+b\,\mathrm{sech}\left(dx+c\right)^{2}\right)^{\frac{5}{2}}dx$$

input
```
int((a+b*sech(d*x+c)^2)^(5/2),x)
```

output
```
int((a+b*sech(d*x+c)^2)^(5/2),x)
```

3.193.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2712 vs. 2(148) = 296.

Time = 0.74 (sec) , antiderivative size = 12452, normalized size of antiderivative = 73.25

$$\int \left(a+b\text{sech}^2(c+dx)\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.193.6 Sympy [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)^{5/2}\,dx=\int \left(a+b\,\text{sech}^2\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sech(d*x+c)**2)**(5/2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**(5/2), x)
```

3.193.7 Maxima [F]

$$\int \left(a+b\text{sech}^2(c+dx)\right)^{5/2}\,dx=\int \left(b\,\text{sech}\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c)^2 + a)^(5/2), x)
```

3.193.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^{5/2} dx = \int \left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^{5/2} dx$$

input
```
int((a + b/cosh(c + d*x)^2)^(5/2),x)
```

output
```
int((a + b/cosh(c + d*x)^2)^(5/2), x)
```

3.194 $\int \frac{\tanh^5(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

3.194.1 Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{(a+2b)\sqrt{a+b\text{sech}^2(x)}}{b^2} - \frac{\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}$$

output
```
-1/3*(a+b*sech(x)^2)^(3/2)/b^2+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1
/2)+(a+2*b)*(a+b*sech(x)^2)^(1/2)/b^2
```

3.194.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \frac{2a(a+3b)+b(a+6b)\text{sech}^2(x)-b^2\text{sech}^4(x)+3b^2\text{arctanh}\left(\sqrt{1+\frac{b\text{sech}^2(x)}{a}}\right)\sqrt{1+\frac{b\text{sech}^2(x)}{a}}}{3b^2\sqrt{a+b\text{sech}^2(x)}}$$

input
```
Integrate[Tanh[x]^5/Sqrt[a + b*Sech[x]^2],x]
```

output
```
(2*a*(a + 3*b) + b*(a + 6*b)*Sech[x]^2 - b^2*Sech[x]^4 + 3*b^2*ArcTanh[Sqr
t[1 + (b*Sech[x]^2)/a]]*Sqrt[1 + (b*Sech[x]^2)/a])/(3*b^2*Sqrt[a + b*Sech[
x]^2])
```

3.194.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\text{sech}^2(x)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i\tan(ix)^5}{\sqrt{a+b\sec(ix)^2}}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \frac{\tan(ix)^5}{\sqrt{b\sec(ix)^2+a}}dx$$
$$\Big\downarrow \text{ 4627}$$
$$-\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)^2}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}(x)$$
$$\Big\downarrow \text{ 354}$$
$$-\frac{1}{2}\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)^2}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)$$
$$\Big\downarrow \text{ 99}$$
$$-\frac{1}{2}\int \left(\frac{-a-2b}{b\sqrt{b\text{sech}^2(x)+a}}+\frac{\sqrt{b\text{sech}^2(x)+a}}{b}+\frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}\right)d\text{sech}^2(x)$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}+\frac{2(a+2b)\sqrt{a+b\text{sech}^2(x)}}{b^2}\right)$$

input
```
Int[Tanh[x]^5/Sqrt[a + b*Sech[x]^2],x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]])/Sqrt[a] + (2*(a + 2*b)*Sqrt[a
+ b*Sech[x]^2])/b^2 - (2*(a + b*Sech[x]^2)^(3/2))/(3*b^2))/2
```

3.194.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.194. $\int \frac{\tanh^5(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.194.4 Maple [F]

$$\int \frac{\tanh(x)^5}{\sqrt{a+\operatorname{sech}(x)^2 b}}dx$$

input
```
int(tanh(x)^5/(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(tanh(x)^5/(a+sech(x)^2*b)^(1/2),x)
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 971 vs. $2(54) = 108$.

Time = 0.39 (sec) , antiderivative size = 2678, normalized size of antiderivative = 40.58

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*
cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b
^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(
x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cos
h(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*
(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2
 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*
a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*si
nh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^
2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 +
10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x
)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)
*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)
^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4
*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh...
```

3.194.6 Sympy [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a + b\text{sech}^2(x)}}\,dx = \int \frac{\tanh^5(x)}{\sqrt{a + b\,\text{sech}^2(x)}}\,dx$$

input
```
integrate(tanh(x)**5/(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**5/sqrt(a + b*sech(x)**2), x)
```

3.194.7 Maxima [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \int \frac{\tanh(x)^5}{\sqrt{b\operatorname{sech}(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^5/sqrt(b*sech(x)^2 + a), x)
```

3.194.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \int \frac{\tanh(x)^5}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

input
```
int(tanh(x)^5/(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^5/(a + b/cosh(x)^2)^(1/2), x)
```

3.195 $\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

3.195.1 Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = -\frac{(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}} + \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b}$$

output
```
-1/2*(a+3*b)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(3/2)+arcta
nh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)+1/2*(a+b-b*tanh(x)^2)^
(1/2)*tanh(x)/b
```

3.195.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.88

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \frac{\text{sech}(x)\left(2\sqrt{2}b^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\sqrt{a+2b+a\cosh(2x)}+\sqrt{a}\left(-\sqrt{2}(a+3b)\arctan\left(\frac{\sqrt{2}\sqrt{b}\sinh}{\sqrt{a+2b+a\cosh}}\right.\right.\right.}{4\sqrt{a}b^{3/2}\sqrt{a+b\text{sech}^2(x)}}$$

input
```
Integrate[Tanh[x]^4/Sqrt[a + b*Sech[x]^2],x]
```

output
```
(Sech[x]*(2*Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b
 + a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*(-(Sqrt[2]*(a + 3*b
)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2
*b + a*Cosh[2*x]]) + Sqrt[b]*(a + 2*b + a*Cosh[2*x])*Sech[x]*Tanh[x])))/(4
*Sqrt[a]*b^(3/2)*Sqrt[a + b*Sech[x]^2])
```

3.195.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4629, 2075, 381, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(ix)^4}{\sqrt{a+b\sec(ix)^2}}dx$$

$$\Big\downarrow \ 4629$$

$$\int \frac{\tanh^4(x)}{(1-\tanh^2(x))\sqrt{a+b(1-\tanh^2(x))}}d\tanh(x)$$

$$\Big\downarrow \ 2075$$

$$\int \frac{\tanh^4(x)}{(1-\tanh^2(x))\sqrt{a-b\tanh^2(x)+b}}d\tanh(x)$$

$$\Big\downarrow \ 381$$

$$\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\int \frac{-((a+3b)\tanh^2(x))+a+b}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{2b}$$

$$\Big\downarrow \ 398$$

$$\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{(a+3b)\int\frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-2b\int\frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{2b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{(a+3b)\int\frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-2b\int\frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{2b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\frac{(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}-2b\int\frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{2b}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\frac{(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}-2b\int\frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}-\frac{\frac{(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}}{2b}$$

input
```
Int[Tanh[x]^4/Sqrt[a + b*Sech[x]^2],x]
```

output
```
-1/2*(((a + 3*b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt
[b] - (2*b*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[a])/
b + (Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/(2*b)
```

3.195. $\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

3.195.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.195. $\int \frac{\tanh^4(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.195.4 Maple [F]

$$\int \frac{\tanh(x)^4}{\sqrt{a+\operatorname{sech}(x)^2 b}}dx$$

input
```
int(tanh(x)^4/(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(tanh(x)^4/(a+sech(x)^2*b)^(1/2),x)
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(72) = 144$.

Time = 0.40 (sec) , antiderivative size = 4569, normalized size of antiderivative = 50.77

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cos
h(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^
2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)
^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 -
 a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))
*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a
^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b
^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh
(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6
 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2
)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
+ b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 +
4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 +
(15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3
*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 +
 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)
^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3
 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 +...
```

3.195.6 Sympy [F]

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\tanh^4(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**4/sqrt(a + b*sech(x)**2), x)
```

3.195.7 Maxima [F]

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\int \frac{\tanh{(x)}^4}{\sqrt{b\,\text{sech}\,(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^4/sqrt(b*sech(x)^2 + a), x)
```

3.195.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\int \frac{\tanh(x)^4}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

input
```
int(tanh(x)^4/(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^4/(a + b/cosh(x)^2)^(1/2), x)
```

3.196 $\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

3.196.1 Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{a+b\text{sech}^2(x)}}{b}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)+(a+b*sech(x)^2)^(1/2)/b
```

3.196.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{a+b\text{sech}^2(x)}}{b}$$

input
```
Integrate[Tanh[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] + Sqrt[a + b*Sech[x]^2]/b
```

3.196.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 26, 4627, 25, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ix)^3}{\sqrt{a+b\sec(ix)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ix)^3}{\sqrt{b\sec(ix)^2+a}}dx$$

$$\Big\downarrow \text{4627}$$

$$\int -\frac{\cosh(x)\left(1-\text{sech}^2(x)\right)}{\sqrt{a+b\text{sech}^2(x)}}d\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+b\text{sech}^2(x)}}{b}-\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+b\text{sech}^2(x)}}{b}-\frac{2\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\sqrt{a+b\text{sech}^2(x)}}{b}\right)$$

input
```
Int[Tanh[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]])/Sqrt[a] + (2*Sqrt[a + b*Sech[x
]^2])/b)/2
```

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

3.196. $\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.196.4 Maple [F]

$$\int \frac{\tanh(x)^3}{\sqrt{a+\operatorname{sech}(x)^2 b}}dx$$

input
```
int(tanh(x)^3/(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(tanh(x)^3/(a+sech(x)^2*b)^(1/2),x)
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(34) = 68$.

Time = 0.32 (sec) , antiderivative size = 1650, normalized size of antiderivative = 39.29

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a)*log(((
a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(
x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 +
b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*
(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a
^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2
 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
+ 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6
+ 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*
a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b
^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b
^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)
*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(
x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^
2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 +
 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a...
```

3.196.6 Sympy [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\tanh^3{(x)}}{\sqrt{a+b\operatorname{sech}^2{(x)}}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**3/sqrt(a + b*sech(x)**2), x)
```

3.196.7 Maxima [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\tanh(x)^3}{\sqrt{b\,\text{sech}(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^3/sqrt(b*sech(x)^2 + a), x)
```

3.196.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\tanh(x)^3}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

input
```
int(tanh(x)^3/(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^3/(a + b/cosh(x)^2)^(1/2), x)
```

3.197 $\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

3.197.1 Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)-arctan(b^(1/2)*ta
nh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.78

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \frac{\left(-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)}{\sqrt{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)}{\sqrt{a}}\right)\sqrt{a+2b+a\cosh(2x)}\text{sech}(x)}{\sqrt{2}\sqrt{a+b\text{sech}^2(x)}}$$

input
```
Integrate[Tanh[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

output
```
((-(ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]/Sqrt[b])
 + ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]/Sqrt[a])
*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[a + b*Sech[x]^2])
```

3.197.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 25, 4629, 25, 2075, 385, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan(ix)^2}{\sqrt{a+b\sec(ix)^2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan(ix)^2}{\sqrt{b\sec(ix)^2+a}}dx$$

$$\Big\downarrow \text{ 4629}$$

$$-\int -\frac{\tanh^2(x)}{(1-\tanh^2(x))\sqrt{a+b(1-\tanh^2(x))}}d\tanh(x)$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\tanh^2(x)}{(1-\tanh^2(x))\sqrt{a+b(1-\tanh^2(x))}}d\tanh(x)$$

$$\Big\downarrow \text{ 2075}$$

$$\int \frac{\tanh^2(x)}{(1-\tanh^2(x))\sqrt{a-b\tanh^2(x)+b}}d\tanh(x)$$

$$\Big\downarrow \text{ 385}$$

$$\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\int \frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)$$

$$\Big\downarrow \; 224$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\int \frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}$$

$$\Big\downarrow \; 216$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

$$\Big\downarrow \; 291$$

$$\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-\frac{\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

$$\Big\downarrow \; 219$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

input
```
Int[Tanh[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

output
```
-(ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(
Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]
```

3.197.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.197. $\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.197.4 Maple [F]

$$\int \frac{\tanh(x)^2}{\sqrt{a+\operatorname{sech}(x)^2 b}}dx$$

input
```
int(tanh(x)^2/(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(tanh(x)^2/(a+sech(x)^2*b)^(1/2),x)
```

3.197. $\int \frac{\tanh^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(48) = 96.

Time = 0.34 (sec) , antiderivative size = 2856, normalized size of antiderivative = 47.60

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*b*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*s
inh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3
)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 +
 (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b
 + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5
 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)
^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2
 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)
*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x
)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cos
h(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh
(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)
^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9
*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(
x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x
)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - 2*a*sqrt(-b)*log(-((a
- b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a +
3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2...
```

3.197.6 Sympy [F]

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\tanh^2(x)}{\sqrt{a+b\,\text{sech}^2(x)}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**2/sqrt(a + b*sech(x)**2), x)
```

3.197.7 Maxima [F]

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\tanh(x)^2}{\sqrt{b\,\text{sech}(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^2/sqrt(b*sech(x)^2 + a), x)
```

3.197.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\tanh(x)^2}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

input
```
int(tanh(x)^2/(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^2/(a + b/cosh(x)^2)^(1/2), x)
```

3.198 $\int \frac{\tanh(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$

3.198.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{\tanh(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Sech[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a]
```

3.198.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 4627, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)}{\sqrt{a+b\sec(ix)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{\sqrt{b\sec(ix)^2+a}}dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Sech[x]^2],x]
```

3.198. $\int \frac{\tanh(x)}{\sqrt{a+b\text{sech}^2(x)}}dx$

output
```
ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a]
```

3.198.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.198. $\int \frac{\tanh(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

3.198.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+\mathrm{sech}(x)^{2}b}}{\mathrm{sech}(x)}\right)}{\sqrt{a}}$	30
default	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+\mathrm{sech}(x)^{2}b}}{\mathrm{sech}(x)}\right)}{\sqrt{a}}$	30

input
```
int(tanh(x)/(a+sech(x)^2*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+sech(x)^2*b)^(1/2))/sech(x))
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(19) = 38.

Time = 0.29 (sec) , antiderivative size = 1430, normalized size of antiderivative = 57.20

$$\int \frac{\tanh(x)}{\sqrt{a+b\mathrm{sech}^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 +
14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh
(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*
cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(
x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a
^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(
2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^
5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5
*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2
+ 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a
^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*
b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b +
b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)
)*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^...
```

3.198.6 Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a + b\mathrm{sech}^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

input
```
integrate(tanh(x)/(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(a + b*sech(x)**2), x)
```

3.198.7 Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\int \frac{\tanh(x)}{\sqrt{b\,\text{sech}(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*sech(x)^2 + a), x)
```

3.198.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.198.9 Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\tanh(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\frac{\text{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(tanh(x)/(a + b/cosh(x)^2)^(1/2),x)
```

output
```
atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.199 $\int \frac{1}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$

3.199.1 Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{1}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)
```

3.199.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int \frac{1}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b+a\sinh^2(x)}}\right)\sqrt{a+2b+a\cosh(2x)}\,\mathrm{sech}(x)}{\sqrt{2}\sqrt{a}\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

input
```
Integrate[1/Sqrt[a + b*Sech[x]^2],x]
```

output
```
(ArcTanh[(Sqrt[a]*Sinh[x])/Sqrt[a + b + a*Sinh[x]^2]]*Sqrt[a + 2*b + a*Cos
h[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2])
```

3.199.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4616, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sec(ix)^2}}dx$$

$$\Big\downarrow \text{4616}$$

$$\int \frac{1}{(1-\tanh^2(x))\sqrt{a-b\tanh^2(x)+b}}d\tanh(x)$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{1-\frac{a\tanh^2(x)}{a-b\tanh^2(x)+b}}d\frac{\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

input
```
Int[1/Sqrt[a + b*Sech[x]^2],x]
```

output
```
ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]
```

3.199.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.199.4 Maple [F]

$$\int \frac{1}{\sqrt{a+\operatorname{sech}\left(x\right)^{2} b}}dx$$

input
```
int(1/(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(1/(a+sech(x)^2*b)^(1/2),x)
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 1059, normalized size of antiderivative = 36.52

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sin
h(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*
sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (
a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b +
 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 -
 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3
 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 -
 b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*s
inh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^
6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(
x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x
)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5
 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
 + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a
*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)
^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^
2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a)*log(-(a*cosh(x)^
4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cos
h(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + si...
```

3.199.6 Sympy [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{a + b\,\text{sech}^2\left(x\right)}}\,dx$$

input
```
integrate(1/(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sech(x)**2), x)
```

3.199.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(x)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sech(x)^2 + a), x)
```

3.199.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

input
```
int(1/(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(1/(a + b/cosh(x)^2)^(1/2), x)
```

3.200 $\int \frac{\coth(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$

3.200.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\coth(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)-arctanh((a+b*sech(x)^2)^(1/
2)/(a+b)^(1/2))/(a+b)^(1/2)
```

3.200.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(56) = 112.

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.21

$$\int \frac{\coth(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$$
$$=\frac{\sqrt{a+2b+a\cosh(2x)}\left(-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+\sqrt{a+b}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh}\right.\right.}{\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

input
```
Integrate[Coth[x]/Sqrt[a + b*Sech[x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cosh[2*x]]*(-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[
x])/Sqrt[a + 2*b + a*Cosh[2*x]]]) + Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x
] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sq
rt[a + b*Sech[x]^2])
```

3.200.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 26, 4627, 25, 354, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\sqrt{a+b\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ix)\sqrt{a+b\sec(ix)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sqrt{b\sec(ix)^2+a}\tan(ix)}dx$$

$$\Big\downarrow \text{4627}$$

$$\int -\frac{\cosh(x)}{\left(1-\text{sech}^2(x)\right)\sqrt{a+b\text{sech}^2(x)}}d\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cosh(x)}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{2}\left(-\int \frac{1}{(1-\text{sech}^2(x))\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)-\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\text{sech}^4(x)}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}-\frac{2\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}\right)$$

input
```
Int[Coth[x]/Sqrt[a + b*Sech[x]^2],x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]])/Sqrt[a] - (2*ArcTanh[Sqrt[a +
b*Sech[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/2
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.200. $\int \frac{\coth(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.200.4 Maple [F]

$$\int \frac{\coth(x)}{\sqrt{a+\operatorname{sech}(x)^2 b}}dx$$

input
```
int(coth(x)/(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(coth(x)/(a+sech(x)^2*b)^(1/2),x)
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(44) = 88$.

Time = 0.36 (sec) , antiderivative size = 3663, normalized size of antiderivative = 65.41

$$\int \frac{\coth(x)}{\sqrt{a + b\text{sech}^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*
a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2
+ b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sin
h(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2
 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 1
0*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^
2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 +
 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*
sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^
4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*
(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^
3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^
2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)
*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(...
```

3.200.6 Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a + b\text{sech}^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a + b\,\text{sech}^2(x)}}\,dx$$

input
```
integrate(coth(x)/(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(coth(x)/sqrt(a + b*sech(x)**2), x)
```

3.200.7 Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\,\text{sech}(x)^2+a}}\,dx$$

input
```
integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/sqrt(b*sech(x)^2 + a), x)
```

3.200.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\coth(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

input
```
int(coth(x)/(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(coth(x)/(a + b/cosh(x)^2)^(1/2), x)
```

3.201 $\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

3.201.1 Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}-\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)-coth(x)*(a+b-b*ta
nh(x)^2)^(1/2)/(a+b)
```

3.201.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.77

$$\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$$
$$=\frac{\sqrt{a+2b+a\cosh(2x)}\text{sech}(x)\left((a+b)\text{arctanh}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b+a\sinh^2(x)}}\right)-\sqrt{a}\text{csch}(x)\sqrt{a+b+a\sinh^2(x)}\right)}{\sqrt{2}\sqrt{a}(a+b)\sqrt{a+b\text{sech}^2(x)}}$$

input
```
Integrate[Coth[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*((a + b)*ArcTanh[(Sqrt[a]*Sinh[x])/Sq
rt[a + b + a*Sinh[x]^2]] - Sqrt[a]*Csch[x]*Sqrt[a + b + a*Sinh[x]^2]))/(Sq
rt[2]*Sqrt[a]*(a + b)*Sqrt[a + b*Sech[x]^2])
```

3.201.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 25, 4629, 25, 2075, 382, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ix)^2\sqrt{a+b\sec(ix)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{b\sec(ix)^2+a}\tan(ix)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\int -\frac{\coth^2(x)}{(1-\tanh^2(x))\sqrt{a+b(1-\tanh^2(x))}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^2(x)}{(1-\tanh^2(x))\sqrt{a+b(1-\tanh^2(x))}}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\coth^2(x)}{(1-\tanh^2(x))\sqrt{a-b\tanh^2(x)+b}}d\tanh(x)$$

$$\Big\downarrow \text{382}$$

$$\frac{\int \frac{a+b}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a+b}-\frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

$$\Big\downarrow \; 27$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

$$\Big\downarrow \; 291$$

$$\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-\frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

$$\Big\downarrow \; 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

input
```
Int[Coth[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

output
```
ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] - (Coth[x]*Sq
rt[a + b - b*Tanh[x]^2])/(a + b)
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.201. $\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.201.4 Maple [F]

$$\int \frac{\coth(x)^2}{\sqrt{a+\operatorname{sech}(x)^2 b}}dx$$

input
```
int(coth(x)^2/(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(coth(x)^2/(a+sech(x)^2*b)^(1/2),x)
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(45) = 90$.

Time = 0.31 (sec) , antiderivative size = 1365, normalized size of antiderivative = 25.75

$$\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
 a - b)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*s
inh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3
)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 +
 (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b
 + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5
 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)
^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2
 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)
*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x
)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cos
h(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh
(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)
^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9
*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(
x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x
)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a + b)*cosh(x)^2 + 2
*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(a)*log(-(a*c...
```

3.201.6 Sympy [F]

$$\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\coth^2(x)}{\sqrt{a+b\,\text{sech}^2(x)}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(coth(x)**2/sqrt(a + b*sech(x)**2), x)
```

3.201.7 Maxima [F]

$$\int \frac{\coth^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \int \frac{\coth(x)^2}{\sqrt{b\operatorname{sech}(x)^2+a}}\,dx$$

input
```
integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/sqrt(b*sech(x)^2 + a), x)
```

3.201.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \int \frac{\coth(x)^2}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

input
```
int(coth(x)^2/(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2/(a + b/cosh(x)^2)^(1/2), x)
```

3.202 $\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

3.202.1 Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}}-\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)}$$

output
```
-1/2*(2*a+3*b)*arctanh((a+b*sech(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)+arct
anh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)-1/2*coth(x)^2*(a+b*sech(x)^2)^(
1/2)/(a+b)
```

3.202.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.77

$$\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \frac{-((a+2b+a\cosh(2x))\text{csch}^2(x))+\frac{\sqrt{2}\sqrt{a+2b+a\cosh(2x)}\left(-\sqrt{a}(2a+3b)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+2(a+b)^{3/2}\log\left(\sqrt{2}\sqrt{a}\right.\right.}{\sqrt{a}\sqrt{a+b}}}{4(a+b)\sqrt{a+b\text{sech}^2(x)}}$$

input
```
Integrate[Coth[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

output
```
(-((a + 2*b + a*Cosh[2*x])*Csch[x]^2) + (Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x
]]*(-(Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2
*b + a*Cosh[2*x]]]) + 2*(a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a
 + 2*b + a*Cosh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b]))/(4*(a + b)*Sqrt[a
+ b*Sech[x]^2])
```

3.202.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 26, 4627, 354, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ix)^3\sqrt{a+b\sec(ix)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sqrt{b\sec(ix)^2+a}\tan(ix)^3}dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \frac{\cosh(x)}{\left(1-\text{sech}^2(x)\right)^2\sqrt{b\text{sech}^2(x)+a}}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)}{\left(1-\text{sech}^2(x)\right)^2\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\cosh(x)\left(b\text{sech}^2(x)+2a+2b\right)}{2\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a+b}-\frac{\sqrt{a+b\text{sech}^2(x)}}{(a+b)\left(1-\text{sech}^2(x)\right)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(-\frac{\int \frac{\cosh(x)\left(b\text{sech}^2(x)+2(a+b)\right)}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{2(a+b)}-\frac{\sqrt{a+b\text{sech}^2(x)}}{(a+b)\left(1-\text{sech}^2(x)\right)}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{2}\left(-\frac{(2a+3b)\int \frac{1}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)+2(a+b)\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{2(a+b)}-\frac{\sqrt{a+b\text{sech}^2(x)}}{(a+b)\left(1-\text{sech}^2(x)\right)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(-\frac{\frac{2(2a+3b)\int \frac{1}{\frac{a+b}{b}-\frac{\text{sech}^4(x)}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}+\frac{4(a+b)\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}}{2(a+b)}-\frac{\sqrt{a+b\text{sech}^2(x)}}{(a+b)\left(1-\text{sech}^2(x)\right)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(-\frac{\frac{2(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{4(a+b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2(a+b)}-\frac{\sqrt{a+b\text{sech}^2(x)}}{(a+b)\left(1-\text{sech}^2(x)\right)}\right)$$

input
```
Int[Coth[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

output
```
(-1/2*((-4*(a + b)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]])/Sqrt[a] + (2*(2
*a + 3*b)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a + b)
 - Sqrt[a + b*Sech[x]^2]/((a + b)*(1 - Sech[x]^2)))/2
```

3.202. $\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

3.202.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.202. $\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.202.4 Maple [F]

$$\int \frac{\coth(x)^3}{\sqrt{a+\operatorname{sech}(x)^2 b}}dx$$

input
```
int(coth(x)^3/(a+sech(x)^2*b)^(1/2),x)
```

output
```
int(coth(x)^3/(a+sech(x)^2*b)^(1/2),x)
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. $2(72) = 144$.

Time = 0.46 (sec) , antiderivative size = 6475, normalized size of antiderivative = 71.94

$$\int \frac{\coth^3(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.202.6 Sympy [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\coth^3(x)}{\sqrt{a+b\,\text{sech}^2(x)}}\,dx$$

input
```
integrate(coth(x)**3/(a+b*sech(x)**2)**(1/2),x)
```

output
```
Integral(coth(x)**3/sqrt(a + b*sech(x)**2), x)
```

3.202.7 Maxima [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\coth(x)^3}{\sqrt{b\,\text{sech}(x)^2+a}}\,dx$$

input
```
integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^3/sqrt(b*sech(x)^2 + a), x)
```

3.202.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \int \frac{\coth(x)^3}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

input
```
int(coth(x)^3/(a + b/cosh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3/(a + b/cosh(x)^2)^(1/2), x)
```

3.203 $\int \frac{\tanh^5(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx$

3.203.1 Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{\tanh^5(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\text{sech}^2(x)}}-\frac{\sqrt{a+b\text{sech}^2(x)}}{b^2}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)-(a+b)^2/a/b^2/(a+b*sech(x)^
2)^(1/2)-(a+b*sech(x)^2)^(1/2)/b^2
```

3.203.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

$$\int \frac{\tanh^5(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{\text{sech}^3(x)\left(\frac{\sqrt{2}(a+2b+a\cosh(2x))^{3/2}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)}{a^{3/2}}-\frac{(a+2b+a\cosh(2x))(2a}{}\right.}{4\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

input
```
Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
(Sech[x]^3*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cos
h[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/a^(3/2) - ((a + 2*b + a*Cosh[2*x])*(2
*a^2 + 4*a*b + b^2 + (2*a^2 + 2*a*b + b^2)*Cosh[2*x])*Sech[x])/(a*b^2)))/(
4*(a + b*Sech[x]^2)^(3/2))
```

3.203.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 4627, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\tan(ix)^5}{(a+b\sec(ix)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\tan(ix)^5}{(b\sec(ix)^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 4627}$$

$$-\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)^2}{\left(b\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)^2}{\left(b\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{ 98}$$

$$-\frac{1}{2}\int \left(-\frac{(a+b)^2}{ab\left(b\text{sech}^2(x)+a\right)^{3/2}}+\frac{\cosh(x)}{a\sqrt{b\text{sech}^2(x)+a}}+\frac{1}{b\sqrt{b\text{sech}^2(x)+a}}\right)d\text{sech}^2(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2(a+b)^2}{ab^2\sqrt{a+b\text{sech}^2(x)}}-\frac{2\sqrt{a+b\text{sech}^2(x)}}{b^2}\right)$$

input
```
Int[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]])/a^(3/2) - (2*(a + b)^2)/(a*b^2
*Sqrt[a + b*Sech[x]^2]) - (2*Sqrt[a + b*Sech[x]^2])/b^2)/2
```

3.203.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.203. $\int \frac{\tanh^5(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx$

3.203.4 Maple [F]

$$\int \frac{\tanh(x)^5}{(a+\operatorname{sech}(x)^2 b)^{\frac{3}{2}}}dx$$

input
```
int(tanh(x)^5/(a+sech(x)^2*b)^(3/2),x)
```

output
```
int(tanh(x)^5/(a+sech(x)^2*b)^(3/2),x)
```

3.203.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1312 vs. $2(58) = 116$.

Time = 0.41 (sec) , antiderivative size = 3360, normalized size of antiderivative = 49.41

$$\int \frac{\tanh^5(x)}{(a+b\operatorname{sech}^2(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*
a*b^2 + 4*b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^
4 + 4*(5*a*b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 +
(3*a*b^2 + 4*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3
*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 +
 4*b^3)*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3
+ 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7
 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)
*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a
^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a
*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b
 + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4
*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b
^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*
(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15
*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*
cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*
sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*...
```

3.203.6 Sympy [F]

$$\int \frac{\tanh^5(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\tanh^5{(x)}}{\left(a+b\,\text{sech}^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**5/(a+b*sech(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**5/(a + b*sech(x)**2)**(3/2), x)
```

3.203.7 Maxima [F]

$$\int \frac{\tanh^5(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\tanh{(x)}^5}{\left(b\,\text{sech}{(x)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^5/(b*sech(x)^2 + a)^(3/2), x)
```

3.203.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^5(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[4,1]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1]%%%
}+%%%{8,[
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx=\int \frac{{\mathrm{tanh}\left(x\right)}^5}{{\left(a+\frac{b}{{\mathrm{cosh}\left(x\right)}^2}\right)}^{3/2}} \,d x$$

input
```
int(tanh(x)^5/(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^5/(a + b/cosh(x)^2)^(3/2), x)
```

3.204 $\int \frac{\tanh^4(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

3.204.1 Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \frac{\tanh^4(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}}$$

output
```
arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh(a^(1/2)*ta
nh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-(a+b)*tanh(x)/a/b/(a+b-b*tanh(x)^2)
^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.97

$$\int \frac{\tanh^4(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx=-\frac{(a+2b+a\cosh(2x))\mathrm{sech}^3(x)\left(-\sqrt{2}b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\sqrt{a+2b+a\cosh(2x)}+\sqrt{a}\left(-\sqrt{2}a\operatorname{ar}\right.\right.}{4a^{3/2}b^{3/2}\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}$$

input
```
Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cosh[2*x])*Sech[x]^3*(-(Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2
]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]
]) + Sqrt[a]*(-(Sqrt[2]*a*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b +
a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]]) + 2*Sqrt[b]*(a + b)*Sinh[x])))/
(a^(3/2)*b^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

3.204.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 4629, 2075, 372, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{(a+b\text{sech}^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(ix)^4}{(a+b\sec(ix)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\int \frac{\tanh^4(x)}{(1-\tanh^2(x))\left(a+b\left(1-\tanh^2(x)\right)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\tanh^4(x)}{(1-\tanh^2(x))\left(a-b\tanh^2(x)+b\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{372}$$

$$\frac{\int \frac{-a\tanh^2(x)+a+b}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{ab}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \text{398}$$

$$\frac{a\int \frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+b\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{ab}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \ 224$$

$$\frac{b\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+a\int \frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}}{ab}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \ 216$$

$$\frac{b\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)+\frac{a\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}}{ab}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \ 291$$

$$\frac{b\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}+\frac{a\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}}{ab}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{a\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}+\frac{b\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}}{ab}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

input
```
Int[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
((a*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[b] + (b*ArcT
anh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[a])/(a*b) - ((a + b
)*Tanh[x])/(a*b*Sqrt[a + b - b*Tanh[x]^2])
```

3.204. $\int \frac{\tanh^4(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx$

3.204.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.204.4 Maple [F]

$$\int \frac{\tanh\left(x\right)^{4}}{\left(a+\operatorname{sech}\left(x\right)^{2} b\right)^{\frac{3}{2}}}dx$$

input
```
int(tanh(x)^4/(a+sech(x)^2*b)^(3/2),x)
```

output
```
int(tanh(x)^4/(a+sech(x)^2*b)^(3/2),x)
```

3.204.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. $2(72) = 144$.

Time = 0.42 (sec) , antiderivative size = 5170, normalized size of antiderivative = 60.12

$$\int \frac{\tanh^4(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.204.6 Sympy [F]

$$\int \frac{\tanh^4(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\tanh^4(x)}{(a+b\,\text{sech}^2(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*sech(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**4/(a + b*sech(x)**2)**(3/2), x)
```

3.204.7 Maxima [F]

$$\int \frac{\tanh^4(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)^4}{(b\,\text{sech}(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^4/(b*sech(x)^2 + a)^(3/2), x)
```

3.204.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^4(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{(a+b\operatorname{sech}^2(x))^{3/2}}\,dx=\int \frac{\tanh(x)^4}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

input
```
int(tanh(x)^4/(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^4/(a + b/cosh(x)^2)^(3/2), x)
```

3.205 $\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx$

3.205.1 Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)+(-a-b)/a/b/(a+b*sech(x)^2)^
(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}$$

input
```
Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)/(a*b*Sqrt[a + b*S
ech[x]^2])
```

3.205.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 26, 4627, 25, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\tan(ix)^3}{\left(a+b\sec(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\tan(ix)^3}{\left(b\sec(ix)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4627}$$

$$\int -\frac{\cosh(x)\left(1-\text{sech}^2(x)\right)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)}{\left(b\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)}{\left(b\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a}-\frac{2(a+b)}{ab\sqrt{a+b\text{sech}^2(x)}}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(-\frac{2\int \frac{1}{\frac{\operatorname{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\operatorname{sech}^2(x)+a}}{ab}-\frac{2(a+b)}{ab\sqrt{a+b\operatorname{sech}^2(x)}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2(a+b)}{ab\sqrt{a+b\operatorname{sech}^2(x)}}\right)$$

input
```
Int[Tanh[x]^3/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]])/a^(3/2) - (2*(a + b))/(a*b*Sqr
t[a + b*Sech[x]^2]))/2
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.205.4 Maple [F]

$$\int \frac{\tanh(x)^3}{(a+\operatorname{sech}(x)^2 b)^{\frac{3}{2}}}dx$$

input
```
int(tanh(x)^3/(a+sech(x)^2*b)^(3/2),x)
```

output
```
int(tanh(x)^3/(a+sech(x)^2*b)^(3/2),x)
```

3.205.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(41) = 82$.

Time = 0.32 (sec) , antiderivative size = 2194, normalized size of antiderivative = 44.78

$$\int \frac{\tanh^3(x)}{(a+b\operatorname{sech}^2(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b +
2*b^2)*cosh(x)^2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(
a*b*cosh(x)^3 + (a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*
b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3
+ 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b
^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^
3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 +
 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3
+ 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2
*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^
6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^
6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 +
 a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^
2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2
 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + ...
```

3.205.6 Sympy [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\mathrm{sech}^2(x)\right)^{3/2}}\,dx=\int \frac{\tanh^3{(x)}}{\left(a+b\,\mathrm{sech}^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*sech(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**3/(a + b*sech(x)**2)**(3/2), x)
```

3.205.7 Maxima [F]

$$\int \frac{\tanh^3(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)^3}{(b\,\text{sech}(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^3/(b*sech(x)^2 + a)^(3/2), x)
```

3.205.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(41) = 82$.

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.90

$$\int \frac{\tanh^3(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = -\frac{\frac{(a^3+2a^2b+ab^2)e^{(2x)}}{a^3b+a^2b^2}+\frac{a^3+2a^2b+ab^2}{a^3b+a^2b^2}}{\sqrt{ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a}}$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^3 + 2*a^2*b + a*b^2)*e^(2*x)/(a^3*b + a^2*b^2) + (a^3 + 2*a^2*b + a*b
^2)/(a^3*b + a^2*b^2))/sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)^3}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

input
```
int(tanh(x)^3/(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^3/(a + b/cosh(x)^2)^(3/2), x)
```

3.206 $\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx$

3.206.1 Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-tanh(x)/a/(a+b-b*
tanh(x)^2)^(1/2)
```

3.206.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(51) = 102.

Time = 0.55 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.51

$$\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx=\frac{(a+2b+a\cosh(2x))\operatorname{sech}^2(x)\left(\operatorname{arcsinh}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)(a+2b+a\cosh(2x))\operatorname{sech}(x)\right.}{4a^{3/2}\sqrt{a+b}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}\sqrt{\frac{a+b+a\sinh^2}{a+b}}}$$

input
```
Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cosh[2*x])*Sech[x]^2*(ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]
*(a + 2*b + a*Cosh[2*x])*Sech[x] - 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + b + a*S
inh[x]^2)/(a + b)]*Tanh[x]))/(4*a^(3/2)*Sqrt[a + b]*(a + b*Sech[x]^2)^(3/2
)*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)])
```

3.206.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 25, 4629, 25, 2075, 373, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ix)^2}{\left(a+b\sec(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ix)^2}{\left(b\sec(ix)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4629}$$

$$-\int -\frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a-b\tanh^2(x)+b\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{373}$$

$$\frac{\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a}-\frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}}{a}-\frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

input
```
Int[Tanh[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - Tanh[x]/(a*
Sqrt[a + b - b*Tanh[x]^2])
```

3.206.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.206. $\int \frac{\tanh^2(x)}{\left(a+b\,\text{sech}^2(x)\right)^{3/2}}\,dx$

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.206.4 Maple [F]

$$\int \frac{\tanh(x)^2}{\left(a+\operatorname{sech}(x)^2 b\right)^{\frac{3}{2}}}dx$$

input
```
int(tanh(x)^2/(a+sech(x)^2*b)^(3/2),x)
```

output
```
int(tanh(x)^2/(a+sech(x)^2*b)^(3/2),x)
```

3.206.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(43) = 86$.

Time = 0.31 (sec) , antiderivative size = 1733, normalized size of antiderivative = 33.98

$$\int \frac{\tanh^2(x)}{(a + b\text{sech}^2(x))^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cos
h(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b
)*cosh(x))*sinh(x) + a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sin
h(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)
^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh
(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4
 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*
cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(
x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a
*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x
)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^
4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^
2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 +
2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqr
t(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(
x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*...
```

3.206.6 Sympy [F]

$$\int \frac{\tanh^2(x)}{(a + b\text{sech}^2(x))^{3/2}} \, dx = \int \frac{\tanh^2(x)}{(a + b\,\text{sech}^2(x))^{\frac{3}{2}}} \, dx$$

input
```
integrate(tanh(x)**2/(a+b*sech(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**2/(a + b*sech(x)**2)**(3/2), x)
```

3.206.7 Maxima [F]

$$\int \frac{\tanh^2(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)^2}{(b\,\text{sech}(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.69

$$\int \frac{\tanh^2(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = -\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+a^2b^2} - \frac{a^2b+ab^2}{a^3b+a^2b^2}}{\sqrt{ae^{(4x)}+2\,ae^{(2x)}+4\,be^{(2x)}+a}}$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b + a*b^2)*e^(2*x)/(a^3*b + a^2*b^2) - (a^2*b + a*b^2)/(a^3*b + a^2
*b^2))/sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)^2}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

input
```
int(tanh(x)^2/(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^2/(a + b/cosh(x)^2)^(3/2), x)
```

3.207 $\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx$

3.207.1 Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)-1/a/(a+b*sech(x)^2)^(1/2)
```

3.207.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(43) = 86.

Time = 0.48 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

$$\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx=$$
$$-\frac{(a+2b+a\cosh(2x))\left(2\sqrt{a}\cosh(x)-\sqrt{2}\sqrt{a+2b+a\cosh(2x)}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(\right.\right.}{4a^{3/2}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

input
```
Integrate[Tanh[x]/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cosh[2*x])*(2*Sqrt[a]*Cosh[x] - Sqrt[2]*Sqrt[a + 2*b +
a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*S
ech[x]^3)/(a^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

3.207.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 4627, 243, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)}{\left(a+b\sec(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{\left(b\sec(ix)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \frac{\cosh(x)}{\left(b\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \frac{\cosh(x)}{\left(b\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a}-\frac{2}{a\sqrt{a+b\text{sech}^2(x)}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{2\int \frac{1}{\frac{\operatorname{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\operatorname{sech}^2(x)+a}}{ab}-\frac{2}{a\sqrt{a+b\operatorname{sech}^2(x)}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a+b\operatorname{sech}^2(x)}}\right)$$

input
```
Int[Tanh[x]/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]])/a^(3/2) - 2/(a*Sqrt[a + b*Sech
[x]^2]))/2
```

3.207.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.207.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$-\frac{1}{a\sqrt{a+\operatorname{sech}(x)^{2} b}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+\operatorname{sech}(x)^{2} b}}{\operatorname{sech}(x)}\right)}{a^{\frac{3}{2}}}$	46
default	$-\frac{1}{a\sqrt{a+\operatorname{sech}(x)^{2} b}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+\operatorname{sech}(x)^{2} b}}{\operatorname{sech}(x)}\right)}{a^{\frac{3}{2}}}$	46

input
```
int(tanh(x)/(a+sech(x)^2*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a/(a+sech(x)^2*b)^(1/2)+1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+sech(x)^2*b)^(1/
2))/sech(x))
```

3.207.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(35) = 70$.

Time = 0.32 (sec) , antiderivative size = 2034, normalized size of antiderivative = 47.30

$$\int \frac{\tanh(x)}{(a+b\mathrm{sech}^2(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cos
h(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b
)*cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8
*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(
x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b
+ 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*co
sh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b
 + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*
b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*
(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^
3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)
*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2
)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2
)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh
(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x)
)*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^
4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 +...
```

3.207.6 Sympy [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{\tanh(x)}{(a+b\mathrm{sech}^2(x))^{3/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{2a\sqrt{a+b\,\mathrm{sech}^2(x)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{-a}}\right)}{2a\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log(\mathrm{sech}^2(x))}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)/(a+b*sech(x)**2)**(3/2),x)
```

output
```
-Piecewise((2*(b/(2*a*sqrt(a + b*sech(x)**2)) + b*atan(sqrt(a + b*sech(x)*
*2)/sqrt(-a))/(2*a*sqrt(-a)))/b, Ne(b, 0)), (log(sech(x)**2)/(2*a**(3/2)),
 True))
```

3.207.7 Maxima [F]

$$\int \frac{\tanh(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)}{(b\,\text{sech}(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*sech(x)^2 + a)^(3/2), x)
```

3.207.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(35) = 70$.

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \frac{\tanh(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = -\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+a^2b^2}+\frac{a^2b+ab^2}{a^3b+a^2b^2}}{\sqrt{ae^{(4x)}+2\,ae^{(2x)}+4\,be^{(2x)}+a}}$$

input
```
integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b + a*b^2)*e^(2*x)/(a^3*b + a^2*b^2) + (a^2*b + a*b^2)/(a^3*b + a^2
*b^2))/sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)
```

3.207.9 Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\tanh(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a\sqrt{a+\frac{b}{\cosh(x)^2}}}$$

input
```
int(tanh(x)/(a + b/cosh(x)^2)^(3/2),x)
```

output
```
atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(3/2) - 1/(a*(a + b/cosh(x)^2)^(1
/2))
```

3.208 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

3.208.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-b*tanh(x)/a/(a+b)
/(a+b-b*tanh(x)^2)^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.88

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx=\frac{(a+2b+a\cosh(2x))\,\mathrm{sech}^3(x)\left((a+b)^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sqrt{\frac{a+2b+a\cosh(2x)}{a+b}}-\right.}{2\sqrt{2}a^{3/2}(a+b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}$$

input
```
Integrate[(a + b*Sech[x]^2)^(-3/2),x]
```

output
```
((a + 2*b + a*Cosh[2*x])*Sech[x]^3*((a + b)^(3/2)*ArcSinh[(Sqrt[a]*Sinh[x]
)/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)] - Sqrt[2]*Sqrt[a]*b*S
inh[x]))/(2*Sqrt[2]*a^(3/2)*(a + b)*(a + b*Sech[x]^2)^(3/2))
```

3.208.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4616, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\sec(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4616}$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\left(a-b\tanh^2(x)+b\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{ 296}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a}-\frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}}{a}-\frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

input
```
Int[(a + b*Sech[x]^2)^(-3/2),x]
```

output
```
ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Tanh[x])
/(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2])
```

3.208.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.208.4 Maple [F]

$$\int \frac{1}{\left(a+\operatorname{sech}\left(x\right)^{2} b\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+sech(x)^2*b)^(3/2),x)
```

output
```
int(1/(a+sech(x)^2*b)^(3/2),x)
```

3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(49) = 98.

Time = 0.33 (sec) , antiderivative size = 2095, normalized size of antiderivative = 36.75

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*
b)*sinh(x)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x
)^2 + a^2 + 3*a*b + 2*b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^
3 + (a^2 + 3*a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 +
 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 +
 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 -
3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 +
 (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^
2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 +
 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2
 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*
(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 +
6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(
x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a
^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b
)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*c
osh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 -
 b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*co
sh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)...
```

3.208.6 Sympy [F]

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\text{sech}^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sech(x)**2)**(3/2),x)
```

output
```
Integral((a + b*sech(x)**2)**(-3/2), x)
```

3.208.7 Maxima [F]

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\,\text{sech}\,(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(x)^2 + a)^(-3/2), x)
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = -\frac{\frac{ab^2e^{(2\,x)}}{a^3b+a^2b^2}-\frac{ab^2}{a^3b+a^2b^2}}{\sqrt{ae^{(4\,x)}+2\,ae^{(2\,x)}+4\,be^{(2\,x)}+a}}$$

input
```
integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-(a*b^2*e^(2*x)/(a^3*b + a^2*b^2) - a*b^2/(a^3*b + a^2*b^2))/sqrt(a*e^(4*x
) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(1/(a + b/cosh(x)^2)^(3/2), x)
```

3.209 $\int \frac{\coth(x)}{\left(a+b\,\text{sech}^2(x)\right)^{3/2}}\,dx$

3.209.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{\coth(x)}{\left(a+b\,\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{b}{a(a+b)\sqrt{a+b\,\text{sech}^2(x)}}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)-arctanh((a+b*sech(x)^2)^(1/
2)/(a+b)^(1/2))/(a+b)^(3/2)-b/a/(a+b)/(a+b*sech(x)^2)^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.96

$$\int \frac{\coth(x)}{\left(a+b\,\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{\text{sech}^2(x)\left(-2b(a+2b+a\cosh(2x))+\frac{\sqrt{2}(a+2b+a\cosh(2x))^{3/2}\left(-a^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a+b}}{\sqrt{a+2b+a}}\right.\right.}{}\right.}{4a(a+b)\left(a+b\,\text{sech}^2(x)\right)^{3}}$$

input
```
Integrate[Coth[x]/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
(Sech[x]^2*(-2*b*(a + 2*b + a*Cosh[2*x]) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x]
)^(3/2)*(-(a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b + a*
Cosh[2*x]]]) + (a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b +
a*Cosh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b])))/(4*a*(a + b)*(a + b*Sech[x
]^2)^(3/2))
```

3.209.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 26, 4627, 25, 354, 96, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{(a+b\text{sech}^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ix)\left(a+b\sec(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(b\sec(ix)^2+a\right)^{3/2}\tan(ix)}dx$$

$$\Big\downarrow \text{4627}$$

$$\int -\frac{\cosh(x)}{(1-\text{sech}^2(x))\,(a+b\text{sech}^2(x))^{3/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cosh(x)}{(1-\text{sech}^2(x))\,(b\text{sech}^2(x)+a)^{3/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)}{(1-\text{sech}^2(x))\,(b\text{sech}^2(x)+a)^{3/2}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\cosh(x)\left(-b\text{sech}^2(x)+a+b\right)}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a(a+b)}-\frac{2b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\cosh(x)\left(-b\text{sech}^2(x)+a+b\right)}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a(a+b)}-\frac{2b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(-\frac{a\int \frac{1}{\left(1-\text{sech}^2(x)\right)\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)+(a+b)\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a(a+b)}-\frac{2b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\frac{2a\int \frac{1}{\frac{a+b}{b}-\frac{\text{sech}^4(x)}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}+\frac{2(a+b)\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}}{a(a+b)}-\frac{2b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{2a\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}-\frac{2b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}\right)$$

input
```
Int[Coth[x]/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
(-(((-2*(a + b)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]])/Sqrt[a] + (2*a*Arc
Tanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b))) - (2*b)
/(a*(a + b)*Sqrt[a + b*Sech[x]^2]))/2
```

3.209. $\int \frac{\coth(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx$

3.209.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.209. $\int \frac{\coth(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx$

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.209.4 Maple [F]

$$\int \frac{\coth(x)}{\left(a+\operatorname{sech}(x)^{2} b\right)^{\frac{3}{2}}}dx$$

input
```
int(coth(x)/(a+sech(x)^2*b)^(3/2),x)
```

output
```
int(coth(x)/(a+sech(x)^2*b)^(3/2),x)
```

3.209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1330 vs. $2(65) = 130$.

Time = 0.47 (sec) , antiderivative size = 6939, normalized size of antiderivative = 87.84

$$\int \frac{\coth(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.209.6 Sympy [F]

$$\int \frac{\coth(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\coth{(x)}}{\left(a+b\operatorname{sech}^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*sech(x)**2)**(3/2),x)
```

output
```
Integral(coth(x)/(a + b*sech(x)**2)**(3/2), x)
```

3.209.7 Maxima [F]

$$\int \frac{\coth(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\coth{(x)}}{\left(b\operatorname{sech}{(x)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/(b*sech(x)^2 + a)^(3/2), x)
```

3.209.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\coth(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx = \int \frac{\coth(x)}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

input
```
int(coth(x)/(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(coth(x)/(a + b/cosh(x)^2)^(3/2), x)
```

3.210 $\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx$

3.210.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}-\frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-b*coth(x)/a/(a+b)
/(a+b-b*tanh(x)^2)^(1/2)-(a-b)*coth(x)*(a+b-b*tanh(x)^2)^(1/2)/a/(a+b)^2
```

3.210.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{\text{sech}^3(x)\left(\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)(a+2b+a\cosh(2x))^{3/2}}{a^{3/2}}-\frac{(a+2b+a\cosh(2x))\left(a(a+2b+a\,\text{c}\cdots\right.}{a(a-\cdots}\right.}{4\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

input
```
Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
(Sech[x]^3*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Co
sh[2*x]]]*(a + 2*b + a*Cosh[2*x])^(3/2))/a^(3/2) - ((a + 2*b + a*Cosh[2*x]
)*(a*(a + 2*b + a*Cosh[2*x])*Csch[x] + 2*b^2*Sinh[x]))/(a*(a + b)^2)))/(4*
(a + b*Sech[x]^2)^(3/2))
```

3.210.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 25, 4629, 25, 2075, 374, 25, 445, 25, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ix)^2\left(a+b\sec(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(b\sec(ix)^2+a\right)^{3/2}\tan(ix)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\int -\frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a-b\tanh^2(x)+b\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{374}$$

$$-\frac{\int -\frac{\coth^2(x)\left(2b\tanh^2(x)+a-b\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a(a+b)}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\coth^2(x)\left(2b\tanh^2(x)+a-b\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a(a+b)}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{\int -\frac{(a+b)^2}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a+b}-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}}{a(a+b)}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{(a+b)^2}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a+b}-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}}{a(a+b)}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}}{a(a+b)}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \; 291$$

$$\frac{(a+b)\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}}{a(a+b)}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}}{a(a+b)}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

3.210. $\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx$

input
```
Int[Coth[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

output
```
-((b*Coth[x])/(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2])) + (((a + b)*ArcTanh[(
Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[a] - ((a - b)*Coth[x]*Sq
rt[a + b - b*Tanh[x]^2])/(a + b))/(a*(a + b))
```

3.210.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.210.4 Maple [F]

$$\int \frac{\coth(x)^2}{\left(a+\operatorname{sech}(x)^2 b\right)^{\frac{3}{2}}}dx$$

input
```
int(coth(x)^2/(a+sech(x)^2*b)^(3/2),x)
```

output
```
int(coth(x)^2/(a+sech(x)^2*b)^(3/2),x)
```

3.210.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. $2(78) = 156$.

Time = 0.43 (sec) , antiderivative size = 3941, normalized size of antiderivative = 44.78

$$\int \frac{\coth^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)*sinh(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^
2 + 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 +
 (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*
b^2 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a
*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^
5 + 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b
^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(
x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*c
osh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3
)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cos
h(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 +
 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a
*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2
*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b
 + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*
sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*si
nh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)...
```

3.210.6 Sympy [F]

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \int \frac{\coth^2(x)}{\left(a+b\,\text{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*sech(x)**2)**(3/2),x)
```

output
```
Integral(coth(x)**2/(a + b*sech(x)**2)**(3/2), x)
```

3.210.7 Maxima [F]

$$\int \frac{\coth^2(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = \int \frac{\coth(x)^2}{(b\,\text{sech}(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

3.210.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(78) = 156$.

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.89

$$\int \frac{\coth^2(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx = -\frac{\frac{(a^2b^3+ab^4)e^{(2x)}}{a^5b+3a^4b^2+3a^3b^3+a^2b^4} - \frac{a^2b^3+ab^4}{a^5b+3a^4b^2+3a^3b^3+a^2b^4}}{\sqrt{ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a}} + \frac{4\left(\sqrt{a}e^{(2x)}-\sqrt{ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a}+\sqrt{a}\right)}{\left(\left(\sqrt{a}e^{(2x)}-\sqrt{ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a}\right)^2-2\left(\sqrt{a}e^{(2x)}-\sqrt{ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a}\right)\sqrt{a}-\right.}$$

input
```
integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b^3 + a*b^4)*e^(2*x)/(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4) - (a
^2*b^3 + a*b^4)/(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4))/sqrt(a*e^(4*x)
+ 2*a*e^(2*x) + 4*b*e^(2*x) + a) + 4*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2
*a*e^(2*x) + 4*b*e^(2*x) + a) + sqrt(a))/(((sqrt(a)*e^(2*x) - sqrt(a*e^(4*
x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))^2 - 2*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*
x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*sqrt(a) - 3*a - 4*b)*(a + b))
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{(a+b\text{sech}^2(x))^{3/2}}\,dx=\int \frac{{\mathrm{coth}\left(x\right)}^2}{{\left(a+\frac{b}{{\mathrm{cosh}\left(x\right)}^2}\right)}^{3/2}}\,dx$$

input
```
int(coth(x)^2/(a + b/cosh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2/(a + b/cosh(x)^2)^(3/2), x)
```

3.211 $\int \frac{\tanh^6(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

3.211.1 Optimal result

Integrand size = 17, antiderivative size = 118

$$\int \frac{\tanh^6(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}$$

output
```
-arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(5/2)+arctanh(a^(1/2)*t
anh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-(1/a^2-1/b^2)*tanh(x)/(a+b-b*tanh(
x)^2)^(1/2)-1/3*(a+b)*tanh(x)^3/a/b/(a+b-b*tanh(x)^2)^(3/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.51

$$\int \frac{\tanh^6(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \frac{\mathrm{sech}^5(x)\left(\frac{\sqrt{2}\left(-a^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\right)(a+2b+a\cosh(2\ldots}{a^{5/2}b^{5/2}}\right.}{8\left(a+b\,\mathrm{sech}^2(x)\right)^{5/\ldots}}$$

input
```
Integrate[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
(Sech[x]^5*((Sqrt[2]*(-(a^(5/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a +
2*b + a*Cosh[2*x]]]) + b^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a +
2*b + a*Cosh[2*x]]])*(a + 2*b + a*Cosh[2*x])^(5/2))/(a^(5/2)*b^(5/2)) + (2
*(a + b)*(a + 2*b + a*Cosh[2*x])*(3*a^2 + 4*a*b - 6*b^2 + a*(3*a - 4*b)*Co
sh[2*x])*Sinh[x])/(3*a^2*b^2)))/(8*(a + b*Sech[x]^2)^(5/2))
```

3.211.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {3042, 25, 4629, 25, 2075, 372, 27, 440, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ix)^6}{\left(a+b\sec(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ix)^6}{\left(b\sec(ix)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4629}$$

$$-\int -\frac{\tanh^6(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^6(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\tanh^6(x)}{\left(1-\tanh^2(x)\right)\left(a-b\tanh^2(x)+b\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow 372$$

$$\frac{\int \frac{3\tanh^2(x)(-a\tanh^2(x)+a+b)}{(1-\tanh^2(x))(-b\tanh^2(x)+a+b)^{3/2}}d\tanh(x)}{3ab}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\tanh^2(x)(-a\tanh^2(x)+a+b)}{(1-\tanh^2(x))(-b\tanh^2(x)+a+b)^{3/2}}d\tanh(x)}{ab}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow 440$$

$$\frac{\frac{(a^2-b^2)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}-\frac{\int \frac{-\tanh^2(x)a^2+a^2-b^2}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{ab}}{ab}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow 398$$

$$\frac{\frac{(a^2-b^2)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}-\frac{a^2\int \frac{1}{\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-b^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{ab}}{ab}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow 224$$

$$\frac{\frac{(a^2-b^2)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}-\frac{a^2\int \frac{1}{\frac{b\tanh^2(x)}{-b\tanh^2(x)+a+b}+1}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-b^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{ab}}{ab}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow 216$$

$$\frac{\frac{(a^2-b^2)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}-\frac{\frac{a^2\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}-b^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{ab}}{ab}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow 291$$

3.211. $\int \frac{\tanh^6(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx$

$$\frac{\frac{(a^2-b^2)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}-\frac{\frac{a^2\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}-b^2\int\frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}}{ab}}{ab}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a^2-b^2)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}-\frac{\frac{a^2\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}}{ab}}{ab}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

input
```
Int[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
-1/3*((a + b)*Tanh[x]^3)/(a*b*(a + b - b*Tanh[x]^2)^(3/2)) + (-(((a^2*ArcT
an[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[b] - (b^2*ArcTanh[(S
qrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[a])/(a*b)) + ((a^2 - b^2)
*Tanh[x])/(a*b*Sqrt[a + b - b*Tanh[x]^2]))/(a*b)
```

3.211.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.211. $\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.211. $\int \frac{\tanh^6(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx$

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.211.4 Maple [F]

$$\int \frac{\tanh\left(x\right)^{6}}{\left(a+\operatorname{sech}\left(x\right)^{2} b\right)^{\frac{5}{2}}}dx$$

input
```
int(tanh(x)^6/(a+sech(x)^2*b)^(5/2),x)
```

output
```
int(tanh(x)^6/(a+sech(x)^2*b)^(5/2),x)
```

3.211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2666 vs. $2(100) = 200$.

Time = 0.71 (sec) , antiderivative size = 11939, normalized size of antiderivative = 101.18

$$\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.211.6 Sympy [F]

$$\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh^{6}{\left(x \right)}}{\left(a+b\operatorname{sech}^{2}{\left(x \right)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**6/(a+b*sech(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**6/(a + b*sech(x)**2)**(5/2), x)
```

3.211.7 Maxima [F]

$$\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh(x)^6}{\left(b\,\text{sech}(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^6/(b*sech(x)^2 + a)^(5/2), x)
```

3.211.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh(x)^6}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

input
```
int(tanh(x)^6/(a + b/cosh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^6/(a + b/cosh(x)^2)^(5/2), x)
```

3.212 $\int \frac{\tanh^5(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

3.212.1 Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \frac{\tanh^5(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{(a+b)^2}{3ab^2\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}} - \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)-1/3*(a+b)^2/a/b^2/(a+b*sech
(x)^2)^(3/2)+(-1/a^2+1/b^2)/(a+b*sech(x)^2)^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int \frac{\tanh^5(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \frac{\left(\frac{4(a+b)\cosh(x)(a+2b+a\cosh(2x))\left(a^2+ab-3b^2+a(a-2b)\cosh(2x)\right)}{3a^2b^2} + \frac{\sqrt{2}(a+2b+a\cosh(2x))^{5/2}\log\left(\sqrt{2}\sqrt{\cdots}\right.}{a^{5/\cdots}}\right.}{8\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}$$

input
```
Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
(((4*(a + b)*Cosh[x]*(a + 2*b + a*Cosh[2*x])*(a^2 + a*b - 3*b^2 + a*(a - 2
*b)*Cosh[2*x]))/(3*a^2*b^2) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Log[S
qrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/a^(5/2))*Sech[x]^5)
/(8*(a + b*Sech[x]^2)^(5/2))
```

3.212.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 4627, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)^5}{\left(a+b\sec(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)^5}{\left(b\sec(ix)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)^2}{\left(b\text{sech}^2(x)+a\right)^{5/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)^2}{\left(b\text{sech}^2(x)+a\right)^{5/2}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{98}$$

$$-\frac{1}{2}\int \left(-\frac{(a+b)^2}{ab\left(b\text{sech}^2(x)+a\right)^{5/2}}+\frac{\cosh(x)}{a^2\sqrt{b\text{sech}^2(x)+a}}+\frac{a^2-b^2}{a^2b\left(b\text{sech}^2(x)+a\right)^{3/2}}\right)d\text{sech}^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)}{\sqrt{a+b\text{sech}^2(x)}}-\frac{2(a+b)^2}{3ab^2\left(a+b\text{sech}^2(x)\right)^{3/2}}\right)$$

input
```
Int[Tanh[x]^5/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]])/a^(5/2) - (2*(a + b)^2)/(3*a*b
^2*(a + b*Sech[x]^2)^(3/2)) - (2*(a^(-2) - b^(-2)))/Sqrt[a + b*Sech[x]^2])
/2
```

3.212.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.212.4 Maple [F]

$$\int \frac{\tanh(x)^5}{\left(a+\operatorname{sech}(x)^2 b\right)^{\frac{5}{2}}}dx$$

input
```
int(tanh(x)^5/(a+sech(x)^2*b)^(5/2),x)
```

output
```
int(tanh(x)^5/(a+sech(x)^2*b)^(5/2),x)
```

3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2224 vs. $2(64) = 128$.

Time = 0.50 (sec) , antiderivative size = 5184, normalized size of antiderivative = 68.21

$$\int \frac{\tanh^5(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.212.6 Sympy [F]

$$\int \frac{\tanh^5(x)}{(a + b\text{sech}^2(x))^{5/2}} dx = \int \frac{\tanh^5(x)}{(a + b\,\text{sech}^2(x))^{\frac{5}{2}}} dx$$

input
```
integrate(tanh(x)**5/(a+b*sech(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**5/(a + b*sech(x)**2)**(5/2), x)
```

3.212.7 Maxima [F]

$$\int \frac{\tanh^5(x)}{(a + b\text{sech}^2(x))^{5/2}} dx = \int \frac{\tanh(x)^5}{(b\,\text{sech}(x)^2 + a)^{\frac{5}{2}}} dx$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^5/(b*sech(x)^2 + a)^(5/2), x)
```

3.212.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(64) = 128$.

Time = 0.40 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.91

$$\int \frac{\tanh^5(x)}{(a + b\text{sech}^2(x))^{5/2}} dx = \frac{2\left(\left(\left(\left(\frac{(a^9+a^8b-3a^7b^2-5a^6b^3-2a^5b^4)e^{(2x)}}{a^8b^2+2a^7b^3+a^6b^4} + \frac{3(a^9+3a^8b+a^7b^2-5a^6b^3-6a^5b^4-2a^4b^5)}{a^8b^2+2a^7b^3+a^6b^4}\right)e^{(2x)} + \frac{3}{\cdots}\right.\right.\right.}{3\left(ae^{(4x)} + 2ae^{(2x)} + 4be\cdots\right.}$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*((((a^9 + a^8*b - 3*a^7*b^2 - 5*a^6*b^3 - 2*a^5*b^4)*e^(2*x)/(a^8*b^2
+ 2*a^7*b^3 + a^6*b^4) + 3*(a^9 + 3*a^8*b + a^7*b^2 - 5*a^6*b^3 - 6*a^5*b^
4 - 2*a^4*b^5)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4))*e^(2*x) + 3*(a^9 + 3*a^8*b
 + a^7*b^2 - 5*a^6*b^3 - 6*a^5*b^4 - 2*a^4*b^5)/(a^8*b^2 + 2*a^7*b^3 + a^6
*b^4))*e^(2*x) + (a^9 + a^8*b - 3*a^7*b^2 - 5*a^6*b^3 - 2*a^5*b^4)/(a^8*b^
2 + 2*a^7*b^3 + a^6*b^4))/(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)^(3/2
)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{{\tanh(x)}^5}{{\left(a+\frac{b}{{\cosh(x)}^2}\right)}^{5/2}}\,dx$$

input
```
int(tanh(x)^5/(a + b/cosh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^5/(a + b/cosh(x)^2)^(5/2), x)
```

3.213 $\int \frac{\tanh^4(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx$

3.213.1 Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{\tanh^4(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)+1/3*(a-3*b)*tanh(
x)/a^2/b/(a+b-b*tanh(x)^2)^(1/2)-1/3*(a+b)*tanh(x)/a/b/(a+b-b*tanh(x)^2)^(
3/2)
```

3.213.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 290 vs. $2(90) = 180$.

Time = 1.46 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.22

$$\int \frac{\tanh^4(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \frac{\text{sech}^4(x)\left(\frac{\sqrt{2}(a+2b+a\cosh(2x))^{5/2}\text{csch}(x)\text{sech}(x)\left(\frac{\sinh^2(x)}{a+b}+\frac{12\sinh^4(x)}{a+b}+\frac{2\sinh^2(x)\left(a+b+a\sinh^2(x)\right)}{(a+b)^2}\right.}{}\right.}{}$$

input
```
Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
(Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x
]^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))
/(a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(
a + b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b
 + a*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a
+ b]]*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/a^3))/(a + b + a*Sinh
[x]^2)^(3/2) + (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x
])/(a + b)^2 - (12*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tan
h[x])/(a + b)^2))/(384*(a + b*Sech[x]^2)^(5/2))
```

3.213.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4629, 2075, 372, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(ix)^4}{(a+b\sec(ix)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\int \frac{\tanh^4(x)}{(1-\tanh^2(x))\left(a+b\left(1-\tanh^2(x)\right)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\tanh^4(x)}{(1-\tanh^2(x))\left(a-b\tanh^2(x)+b\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{372}$$

$$\frac{\int \frac{-((a-2b)\tanh^2(x))+a+b}{(1-\tanh^2(x))(-b\tanh^2(x)+a+b)^{3/2}}d\tanh(x)}{3ab}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{(a-3b)\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}-\frac{\int -\frac{3b(a+b)}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a(a+b)}}{3ab}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3b\int \frac{1}{(1-\tanh^2(x))\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a}+\frac{(a-3b)\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}}{3ab}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3b\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}}{a}+\frac{(a-3b)\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}}{3ab}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3b\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}+\frac{(a-3b)\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}}{3ab}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

input
```
Int[Tanh[x]^4/(a + b*Sech[x]^2)^(5/2),x]
```

3.213. $\int \frac{\tanh^4(x)}{\left(a+b\,\text{sech}^2(x)\right)^{5/2}}\,dx$

output
```
-1/3*((a + b)*Tanh[x])/(a*b*(a + b - b*Tanh[x]^2)^(3/2)) + ((3*b*ArcTanh[(
Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/a^(3/2) + ((a - 3*b)*Tanh[x])
/(a*Sqrt[a + b - b*Tanh[x]^2]))/(3*a*b)
```

3.213.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.213. $\int \frac{\tanh^4(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.213.4 Maple [F]

$$\int \frac{\tanh(x)^4}{\left(a+\operatorname{sech}(x)^2 b\right)^{\frac{5}{2}}}dx$$

input
```
int(tanh(x)^4/(a+sech(x)^2*b)^(5/2),x)
```

output
```
int(tanh(x)^4/(a+sech(x)^2*b)^(5/2),x)
```

3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. 2(76) = 152.

Time = 0.47 (sec) , antiderivative size = 3559, normalized size of antiderivative = 39.54

$$\int \frac{\tanh^4(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2
 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a
^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b
^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 +
 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^
3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2
 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*
b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2
+ 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cos
h(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 +
a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^
2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh
(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 +
4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*co
sh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15
*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cos
h(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2
*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*
b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (1...
```

3.213.6 Sympy [F]

$$\int \frac{\tanh^4(x)}{(a+b\text{sech}^2(x))^{5/2}}\,dx = \int \frac{\tanh^4(x)}{(a+b\,\text{sech}^2(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*sech(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**4/(a + b*sech(x)**2)**(5/2), x)
```

3.213.7 Maxima [F]

$$\int \frac{\tanh^4(x)}{(a+b\mathrm{sech}^2(x))^{5/2}}\,dx = \int \frac{\tanh(x)^4}{(b\,\mathrm{sech}(x)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^4/(b*sech(x)^2 + a)^(5/2), x)
```

3.213.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(76) = 152.

Time = 0.41 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.69

$$\int \frac{\tanh^4(x)}{(a+b\mathrm{sech}^2(x))^{5/2}}\,dx = -\frac{4\left(\left(\left(\frac{(a^7b^2+2\,a^6b^3+a^5b^4)e^{(2\,x)}}{a^8b^2+2\,a^7b^3+a^6b^4}+\frac{3\,(a^6b^3+2\,a^5b^4+a^4b^5)}{a^8b^2+2\,a^7b^3+a^6b^4}\right)e^{(2\,x)}-\frac{3\,(a^6b^3+2\,a^5b^4+a^4b^5)}{a^8b^2+2\,a^7b^3+a^6b^4}\right)e^{(2\,x)}-\frac{a^7b^2+2\,a^6b^3+a^5b^4}{a^8b^2+2\,a^7b^3+a^6b^4}\right)}{3\,(ae^{(4\,x)}+2\,ae^{(2\,x)}+4\,be^{(2\,x)}+a)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*((((a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*e^(2*x)/(a^8*b^2 + 2*a^7*b^3 + a^6
*b^4) + 3*(a^6*b^3 + 2*a^5*b^4 + a^4*b^5)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4))
*e^(2*x) - 3*(a^6*b^3 + 2*a^5*b^4 + a^4*b^5)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^
4))*e^(2*x) - (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)/(a^8*b^2 + 2*a^7*b^3 + a^6*b
^4))/(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)^(3/2)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{(a+b\mathrm{sech}^2(x))^{5/2}}\,dx = \int \frac{\tanh(x)^4}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

input
```
int(tanh(x)^4/(a + b/cosh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^4/(a + b/cosh(x)^2)^(5/2), x)
```

3.213. $\int \frac{\tanh^4(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx$

3.214 $\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx$

3.214.1 Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)+1/3*(-a-b)/a/b/(a+b*sech(x)
^2)^(3/2)-1/a^2/(a+b*sech(x)^2)^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.82

$$\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx=-\frac{(a+2b+a\cosh(2x))\left(3\sqrt{a}(a+2b)^2\cosh(x)+a^{3/2}(a+4b)\cosh(3x)-3\sqrt{2}b(a+2b+a\cosh(2x))^{3/2}\log\right.}{24a^{5/2}b\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}$$

input
```
Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
-1/24*((a + 2*b + a*Cosh[2*x])*(3*Sqrt[a]*(a + 2*b)^2*Cosh[x] + a^(3/2)*(a
 + 4*b)*Cosh[3*x] - 3*Sqrt[2]*b*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*
Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x]^5)/(a^(5/2)*b*(a +
 b*Sech[x]^2)^(5/2))
```

3.214.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 26, 4627, 25, 354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ix)^3}{\left(a+b\sec(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ix)^3}{\left(b\sec(ix)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\int -\frac{\cosh(x)\left(1-\text{sech}^2(x)\right)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)}{\left(b\text{sech}^2(x)+a\right)^{5/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)\left(1-\text{sech}^2(x)\right)}{\left(b\text{sech}^2(x)+a\right)^{5/2}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\cosh(x)}{\left(b\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}^2(x)}{a}-\frac{2(a+b)}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a}+\frac{2}{a\sqrt{a+b\text{sech}^2(x)}}}{a}-\frac{2(a+b)}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{\frac{2\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{ab}+\frac{2}{a\sqrt{a+b\text{sech}^2(x)}}}{a}-\frac{2(a+b)}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{\frac{2}{a\sqrt{a+b\text{sech}^2(x)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}-\frac{2(a+b)}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}}\right)$$

input
```
Int[Tanh[x]^3/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
((-2*(a + b))/(3*a*b*(a + b*Sech[x]^2)^(3/2)) - ((-2*ArcTanh[Sqrt[a + b*Se
ch[x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sech[x]^2]))/a)/2
```

3.214. $\int \frac{\tanh^3(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

3.214.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.214. $\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.214.4 Maple [F]

$$\int \frac{\tanh(x)^3}{\left(a+\operatorname{sech}(x)^2 b\right)^{\frac{5}{2}}}dx$$

input
```
int(tanh(x)^3/(a+sech(x)^2*b)^(5/2),x)
```

output
```
int(tanh(x)^3/(a+sech(x)^2*b)^(5/2),x)
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1954 vs. $2(56) = 112$.

Time = 0.48 (sec) , antiderivative size = 4644, normalized size of antiderivative = 68.29

$$\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 +
4*(a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*si
nh(x)^6 + 8*(7*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 +
2*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b
+ 8*a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b
*cosh(x)^5 + 10*(a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*
cosh(x))*sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*co
sh(x)^6 + 15*(a^2*b + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b +
8*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*
a*b^2)*cosh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^
2)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a
^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^
8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4
*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^
3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(
x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b +
4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b +
 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*...
```

3.214.6 Sympy [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh^3{(x)}}{\left(a+b\operatorname{sech}^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*sech(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**3/(a + b*sech(x)**2)**(5/2), x)
```

3.214.7 Maxima [F]

$$\int \frac{\tanh^3(x)}{(a+b\operatorname{sech}^2(x))^{5/2}}\,dx = \int \frac{\tanh(x)^3}{(b\operatorname{sech}(x)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^3/(b*sech(x)^2 + a)^(5/2), x)
```

3.214.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(56) = 112$.

Time = 0.40 (sec) , antiderivative size = 285, normalized size of antiderivative = 4.19

$$\int \frac{\tanh^3(x)}{(a+b\operatorname{sech}^2(x))^{5/2}}\,dx =$$
$$-\frac{\left(\left(\frac{(a^8b+6a^7b^2+9a^6b^3+4a^5b^4)e^{(2x)}}{a^8b^2+2a^7b^3+a^6b^4}+\frac{3(a^8b+6a^7b^2+13a^6b^3+12a^5b^4+4a^4b^5)}{a^8b^2+2a^7b^3+a^6b^4}\right)e^{(2x)}+\frac{3(a^8b+6a^7b^2+13a^6b^3+12a^5b^4+4a^4b^5)}{a^8b^2+2a^7b^3+a^6b^4}\right)e^{(2}}{3\left(ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a\right)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((((a^8*b + 6*a^7*b^2 + 9*a^6*b^3 + 4*a^5*b^4)*e^(2*x)/(a^8*b^2 + 2*a
^7*b^3 + a^6*b^4) + 3*(a^8*b + 6*a^7*b^2 + 13*a^6*b^3 + 12*a^5*b^4 + 4*a^4
*b^5)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4))*e^(2*x) + 3*(a^8*b + 6*a^7*b^2 + 13
*a^6*b^3 + 12*a^5*b^4 + 4*a^4*b^5)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4))*e^(2*x
) + (a^8*b + 6*a^7*b^2 + 9*a^6*b^3 + 4*a^5*b^4)/(a^8*b^2 + 2*a^7*b^3 + a^6
*b^4))/(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)^(3/2)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{{\tanh(x)}^3}{{\left(a+\frac{b}{{\cosh(x)}^2}\right)}^{5/2}}\,dx$$

input
```
int(tanh(x)^3/(a + b/cosh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^3/(a + b/cosh(x)^2)^(5/2), x)
```

3.215 $\int \frac{\tanh^2(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

3.215.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\tanh^2(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*(2*a+3*b)*tan
h(x)/a^2/(a+b)/(a+b-b*tanh(x)^2)^(1/2)-1/3*tanh(x)/a/(a+b-b*tanh(x)^2)^(3/
2)
```

3.215.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 290 vs. $2(88) = 176$.

Time = 0.44 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.30

$$\int \frac{\tanh^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \frac{\text{sech}^4(x)\left(\frac{\sqrt{2}(a+2b+a\cosh(2x))^{5/2}\text{csch}(x)\text{sech}(x)\left(\frac{\sinh^2(x)}{a+b}+\frac{12\sinh^4(x)}{a+b}+\frac{2\sinh^2(x)\left(a+b+a\sinh^2(x)\right)}{(a+b)^2}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
(Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x
]^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))
/(a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(
a + b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b
 + a*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a
+ b]]*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/a^3))/(a + b + a*Sinh
[x]^2)^(3/2) - (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x
])/(a + b)^2 + (4*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tanh
[x])/(a + b)^2))/(384*(a + b*Sech[x]^2)^(5/2))
```

3.215.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 25, 4629, 25, 2075, 373, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan(ix)^2}{\left(a+b\sec(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan(ix)^2}{\left(b\sec(ix)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4629}$$

$$-\int -\frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{ 2075}$$

$$\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a-b\tanh^2(x)+b\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{ 373}$$

$$\frac{\int \frac{2\tanh^2(x)+1}{\left(1-\tanh^2(x)\right)\left(-b\tanh^2(x)+a+b\right)^{3/2}}d\tanh(x)}{3a}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{-\frac{\int -\frac{3(a+b)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a(a+b)}-\frac{(2a+3b)\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a}-\frac{(2a+3b)\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{3\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}}{a}-\frac{(2a+3b)\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

3.215. $\int \frac{\tanh^2(x)}{\left(a+b\,\text{sech}^2(x)\right)^{5/2}}\,dx$

$$\frac{\frac{3\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{(2a+3b)\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

input
```
Int[Tanh[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
-1/3*Tanh[x]/(a*(a + b - b*Tanh[x]^2)^(3/2)) + ((3*ArcTanh[(Sqrt[a]*Tanh[x
])/Sqrt[a + b - b*Tanh[x]^2]])/a^(3/2) - ((2*a + 3*b)*Tanh[x])/(a*(a + b)*
Sqrt[a + b - b*Tanh[x]^2]))/(3*a)
```

3.215.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.215.4 Maple [F]

$$\int \frac{\tanh(x)^2}{\left(a+\operatorname{sech}(x)^2 b\right)^{\frac{5}{2}}}dx$$

input
```
int(tanh(x)^2/(a+sech(x)^2*b)^(5/2),x)
```

output
```
int(tanh(x)^2/(a+sech(x)^2*b)^(5/2),x)
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. $2(74) = 148$.

Time = 0.51 (sec) , antiderivative size = 4989, normalized size of antiderivative = 56.69

$$\int \frac{\tanh^2(x)}{(a+b\text{sech}^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a
^3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(a^3 + 3
*a^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + a^2*
b)*cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^3 +
 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cosh(x)^4 +
3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)
^2)*sinh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b + 2*a*b^2
)*cosh(x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a
^3 + a^2*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*b)*co
sh(x)^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a*b^2
 + 3*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^3
+ a^2*b)*cosh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11*a
^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*si
nh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*si
nh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)
*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 +
(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b
+ 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5
- 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^
3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b...
```

3.215.6 Sympy [F]

$$\int \frac{\tanh^2(x)}{(a+b\text{sech}^2(x))^{5/2}}\,dx = \int \frac{\tanh^2{(x)}}{(a+b\,\text{sech}^2{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*sech(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**2/(a + b*sech(x)**2)**(5/2), x)
```

3.215.7 Maxima [F]

$$\int \frac{\tanh^2(x)}{(a+b\text{sech}^2(x))^{5/2}}\,dx = \int \frac{\tanh(x)^2}{(b\,\text{sech}(x)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

3.215.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(74) = 148.

Time = 0.40 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.00

$$\int \frac{\tanh^2(x)}{(a+b\text{sech}^2(x))^{5/2}}\,dx = -\frac{\left(\left(\frac{(3a^7b^2+7a^6b^3+4a^5b^4)e^{(2x)}}{a^8b^2+2a^7b^3+a^6b^4}+\frac{3(a^7b^2+5a^6b^3+8a^5b^4+4a^4b^5)}{a^8b^2+2a^7b^3+a^6b^4}\right)e^{(2x)}-\frac{3(a^7b^2+5a^6b^3+8a^5b^4+4a^4b^5)}{a^8b^2+2a^7b^3+a^6b^4}\right)e^{(2x)}-\frac{3a^7b^2+7a^6b^3+\cdots}{a^8b^2+2a^7b^3+\cdots}}{3\left(ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a\right)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((((3*a^7*b^2 + 7*a^6*b^3 + 4*a^5*b^4)*e^(2*x)/(a^8*b^2 + 2*a^7*b^3 +
 a^6*b^4) + 3*(a^7*b^2 + 5*a^6*b^3 + 8*a^5*b^4 + 4*a^4*b^5)/(a^8*b^2 + 2*a
^7*b^3 + a^6*b^4))*e^(2*x) - 3*(a^7*b^2 + 5*a^6*b^3 + 8*a^5*b^4 + 4*a^4*b^
5)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4))*e^(2*x) - (3*a^7*b^2 + 7*a^6*b^3 + 4*a
^5*b^4)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4))/(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^
(2*x) + a)^(3/2)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\left(a+b\mathrm{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh(x)^2}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

input
```
int(tanh(x)^2/(a + b/cosh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^2/(a + b/cosh(x)^2)^(5/2), x)
```

3.216 $\int \frac{\tanh(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

3.216.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{\tanh(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{1}{3a\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)-1/3/a/(a+b*sech(x)^2)^(3/2)
-1/a^2/(a+b*sech(x)^2)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \frac{\tanh(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx = -\frac{(a+2b+a\cosh(2x))\left(12\sqrt{a}(a+b)\cosh(x)+4a^{3/2}\cosh(3x)-3\sqrt{2}(a+2b+a\cosh(2x))^{3/2}\log\left(\sqrt{2}\sqrt{a}\right.\right.}{24a^{5/2}\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}$$

input
```
Integrate[Tanh[x]/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
-1/24*((a + 2*b + a*Cosh[2*x])*(12*Sqrt[a]*(a + b)*Cosh[x] + 4*a^(3/2)*Cos
h[3*x] - 3*Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[
x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x]^5)/(a^(5/2)*(a + b*Sech[x]^2)^(
5/2))
```

3.216.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 26, 4627, 243, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)}{\left(a+b\sec(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{\left(b\sec(ix)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \frac{\cosh(x)}{\left(b\text{sech}^2(x)+a\right)^{5/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \frac{\cosh(x)}{\left(b\text{sech}^2(x)+a\right)^{5/2}}d\text{sech}^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\cosh(x)}{\left(b\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}^2(x)}{a}-\frac{2}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{\cosh(x)}{\sqrt{b\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a}+\frac{2}{a\sqrt{a+b\text{sech}^2(x)}}}{a}-\frac{2}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{\frac{2\int \frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{ab}+\frac{2}{a\sqrt{a+b\text{sech}^2(x)}}}{a}-\frac{2}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{\frac{2}{a\sqrt{a+b\text{sech}^2(x)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}-\frac{2}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}\right)$$

input
```
Int[Tanh[x]/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
(-2/(3*a*(a + b*Sech[x]^2)^(3/2)) - ((-2*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqr
t[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sech[x]^2]))/a)/2
```

3.216. $\int \frac{\tanh(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

3.216.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.216. $\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx$

3.216.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$-\frac{1}{3a\left(a+\operatorname{sech}(x)^{2} b\right)^{\frac{3}{2}}}-\frac{1}{a^{2} \sqrt{a+\operatorname{sech}(x)^{2} b}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+\operatorname{sech}(x)^{2} b}}{\operatorname{sech}(x)}\right)}{a^{\frac{5}{2}}}$	61
default	$-\frac{1}{3a\left(a+\operatorname{sech}(x)^{2} b\right)^{\frac{3}{2}}}-\frac{1}{a^{2} \sqrt{a+\operatorname{sech}(x)^{2} b}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+\operatorname{sech}(x)^{2} b}}{\operatorname{sech}(x)}\right)}{a^{\frac{5}{2}}}$	61

input
```
int(tanh(x)/(a+sech(x)^2*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/(a+sech(x)^2*b)^(3/2)-1/a^2/(a+sech(x)^2*b)^(1/2)+1/a^(5/2)*ln((2*a
+2*a^(1/2)*(a+sech(x)^2*b)^(1/2))/sech(x))
```

3.216.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1629 vs. 2(50) = 100.

Time = 0.46 (sec) , antiderivative size = 3994, normalized size of antiderivative = 64.42

$$\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2
 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a
^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b
^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 +
 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^
3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2
 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*
b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2
+ 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cos
h(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*
(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2
 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*
a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*si
nh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^
2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 +
10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cos...
```

3.216.6 Sympy [A] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{\tanh(x)}{\left(a+b\mathrm{sech}^2(x)\right)^{5/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{6a\left(a+b\,\mathrm{sech}^2(x)\right)^{\frac{3}{2}}}+\frac{b}{2a^2\sqrt{a+b\,\mathrm{sech}^2(x)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{-a}}\right)}{2a^2\sqrt{-a}}\right)}{b} & \text{for}\ b\neq 0 \\ \frac{\log\left(\mathrm{sech}^2(x)\right)}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)/(a+b*sech(x)**2)**(5/2),x)
```

output
```
-Piecewise((2*(b/(6*a*(a + b*sech(x)**2)**(3/2)) + b/(2*a**2*sqrt(a + b*se
ch(x)**2)) + b*atan(sqrt(a + b*sech(x)**2)/sqrt(-a))/(2*a**2*sqrt(-a)))/b,
 Ne(b, 0)), (log(sech(x)**2)/(2*a**(5/2)), True))
```

3.216.7 Maxima [F]

$$\int \frac{\tanh(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh\left(x\right)}{\left(b\,\text{sech}\left(x\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*sech(x)^2 + a)^(5/2), x)
```

3.216.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(50) = 100$.

Time = 0.41 (sec) , antiderivative size = 257, normalized size of antiderivative = 4.15

$$\int \frac{\tanh(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=$$
$$-\frac{4\left(\left(\left(\frac{(a^7b^2+2\,a^6b^3+a^5b^4)e^{(2\,x)}}{a^8b^2+2\,a^7b^3+a^6b^4}+\frac{3\,(a^7b^2+3\,a^6b^3+3\,a^5b^4+a^4b^5)}{a^8b^2+2\,a^7b^3+a^6b^4}\right)e^{(2\,x)}+\frac{3\,(a^7b^2+3\,a^6b^3+3\,a^5b^4+a^4b^5)}{a^8b^2+2\,a^7b^3+a^6b^4}\right)e^{(2\,x)}+\frac{a^7b^2+2\,a^6b^3+a^5b^4}{a^8b^2+2\,a^7b^3+a^6b^4}\right)}{3\left(ae^{(4\,x)}+2\,ae^{(2\,x)}+4\,be^{(2\,x)}+a\right)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*((((a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*e^(2*x)/(a^8*b^2 + 2*a^7*b^3 + a^6
*b^4) + 3*(a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)/(a^8*b^2 + 2*a^7*b^3
 + a^6*b^4))*e^(2*x) + 3*(a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)/(a^8*
b^2 + 2*a^7*b^3 + a^6*b^4))*e^(2*x) + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)/(a^8
*b^2 + 2*a^7*b^3 + a^6*b^4))/(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)^(
3/2)
```

3.216.9 Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{1}{3\,a}+\frac{a+\frac{b}{\cosh(x)^2}}{a^2}}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}$$

input
```
int(tanh(x)/(a + b/cosh(x)^2)^(5/2),x)
```

output
```
atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(5/2) - (1/(3*a) + (a + b/cosh(x)
^2)/a^2)/(a + b/cosh(x)^2)^(3/2)
```

3.217 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

3.217.1 Optimal result

Integrand size = 12, antiderivative size = 95

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}}-\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*b*(5*a+3*b)*t
anh(x)/a^2/(a+b)^2/(a+b-b*tanh(x)^2)^(1/2)-1/3*b*tanh(x)/a/(a+b)/(a+b-b*ta
nh(x)^2)^(3/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx=\frac{\mathrm{sech}^5(x)\left(\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)(a+2b+a\cosh(2x))^{5/2}}{a^{5/2}}-\frac{4b(a+2b+a\cosh(2x))\left(3a^2+7ab+\cdots\right.}{3a^2(\cdots}\right.}{8\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}$$

input
```
Integrate[(a + b*Sech[x]^2)^(-5/2),x]
```

output
```
(Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Co
sh[2*x]]]*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - (4*b*(a + 2*b + a*Cosh[
2*x])*(3*a^2 + 7*a*b + 3*b^2 + a*(3*a + 2*b)*Cosh[2*x])*Sinh[x])/(3*a^2*(a
 + b)^2)))/(8*(a + b*Sech[x]^2)^(5/2))
```

3.217.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4616, 316, 25, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sec(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4616}$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\left(a-b\tanh^2(x)+b\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{316}$$

$$-\frac{\int -\frac{2b\tanh^2(x)+3a+b}{\left(1-\tanh^2(x)\right)\left(-b\tanh^2(x)+a+b\right)^{3/2}}d\tanh(x)}{3a(a+b)}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2b\tanh^2(x)+3a+b}{\left(1-\tanh^2(x)\right)\left(-b\tanh^2(x)+a+b\right)^{3/2}}d\tanh(x)}{3a(a+b)}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{-\frac{\int -\frac{3(a+b)^2}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a(a+b)}-\frac{b(5a+3b)\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a}-\frac{b(5a+3b)\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3(a+b)\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}}{a}-\frac{b(5a+3b)\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3(a+b)\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{b(5a+3b)\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

input
```
Int[(a + b*Sech[x]^2)^(-5/2),x]
```

output
```
-1/3*(b*Tanh[x])/(a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) + ((3*(a + b)*Arc
Tanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/a^(3/2) - (b*(5*a + 3*b
)*Tanh[x])/(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2]))/(3*a*(a + b))
```

3.217.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.217.4 Maple [F]

$$\int \frac{1}{\left(a+\operatorname{sech}\left(x\right)^{2} b\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+sech(x)^2*b)^(5/2),x)
```

output
```
int(1/(a+sech(x)^2*b)^(5/2),x)
```

3.217.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2870 vs. $2(81) = 162$.

Time = 0.52 (sec) , antiderivative size = 6299, normalized size of antiderivative = 66.31

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.217.6 Sympy [F]

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\,\text{sech}^2\,(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sech(x)**2)**(5/2),x)
```

output
```
Integral((a + b*sech(x)**2)**(-5/2), x)
```

3.217.7 Maxima [F]

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\,\text{sech}\,(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(x)^2 + a)^(-5/2), x)
```

3.217.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(81) = 162$.

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.44

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = -\frac{2\left(\left(\left(\frac{(3a^6b^3+2a^5b^4)e^{(2x)}}{a^8b^2+2a^7b^3+a^6b^4}+\frac{3(a^6b^3+4a^5b^4+2a^4b^5)}{a^8b^2+2a^7b^3+a^6b^4}\right)e^{(2x)}-\frac{3(a^6b^3+4a^5b^4+2a^4b^5)}{a^8b^2+2a^7b^3+a^6b^4}\right)e^{(2x)}-\frac{3a^6b^3+2a^5b^4}{a^8b^2+2a^7b^3+a^6b^4}\right)}{3\left(ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-2/3*((((3*a^6*b^3 + 2*a^5*b^4)*e^(2*x)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4) +
3*(a^6*b^3 + 4*a^5*b^4 + 2*a^4*b^5)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4))*e^(2*
x) - 3*(a^6*b^3 + 4*a^5*b^4 + 2*a^4*b^5)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4))*
e^(2*x) - (3*a^6*b^3 + 2*a^5*b^4)/(a^8*b^2 + 2*a^7*b^3 + a^6*b^4))/(a*e^(4
*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)^(3/2)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/cosh(x)^2)^(5/2),x)
```

output
```
int(1/(a + b/cosh(x)^2)^(5/2), x)
```

3.218 $\int \frac{\coth(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

3.218.1 Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{\coth(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{b}{3a(a+b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

output
```
arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)-arctanh((a+b*sech(x)^2)^(1/
2)/(a+b)^(1/2))/(a+b)^(5/2)-1/3*b/a/(a+b)/(a+b*sech(x)^2)^(3/2)-b*(2*a+b)/
a^2/(a+b)^2/(a+b*sech(x)^2)^(1/2)
```

3.218.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 242 vs. 2(109) = 218.

Time = 0.96 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.22

$$\int \frac{\coth(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \frac{\left(-\frac{2b\cosh(x)(a+2b+a\cosh(2x))\left(7a^2+16ab+6b^2+a(7a+4b)\cosh(2x)\right)}{3a^2(a+b)^2} - \frac{(a+2b+a\cosh(2x))^{5/2}\left(\sqrt{a}(a^2-\right.}{}\right.}{}$$

input
```
Integrate[Coth[x]/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
(((-2*b*Cosh[x]*(a + 2*b + a*Cosh[2*x])*(7*a^2 + 16*a*b + 6*b^2 + a*(7*a +
 4*b)*Cosh[2*x]))/(3*a^2*(a + b)^2) - ((a + 2*b + a*Cosh[2*x])^(5/2)*(Sqrt
[a]*(a^2 - 2*a*b - b^2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b
 + a*Cosh[2*x]]] + (a + b)^2*(Sqrt[a]*ArcTanh[(Sqrt[2*a + 2*b]*Cosh[x])/Sq
rt[a + 2*b + a*Cosh[2*x]]] - 2*Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + S
qrt[a + 2*b + a*Cosh[2*x]]])))/(Sqrt[2]*a^(5/2)*(a + b)^(5/2)))*Sech[x]^5)
/(8*(a + b*Sech[x]^2)^(5/2))
```

3.218.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 26, 4627, 25, 354, 96, 25, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ix)\left(a+b\sec(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(b\sec(ix)^2+a\right)^{5/2}\tan(ix)}dx$$

$$\Big\downarrow \text{4627}$$

$$\int -\frac{\cosh(x)}{\left(1-\text{sech}^2(x)\right)\left(a+b\text{sech}^2(x)\right)^{5/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cosh(x)}{\left(1-\text{sech}^2(x)\right)\left(b\text{sech}^2(x)+a\right)^{5/2}}d\text{sech}(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\cosh(x)}{\left(1-\text{sech}^2(x)\right)\left(b\,\text{sech}^2(x)+a\right)^{5/2}}d\text{sech}^2(x)$$

$$\Big\downarrow \; 96$$

$$\frac{1}{2}\left(\frac{\int -\frac{\cosh(x)\left(-b\,\text{sech}^2(x)+a+b\right)}{\left(1-\text{sech}^2(x)\right)\left(b\,\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}^2(x)}{a(a+b)}-\frac{2b}{3a(a+b)\left(a+b\,\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{\cosh(x)\left(-b\,\text{sech}^2(x)+a+b\right)}{\left(1-\text{sech}^2(x)\right)\left(b\,\text{sech}^2(x)+a\right)^{3/2}}d\text{sech}^2(x)}{a(a+b)}-\frac{2b}{3a(a+b)\left(a+b\,\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 169$$

$$\frac{1}{2}\left(-\frac{\frac{2\int \frac{\cosh(x)\left((a+b)^2-b(2a+b)\text{sech}^2(x)\right)}{2\left(1-\text{sech}^2(x)\right)\sqrt{b\,\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\,\text{sech}^2(x)}}}{a(a+b)}-\frac{2b}{3a(a+b)\left(a+b\,\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{\cosh(x)\left((a+b)^2-b(2a+b)\text{sech}^2(x)\right)}{\left(1-\text{sech}^2(x)\right)\sqrt{b\,\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\,\text{sech}^2(x)}}}{a(a+b)}-\frac{2b}{3a(a+b)\left(a+b\,\text{sech}^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(-\frac{\frac{a^2\int \frac{1}{\left(1-\text{sech}^2(x)\right)\sqrt{b\,\text{sech}^2(x)+a}}d\text{sech}^2(x)+(a+b)^2\int \frac{\cosh(x)}{\sqrt{b\,\text{sech}^2(x)+a}}d\text{sech}^2(x)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\,\text{sech}^2(x)}}}{a(a+b)}-\frac{2b}{3a(a+b)\left(a+b\text{se}\right.}\right.$$

3.218. $\int \frac{\coth(x)}{\left(a+b\,\text{sech}^2(x)\right)^{5/2}}\,dx$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2a^2\int \frac{1}{\frac{a+b}{b}-\frac{\text{sech}^4(x)}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}+\frac{2(a+b)^2\int\frac{1}{\frac{\text{sech}^4(x)}{b}-\frac{a}{b}}d\sqrt{b\text{sech}^2(x)+a}}{b}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}}{a(a+b)}-\frac{2b}{3a(a+b)\left(a+b\text{sech}\right.}\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)^2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}}{a(a+b)}-\frac{2b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/}}\right.$$

input
```
Int[Coth[x]/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
((-2*b)/(3*a*(a + b)*(a + b*Sech[x]^2)^(3/2)) - (((-2*(a + b)^2*ArcTanh[Sq
rt[a + b*Sech[x]^2]/Sqrt[a]])/Sqrt[a] + (2*a^2*ArcTanh[Sqrt[a + b*Sech[x]^
2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b)) + (2*b*(2*a + b))/(a*(a + b)*Sqr
t[a + b*Sech[x]^2]))/(a*(a + b)))/2
```

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.218. $\int \frac{\coth(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.218. $\int \frac{\coth(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx$

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.218.4 Maple [F]

$$\int \frac{\coth(x)}{\left(a+\operatorname{sech}(x)^{2} b\right)^{\frac{5}{2}}}dx$$

input
```
int(coth(x)/(a+sech(x)^2*b)^(5/2),x)
```

output
```
int(coth(x)/(a+sech(x)^2*b)^(5/2),x)
```

3.218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4236 vs. $2(91) = 182$.

Time = 0.92 (sec) , antiderivative size = 18563, normalized size of antiderivative = 170.30

$$\int \frac{\coth(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(coth(x)/(a+b*sech(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.218.7 Maxima [F]

$$\int \frac{\coth(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{\coth\left(x\right)}{\left(b\,\text{sech}\left(x\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/(b*sech(x)^2 + a)^(5/2), x)
```

3.218.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\coth(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{(a+b\,\mathrm{sech}^2(x))^{5/2}}\,dx = \int \frac{\coth(x)}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

input
```
int(coth(x)/(a + b/cosh(x)^2)^(5/2),x)
```

output
```
int(coth(x)/(a + b/cosh(x)^2)^(5/2), x)
```

3.219 $\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

3.219.1 Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3b)(3a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3a^2(a+b)^3}$$

output

```
arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*b*(7*a+3*b)*c
oth(x)/a^2/(a+b)^2/(a+b-b*tanh(x)^2)^(1/2)-1/3*(a-3*b)*(3*a+b)*coth(x)*(a+
b-b*tanh(x)^2)^(1/2)/a^2/(a+b)^3-1/3*b*coth(x)/a/(a+b)/(a+b-b*tanh(x)^2)^(
3/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \frac{\text{sech}^5(x)\left(\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)(a+2b+a\cosh(2x))^{5/2}}{a^{5/2}} - \frac{(a+2b+a\cosh(2x))\left(3a^2(a+2b+\right.}{\cdots}\right.}{8\left(a+b\text{sech}^2(x)\right)^{5/}}$$

input
```
Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
(Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Co
sh[2*x]]]*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - ((a + 2*b + a*Cosh[2*x]
)*(3*a^2*(a + 2*b + a*Cosh[2*x])^2*Csch[x] - 4*b^3*(a + b)*Sinh[x] + 2*b^2
*(9*a + 4*b)*(a + 2*b + a*Cosh[2*x])*Sinh[x]))/(3*a^2*(a + b)^3)))/(8*(a +
 b*Sech[x]^2)^(5/2))
```

3.219.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {3042, 25, 4629, 25, 2075, 374, 25, 441, 25, 445, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ix)^2\left(a+b\sec(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(b\sec(ix)^2+a\right)^{5/2}\tan(ix)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$-\int -\frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\left(1-\tanh^2(x)\right)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{2075}$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a-b\tanh^2(x)+b\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{374}$$

$$-\frac{\int -\frac{\coth^2(x)(4b\tanh^2(x)+3a-b)}{(1-\tanh^2(x))(-b\tanh^2(x)+a+b)^{3/2}}d\tanh(x)}{3a(a+b)}-\frac{b\coth(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^2(x)(4b\tanh^2(x)+3a-b)}{(1-\tanh^2(x))(-b\tanh^2(x)+a+b)^{3/2}}d\tanh(x)}{3a(a+b)}-\frac{b\coth(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{441}$$

$$\frac{-\frac{\int -\frac{\coth^2(x)\left(2b(7a+3b)\tanh^2(x)+(a-3b)(3a+b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a(a+b)}-\frac{b(7a+3b)\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\coth(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\coth^2(x)\left(2b(7a+3b)\tanh^2(x)+(a-3b)(3a+b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a(a+b)}-\frac{b(7a+3b)\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\coth(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int -\frac{3(a+b)^3}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)}{a+b}-\frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}}{a(a+b)}-\frac{b(7a+3b)\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\coth(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{-b\tanh^2(x)+a+b}}d\tanh(x)-\frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}}{a(a+b)}-\frac{b(7a+3b)\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\coth(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{291}$$

3.219. $\displaystyle\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

$$\frac{\frac{3(a+b)^2\int \frac{1}{1-\frac{a\tanh^2(x)}{-b\tanh^2(x)+a+b}}d\frac{\tanh(x)}{\sqrt{-b\tanh^2(x)+a+b}}-\frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}}{a(a+b)}-\frac{b(7a+3b)\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\coth(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{3(a+b)^2\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}}{a(a+b)}-\frac{b(7a+3b)\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}}{3a(a+b)}-\frac{b\coth(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

input
```
Int[Coth[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

output
```
-1/3*(b*Coth[x])/(a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) + (-((b*(7*a + 3*
b)*Coth[x])/(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2])) + ((3*(a + b)^2*ArcTanh
[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[a] - ((a - 3*b)*(3*a +
 b)*Coth[x]*Sqrt[a + b - b*Tanh[x]^2])/(a + b))/(a*(a + b)))/(3*a*(a + b))
```

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.219. $\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.219. $\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

3.219.4 Maple [F]

$$\int \frac{\coth(x)^2}{\left(a+\operatorname{sech}(x)^2 b\right)^{\frac{5}{2}}}dx$$

input
```
int(coth(x)^2/(a+sech(x)^2*b)^(5/2),x)
```

output
```
int(coth(x)^2/(a+sech(x)^2*b)^(5/2),x)
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5323 vs. 2(115) = 230.

Time = 0.98 (sec) , antiderivative size = 11205, normalized size of antiderivative = 84.25

$$\int \frac{\coth^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(coth(x)**2/(a+b*sech(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.219.7 Maxima [F]

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{\coth\left(x\right)^{2}}{\left(b\,\text{sech}\left(x\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

3.219.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(115) = 230$.

Time = 0.58 (sec) , antiderivative size = 854, normalized size of antiderivative = 6.42

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx=$$

$$-\frac{\left(\left(\frac{\left(9\,a^{13}b^{4}+67\,a^{12}b^{5}+217\,a^{11}b^{6}+399\,a^{10}b^{7}+455\,a^{9}b^{8}+329\,a^{8}b^{9}+147\,a^{7}b^{10}+37\,a^{6}b^{11}+4\,a^{5}b^{12}\right)e^{(2\,x)}}{a^{16}b^{2}+10\,a^{15}b^{3}+45\,a^{14}b^{4}+120\,a^{13}b^{5}+210\,a^{12}b^{6}+252\,a^{11}b^{7}+210\,a^{10}b^{8}+120\,a^{9}b^{9}+45\,a^{8}b^{10}+10\,a^{7}b^{11}+a^{6}b^{12}}+\frac{3\left(3\,a^{13}b^{4}+33\,a^{12}b^{5}+151\cdots}{a^{16}b^{2}+10\,a^{15}b^{3}+45\,a^{14}b^{4}+\cdots}\cdots}{\cdots}$$

$$+\frac{4\left(\sqrt{a}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+2\,ae^{(2\,x)}+4\,be^{(2\,x)}+a}+\sqrt{a}\right)}{\left(\left(\sqrt{a}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+2\,ae^{(2\,x)}+4\,be^{(2\,x)}+a}\right)^{2}-2\left(\sqrt{a}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+2\,ae^{(2\,x)}+4\,be^{(2\,x)}+a}\right)\sqrt{a}-\cdots}$$

input
```
integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((((9*a^13*b^4 + 67*a^12*b^5 + 217*a^11*b^6 + 399*a^10*b^7 + 455*a^9*
b^8 + 329*a^8*b^9 + 147*a^7*b^10 + 37*a^6*b^11 + 4*a^5*b^12)*e^(2*x)/(a^16
*b^2 + 10*a^15*b^3 + 45*a^14*b^4 + 120*a^13*b^5 + 210*a^12*b^6 + 252*a^11*
b^7 + 210*a^10*b^8 + 120*a^9*b^9 + 45*a^8*b^10 + 10*a^7*b^11 + a^6*b^12) +
 3*(3*a^13*b^4 + 33*a^12*b^5 + 151*a^11*b^6 + 385*a^10*b^7 + 609*a^9*b^8 +
 623*a^8*b^9 + 413*a^7*b^10 + 171*a^6*b^11 + 40*a^5*b^12 + 4*a^4*b^13)/(a^
16*b^2 + 10*a^15*b^3 + 45*a^14*b^4 + 120*a^13*b^5 + 210*a^12*b^6 + 252*a^1
1*b^7 + 210*a^10*b^8 + 120*a^9*b^9 + 45*a^8*b^10 + 10*a^7*b^11 + a^6*b^12)
)*e^(2*x) - 3*(3*a^13*b^4 + 33*a^12*b^5 + 151*a^11*b^6 + 385*a^10*b^7 + 60
9*a^9*b^8 + 623*a^8*b^9 + 413*a^7*b^10 + 171*a^6*b^11 + 40*a^5*b^12 + 4*a^
4*b^13)/(a^16*b^2 + 10*a^15*b^3 + 45*a^14*b^4 + 120*a^13*b^5 + 210*a^12*b^
6 + 252*a^11*b^7 + 210*a^10*b^8 + 120*a^9*b^9 + 45*a^8*b^10 + 10*a^7*b^11
+ a^6*b^12))*e^(2*x) - (9*a^13*b^4 + 67*a^12*b^5 + 217*a^11*b^6 + 399*a^10
*b^7 + 455*a^9*b^8 + 329*a^8*b^9 + 147*a^7*b^10 + 37*a^6*b^11 + 4*a^5*b^12
)/(a^16*b^2 + 10*a^15*b^3 + 45*a^14*b^4 + 120*a^13*b^5 + 210*a^12*b^6 + 25
2*a^11*b^7 + 210*a^10*b^8 + 120*a^9*b^9 + 45*a^8*b^10 + 10*a^7*b^11 + a^6*
b^12))/(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a)^(3/2) + 4*(sqrt(a)*e^(2
*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a) + sqrt(a))/(((sqrt(a
)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))^2 - 2*(sqrt(a
)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*sqrt(a) - ...
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx=\int \frac{\coth(x)^2}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

input
```
int(coth(x)^2/(a + b/cosh(x)^2)^(5/2),x)
```

output
```
int(coth(x)^2/(a + b/cosh(x)^2)^(5/2), x)
```

3.220 $\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}} dx$

3.220.1 Optimal result

Integrand size = 16, antiderivative size = 183

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}} dx = \frac{\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+dx)}{15a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)^{3/2}} - \frac{b(33a^2+40ab+15b^2)\tanh(c+dx)}{15a^3(a+b)^3d\sqrt{a+b-b\tanh^2(c+dx)}}$$

output
```
arctanh(a^(1/2)*tanh(d*x+c)/(a+b-b*tanh(d*x+c)^2)^(1/2))/a^(7/2)/d-1/15*b*
(33*a^2+40*a*b+15*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)^(1/
2)-1/5*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^(5/2)-1/15*b*(9*a+5*b
)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)^(3/2)
```

3.220.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}}\,dx = \frac{\text{sech}^7(c+dx)\left(\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b+a\sinh^2(c+dx)}}\right)(a+2b+a\cosh(2(c+dx)))^{7/2}}{a^{7/2}d}-\frac{b(a+2b+ \cdots}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(a + b*Sech[c + d*x]^2)^(-7/2),x]
```

output
```
(Sech[c + d*x]^7*((Sqrt[2]*ArcTanh[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b + a*
Sinh[c + d*x]^2]]*(a + 2*b + a*Cosh[2*(c + d*x)])^(7/2))/(a^(7/2)*d) - (b*
(a + 2*b + a*Cosh[2*(c + d*x)])*(135*a^4 + 480*a^3*b + 709*a^2*b^2 + 460*a
*b^3 + 120*b^4 + 4*a*(45*a^3 + 135*a^2*b + 117*a*b^2 + 35*b^3)*Cosh[2*(c +
 d*x)] + a^2*(45*a^2 + 60*a*b + 23*b^2)*Cosh[4*(c + d*x)])*Sinh[c + d*x])/
(15*a^3*(a + b)^3*d)))/(16*(a + b*Sech[c + d*x]^2)^(7/2))
```

3.220.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 4616, 316, 25, 402, 25, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+b\sec(ic+idx)^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 4616}$$

$$\frac{\int \frac{1}{(1-\tanh^2(c+dx))\left(-b\tanh^2(c+dx)+a+b\right)^{7/2}}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 316}$$

$$\frac{-\frac{\int -\frac{4b\tanh^2(c+dx)+5a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^{5/2}}d\tanh(c+dx)}{5a(a+b)}-\frac{b\tanh(c+dx)}{5a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{5/2}}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{4b\tanh^2(c+dx)+5a+b}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^{5/2}}d\tanh(c+dx)}{5a(a+b)}-\frac{b\tanh(c+dx)}{5a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{5/2}}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{-\frac{\int -\frac{15a^2+12ba+5b^2+2b(9a+5b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^{3/2}}d\tanh(c+dx)}{3a(a+b)}-\frac{b(9a+5b)\tanh(c+dx)}{3a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tanh(c+dx)}{5a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{5/2}}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{15a^2+12ba+5b^2+2b(9a+5b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(-b\tanh^2(c+dx)+a+b\right)^{3/2}}d\tanh(c+dx)}{3a(a+b)}-\frac{b(9a+5b)\tanh(c+dx)}{3a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tanh(c+dx)}{5a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{5/2}}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{-\frac{\int -\frac{15(a+b)^3}{\left(1-\tanh^2(c+dx)\right)\sqrt{-b\tanh^2(c+dx)+a+b}}d\tanh(c+dx)}{a(a+b)}-\frac{b\left(33a^2+40ab+15b^2\right)\tanh(c+dx)}{a(a+b)\sqrt{a-b\tanh^2(c+dx)+b}}}{3a(a+b)}-\frac{b(9a+5b)\tanh(c+dx)}{3a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tanh(c+dx)}{5a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{5/2}}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{15(a+b)^2\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\sqrt{-b\tanh^2(c+dx)+a+b}}d\tanh(c+dx)}{a}-\frac{b\left(33a^2+40ab+15b^2\right)\tanh(c+dx)}{a(a+b)\sqrt{a-b\tanh^2(c+dx)+b}}}{3a(a+b)}-\frac{b(9a+5b)\tanh(c+dx)}{3a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tanh(c+dx)}{5a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{5/2}}}{d}$$

$\Big\downarrow$ 291

3.220. $\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^{7/2}}\,dx$

$$\frac{\frac{\frac{\frac{15(a+b)^2\int \frac{1}{1-\frac{a\tanh^2(c+dx)}{-b\tanh^2(c+dx)+a+b}}d\frac{\tanh(c+dx)}{\sqrt{-b\tanh^2(c+dx)+a+b}}}{a}-\frac{b(33a^2+40ab+15b^2)\tanh(c+dx)}{a(a+b)\sqrt{a-b\tanh^2(c+dx)+b}}}{3a(a+b)}-\frac{b(9a+5b)\tanh(c+dx)}{3a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tanh(c}{5a(a+b)(a-b\tanh}}{d}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\frac{15(a+b)^2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{a^{3/2}}-\frac{b(33a^2+40ab+15b^2)\tanh(c+dx)}{a(a+b)\sqrt{a-b\tanh^2(c+dx)+b}}}{3a(a+b)}-\frac{b(9a+5b)\tanh(c+dx)}{3a(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tanh(c+dx)}{5a(a+b)(a-b\tanh^2(c+dx)+b)^{5/}}}{d}$$

input
```
Int[(a + b*Sech[c + d*x]^2)^(-7/2),x]
```

output
```
(-1/5*(b*Tanh[c + d*x])/(a*(a + b)*(a + b - b*Tanh[c + d*x]^2)^(5/2)) + (-
1/3*(b*(9*a + 5*b)*Tanh[c + d*x])/(a*(a + b)*(a + b - b*Tanh[c + d*x]^2)^(
3/2)) + ((15*(a + b)^2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + b - b*Tanh
[c + d*x]^2]])/a^(3/2) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tanh[c + d*x])/(a*(
a + b)*Sqrt[a + b - b*Tanh[c + d*x]^2]))/(3*a*(a + b)))/(5*a*(a + b)))/d
```

3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.220. $\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}}\,dx$

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.220.4 Maple [F]

$$\int \frac{1}{\left(a+b\,\mathrm{sech}\left(dx+c\right)^{2}\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*sech(d*x+c)^2)^(7/2),x)
```

output
```
int(1/(a+b*sech(d*x+c)^2)^(7/2),x)
```

3.220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8940 vs. 2(165) = 330.

Time = 1.83 (sec) , antiderivative size = 18565, normalized size of antiderivative = 101.45

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.220.6 Sympy [F]

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}}\,dx = \int \frac{1}{\left(a+b\,\text{sech}^2\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)**2)**(7/2),x)
```

output
```
Integral((a + b*sech(c + d*x)**2)**(-7/2), x)
```

3.220.7 Maxima [F]

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}}\,dx = \int \frac{1}{\left(b\,\text{sech}\left(dx+c\right)^2+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c)^2 + a)^(-7/2), x)
```

3.220.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^{7/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(a + b/cosh(c + d*x)^2)^(7/2),x)
```

output
```
int(1/(a + b/cosh(c + d*x)^2)^(7/2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif is_sqrt(expn):
      if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```